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EFFECTS OF CHEMICAL REACTION ON MHD BOUNDARY LAYER FLOW OVER AN EXPONENTIALLY STRETCHING SHEET WITH JOULE HEATING AND THERMAL RADIATION

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Abstract

A numerical analysis has been carried out to study a steady two-dimensional flow of a viscous incompressible, electrically conducting dissipating fluid past an exponentially stretching surface in presence of magnetic field and thermal radiation. The present work examines the effects of chemical reaction and magnetic field on velocity, temperature and concentration fields. Effects of viscous dissipation and Joule heating are taken into consideration in the energy equation. Non-linear partial differential equations governing the motion are reduced to a system of ordinary differential equations using suitable similarity transformations. Resultant equations are then solved numerically using MATLAB's built in solver bvp4c. The effects of various flow governing parameters on velocity, temperature and species concentration profile are depicted graphically. Finally, numerical values of coefficient of skin-friction, Nusselt number and Sherwood number are presented in tables for various physical parameters.

Key Words: *Chemical reaction, magnetic field, Joule heating, heat transfer, mass transfer.*

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1. INTRODUCTION

Heat and mass transfer study on fluids with chemical reaction effect over an exponentially stretching sheet have important role in metallurgy and chemical engineering industries, such as food processing and polymer production. Moreover, coupled heat and mass transfer problems in the presence of chemical reaction are of importance in many processes, and have therefore received a considerable amount of attention in recent times. Possible applications can be found in processes such as drying, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, evaporation at the surface of a water body and energy transfer in a wet cooling tower, and flow in a desert cooler.

Chambre and Acrivos[1] analyzed catalytic surface reactions in hydrodynamic flows. The paper was concerned with its counterpart, namely, an investigation of a certain special class of homogeneous volume reactions in flow systems. Chambre et al.[2] had studied the diffusion of a chemically reactive species in a laminar boundary layer flow. Goddard and Acrivos [3] analyzed the laminar forced convection mass transfer with homogeneous chemical reaction. A unified boundary layer analysis was applied to the problem of steady state mass transfer of a chemical species, diffusing from a surface and reacting isothermally in a linear fluid stream.

Yih [4] presented an analysis of the forced convection boundary layer flow over a wedge with uniform suction/blowing, whereas Watanabe [5] investigated the behavior of the boundary layer over a wedge with suction or injection in forced flow. MHD laminar boundary layer flow over a wedge with suction or injection had been discussed by Kafoussias and Nanousis [6] and Kumari[7] discussed the effect of large blowing rates on the steady laminar incompressible electrically conducting fluid over an infinite wedge with a magnetic field applied parallel to the wedge. Anjali Devi and Kandasamy [8] have studied the effects of heat and mass transfer on nonlinear boundary layer flow over a wedge with suction or injection. The effect of induced magnetic field is included in the analysis. Chamkha and Khaled [9] investigated the problem of coupled heat and mass transfer by MHD free convection from an inclined plate in the presence of internal heat generation or absorption. For the problem of coupled heat and mass transfer in MHD free convection, the effects of viscous dissipation and Ohmic heating with chemical reaction are not studied in the above investigation. Hossain et al.[10] analyzed the effects of radiation on free convection from a porous plate.

During chemical reaction between two species concentration, heat is also generated [11]. Here it has been assumed that the level of species concentration is very low and that the heat generated during chemical reaction can be neglected. The effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction were studied [12]. Hence these results are found useful in chemical technology where ammonia and ethyl alcohol are used. The present investigation deals with the study of flow of a electrically conducting fluid in the presence of chemical reaction with a species concentration due to the MHD flow over an exponentially stretching sheet.

2. MATHEMATICAL FORMULATION

Consider a steady two-dimensional flow of an incompressible viscous fluid bounded by a stretching sheet in which *x*-axis is taken along the stretching sheet in the direction of the motion and *y*-axis is perpendicular to it. A uniform magnetic field of strength B_0 is assumed to be applied in the y-direction. Under the usual boundary layer approximations, the flow, heat and mass transfer in the presence of a chemical reaction effects with Joule heating and thermal radiation are governed by the following equations (Sajid and Hayat [13]):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad ...(1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_o^2}{\rho}u \qquad ...(2)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\lambda\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\vec{J}^2}{\sigma}-\frac{\partial q_r}{\partial y} \qquad ...(3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-k_1 C \qquad ...(4)$$

where u and v are the velocities in the x- and y-directions respectively, ρ is the fluid density, υ is the dynamic viscosity, T is the temperature, λ is the thermal conductivity, c_p is the specific heat and q_r is the radiative heat flux, C is the species concentration of the fluid within the boundary layer, D_m is the molecular diffusivity of the species concentration.

The boundary conditions are given by:

$$\left.\begin{array}{l} u(0)=U_0 e^{x/L}, v(0)=0, T(0)=T_\infty+T_0 e^{x/2L}, \\ C(0)=C_\infty+C_0 e^{x/2L} \\ u\to 0,\ T\to 0,\ C\to 0 \text{ as } y\to\infty. \end{array}\right\} ...(5)$$

where U_0 is the reference velocity, T_0 and C_0 are the temperature and species concentration at the plate. and T_∞ and C_∞ are the temperature and species concentration far away from the plate, L is a constant. Employing Rosseland approximation of radiation for an optically thick layer one has (Sajid and Hayat [13])

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \qquad ...(6)$$

where k^* is the mean absorption coefficient and σ^* is the Stefan-Boltzmann constant, T^4 is expressed as a linear function of temperature, hence

$$T^4=4T_\infty^3 T-3T_\infty^4 \qquad ...(7)$$

Let us introduce the following transformations:

$$\left.\begin{array}{l} u=U_0 e^{x/L} f'(\eta),\quad v=-\sqrt{\frac{\upsilon U_0}{2L}}e^{x/2L}\{f(\eta)+\eta f'(\eta)\}, \\ \eta=\sqrt{\frac{U_0}{2\upsilon L}}e^{x/2L}y,\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \text{ where} \\ T-T_\infty=T_0 e^{x/2L}\theta(\eta), \\ \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \text{ where } C-C_\infty=C_0 e^{x/2L}\varphi(\eta). \end{array}\right\}$$

...(8)

where the prime $(')$ denotes derivative with respect to η.

Using equations (6)-(8), (2)-(4) reduces to

$$f'''+ff''-2f'^2+Mf'=0 \qquad ...(9)$$

$$\left(1+\frac{4}{3}Nr\right)\theta''+\Pr\left(f\theta'-f'\theta+Ecf''^2+MEcf'^2\right)=0 \qquad ...(10)$$

$$\varphi''+Sc(f\varphi'-f'\varphi-\gamma_1\varphi)=0 \qquad ...(11)$$

with the boundary conditions (5) become

$$\left.\begin{array}{l} f(0)=0,\ f'(0)=1,\ \theta(0)=1,\ \varphi(0)=1, \\ f'\to 0, \theta\to 0, \varphi\to 0 \quad \text{as} \quad \eta\to\infty \end{array}\right\} . \quad \ldots(12)$$

where dimensionless parameters are:

$\Pr = \frac{\mu c_p}{\lambda}$ is the Prandtl number, $Ec = \frac{U_0^2}{T_0 c_p}$ is the Eckert number, $Nr = \frac{4\sigma^* T_\infty^3}{k^* \lambda}$ is the radiation parameter, $\gamma = \frac{\upsilon k_1}{U_0^2}$ is the chemical reaction parameter, $M = \frac{\sigma B_0^2}{\rho c}$ is the magnetic field parameter, $Ec = \frac{u_0^2}{c_p(T_w - T_\infty)}$ is the Eckert number, $Sc = \frac{\upsilon}{D_m}$ is the Schmidt number.

For the present problem the coefficient of skin-friction and Nusselt number are the parameters of engineering interest which are given respectively as below:

Coefficient of skin-friction:

Coefficient of skin-friction is given by $C_f = \frac{2\tau_w}{\rho U_0^2}$, where $\tau_w = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0}$ is the shearing stress.
Using the non-dimensional variables, we get

$C_f (\text{Re})^{1/2} = f''(0)$, where $\text{Re} = \frac{U_0 x}{\upsilon}$ is the Renolds number.

Nusselt number:

The Nusselt number which is defined as

$Nu = \frac{x q_w}{T_w - T_\infty}$, where q_w is the heat transfer from the sheet is given by

$$q_w = -\lambda \left.\frac{\partial T}{\partial y}\right)_{y=0} .$$

Using the non-dimensional variables, we get

$$Nu(\text{Re})^{1/2} = -\theta'(0).$$

Sherwood number:
The Sherwood number which is defined as

$Sh = \frac{x m_w}{Dm(C_w - C_\infty)}$, where $m_w = -Dm \left.\frac{\partial C}{\partial y}\right)_{y=0}$ is the mass flux at the surface .
Using the non-dimensional variables, we get

$$Sh(\text{Re})^{1/2} = -\varphi'(0).$$

3. RESULTS AND DISCUSSION

The set of equations (9) to (11) under boundary conditions (12) are non-linear coupled ordinary differential equations so their solutions cannot be obtained in closed form therefore these equations are solved numerically with MATLAB's built – in solver bvp4c.

Computations have been carried out for different values of magnetic parameter (M) , Eckert number (Ec) and Prandtl number(Pr), chemical reaction parameter(γ), radiation parameter and Schimdt number(Sc). Results are presented through graphs for velocity $f'(\eta)$, temperature $\theta(\eta)$ and species concentration $\varphi(\eta)$ in Fig.1-Fig.7 for various values of flow governing parameters. Values of the parameters are taken as: *M=0.5, Pr=0.71, Nr=0.5, Ec=0.05,* γ *=0.1* and *Sc=0.22* unless otherwise stated.

Fig.1 and Fig.2 show the effect of magnetic parameter (M) on dimensionless velocity and temperature distributions, respectively. In the presence of a magnetic field in an electrically conducting fluid induces a force called Lorentz force, which opposes the flow. This resistive force tends to slow down the flow, so the effect of increase in M is to decrease the velocity and also causes increases in its temperature.

Fig. 3 depicts the variation of concentration profiles with chemical reaction parameter (γ). It is seen that an increase in the value of chemical reaction parameter decreases the concentration of species in the boundary layer. This is due to the fact that chemical reaction in this system results in consumption of the chemical and hence results in decrease of concentration profile. The variation of concentration profiles for different values of Schimdt number(Sc) is shown in Fig.4. It is observed that concentration of species decreases with increasing value of Sc.

The effects of Prandtl number(Pr) on the temperature field is shown in Fig.5. It is seen that increase in the Pr leads to a fall in the fluid. This is due to the fact that thermal boundary layer thickness decreases compared to velocity boundary layer thickness for increasing Pr. From Fig.6, it is observed that temperature profile increases for increasing value of radiation parameter(Nr). This is because an increase in the radiation parameter Nr leads to increase in the boundary layer thickness and enhances the heat transfer rate on the surface in the presence of chemical effect. Fig.7 shows that temperature of the fluid increases with the increasing value of Eckert number(Ec).

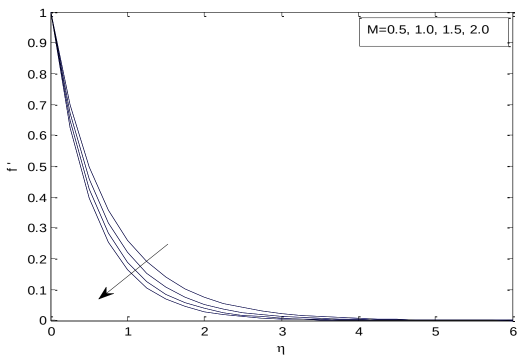

Fig.1: Velocity profile for different M

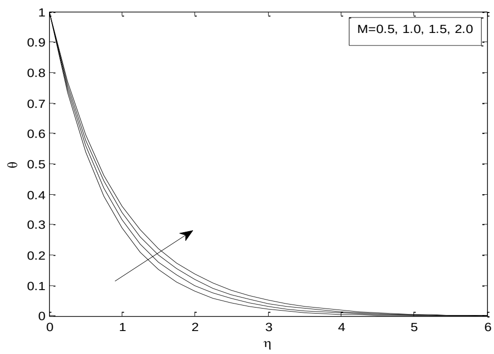

Fig.2: Temperature profile for different M

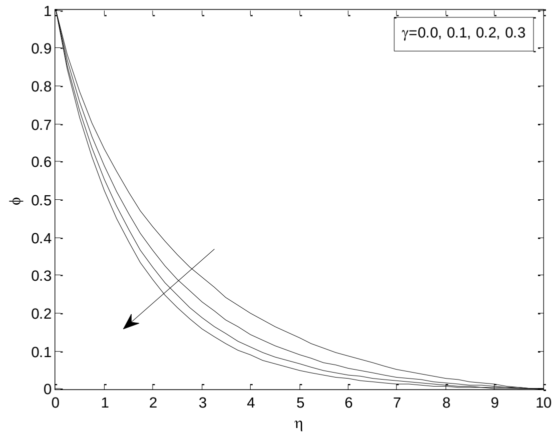

Fig.3: Concentration profile for different γ

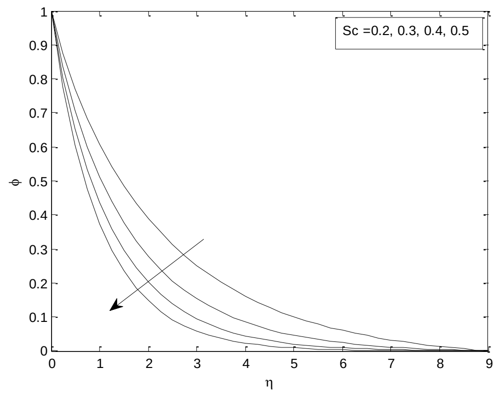

Fig.4: Concentration profile for different Sc

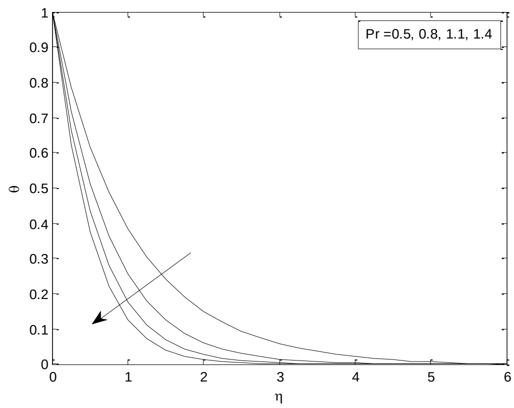

Fig.5: Temperature profile for different Pr

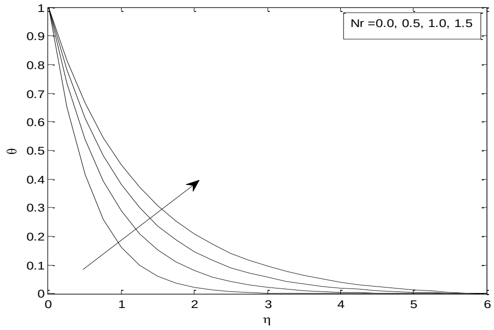

Fig.6: Temperature profile for different *Nr*

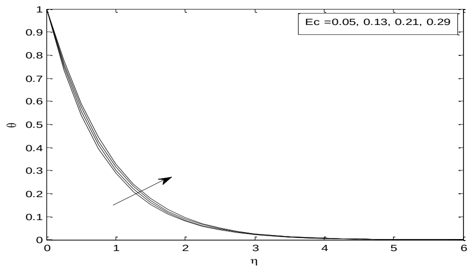

Fig.7: Temperature profile for different *Ec*

TABLE 1

Effects of M, γ and Sc on coefficient of skin-friction (C_f), Nusselt number(Nu) and Sherwood number(Sh)

M	γ	Sc	C_f	Nu	Sh
0			-1.28227	1.298839	0.602313
0.5	0.1	0.22	-1.46667	1.228968	0.57269
1			-1.62946	1.16816	0.550054
	0		-1.46667	1.228968	0.5214
0.5	0.1	0.22	-1.46667	1.228968	0.57269
	0.2		-1.46667	1.228968	0.618014
		0.2	-1.46667	1.228968	0.540241
0.5	0.1	0.3	-1.46667	1.228968	0.69501
		0.4	-1.46667	1.228968	0.833221

Table 1 demonstrates the effects of M, γ and Sc on coefficient of skin-friction C_f, which represent plate shearing stress, rate of heat transfer from the plate to the

fluid in terms of Nusselt number Nu and rate of mass transfer in terms of Sherwood number Sh. It is seen that the value of Nu and Sh decreases with the increase of magnetic parameter(M). But magnitude of C_f increases with M. Physically negative values of C_f mean that surface exerts a drag force on the fluid, so that stretching surface will induce the flow. Also it is observed that Sherwood number increases for increasing values of chemical reaction parameter and Schimdt number.

4. CONCLUSIONS

On the basis of the above study following observations were made:

(1) Velocity of the fluid decreases and temperature increases with the increasing magnetic field intensity.

(2) The increase in the chemical reaction parameter and Schimdt number lead to a fall in the species concentration.

(3) Temperature of the fluid increases for increasing radiation parameter and Eckert number, while that of decreases with the increase of Prandtl number.

(4) Wall shear stress of fluid increases with the increase of magnetic field strength. But rate of heat and mass transfer rise due to the increase of magnetic field strength.

(5) Rate of mass transfer increases with the rise of chemical reaction parameter and Schimdt number.

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