# Some new sets in Ideal topological spaces

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***ABSTRACT*** *In this paper, some related generalized sets of $\tau$ * namely R*-I closed sets, W-R*-I closed sets, I-R* closed sets in Ideal topological space are introduced. The relationships between these sets are investigated and some of the properties are also studied.*

KEYWORDS IR*-closed,R*-I closed,Weakly R*-I closed

## 1. INTRODUCTION

The notion of generalized closed sets in Ideal topological spaces was studied by Dontchev et. al [4] in 1999.Further closed sets like $I_{rg}$ ,$I_{rw}$ were further developed by Navaneethakrishnan [10] and A.Vadivel [12] in 2009 and 2013 respectively. The main aim of this paper is to introduce some new related closed sets in the same space and study the relationships between them.

## *2.* PRELIMINARIES

An ideal on a topological space $(X,\tau)$ is a non empty collection of subsets of X which satisfies the following properties (i) $A\in I$ and $B\subset A$ implies $B\in I$ (ii) $A\in I$ and $B\in I$ implies $A\cup B\in I$. An ideal topological space is a topological space with an ideal I on X and is denoted by $(X,\tau,I)$.For a subset $A\subset X$, $A^*(\tau,I) = \{x \in X: A\cap U\notin I$ for every $U\in\tau(X, x)\}$ is called the local function of A with respect to I and $\tau$ .A Kuratowski's closure operator cl*(.) for a topology $\tau^*(I,\tau)$,called the *-topology, finer than $\tau$ is defined by $cl^*(A) = A\cup A^*(I,\tau)$.Moreover $(G,\tau_G,I_G)$ represents the relative topology on G denoted by $\tau_G$ and $I_G = \{G\cap J, J\in I\}$is an ideal topological space for $(X,\tau,I)$ and $G\subset X$.

## Definition 2.1

A subset A of a space $(X,\tau)$ is called

1. Regular open[10] if int(cl(A)) = A
2. Regular semi open [4] if there is a regular open set U such that $U\subset A \subset cl(U)$. Also X\A is regular semi open.

**Definition 2.2[7]** The intersection of all regular closed subset of $(X,\tau)$ containing A is called the regular closure of A and is denoted by rcl(A).

**Definition 2.3 [7]** A subset A of a space $(X,\tau)$ is called R*- closed if $rcl(A)\subset U$ whenever $A \subset U$ and U is regular semiopen in $(X,\tau)$. We denote the set of all R*-closed sets in $(X,\tau)$ by R*-C(X).

**Definition 2.4**

A subset A of a space $(X,\tau, I)$ is called

1. *-closed [8] if $A^*\subset A$
2. I-R closed [1] if A= cl*(int (A))
3. Regular-I closed [9] if A = (int(A))*
4. $Pre_I^*$-open [6] if $A \subset Int^*(cl(A))$
5. $Pre_I^*$-closed[6] if $cl^*(int(A)) \subset A$

## 3. R*-I-CLOSED SETS

**Definition 3.1**
The intersection of all regular –I closed sets containing A is called the regular-I-closure and is denoted by $r_I^*cl(A)$.

**Definition 3.2**
A subset A of an ideal space $(X,\tau, I)$ is said to be R*-I closed if $r_I^*cl(A)\subset U$ whenever $A\subset U$ and U is regular semi open.

**Definition 3.3**
A subset A of a space $(X,\tau, I)$ is called R*-I open if X\A is R*-I closed.

**Theorem 3.4:** The union of two R*-I closed sets is R*-I closed.

**Proof:** Assume A and B are R*-I closed sets in $(X,\tau, I)$.Let U be a regular semi open in X such that $A\cup B\subset U$. Then $A\subset U$ and $B\subset U$. Since A and B are R*-I closed sets, $r_I^*cl(A) \subset U$ and $r_I^*cl(B) \subset U$ respectively, hence $r_I^*cl(A\cup B) \subset U$. Therefore $A\cup B$ is R*-I closed.

**Remark 3.5:** The finite intersection of two R*-I closed need not be R*-I closed.

**Example 3.6:** Let X = {a, b, c} $\tau=\{X,\varphi,\{a\},\{c\},\{a,c\}\}$ I = $\{\varphi,\{a\}\}$
A= {a,c} and B= {b,c} are R*-I closed sets ,while $A\cap B$= {c} is not an R*-I closed set. **Remark3.7:** Every regular- I closed set is R*-I closed while the converse is not true.

**Example 3.8:** Let X = {a, b, c, d} $\tau=\{X,\varphi,\{a\},\{b\},\{a, b\},\{b, c\},\{a, b, c\}\}$

I = { $\varphi$ ,{a}},regular-I-closed sets = { X, $\varphi$ ,{b, c, d}}and R*-I-closed sets are
{X, $\varphi$ ,{a, b},{a, c},{b, d},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}}

**Remark3.9**: Every regular- I closed set is I-R closed but not conversely.

**Example3.10**: In the above example 3.8, I-R closed sets are {X, $\varphi$ ,{a},{b, c, d}}.The set {a} is not regular-I-closed.

**Theorem3.11:** Let (X, $\tau$ , I) be an ideal space and A $\subset$ X. If A is R*-I closed, then $r_I^* cl(A) \backslash A$ does not contains any nonempty regular semi open set.

**Proof:** Suppose A is R*-I closed set in (X, $\tau$ , I).Also let F be a regular semi closed set contained in $r_I^*$ cl (A) \A. It implies F $\subset$ $r_I^*$ cl (A)\A $\cap$ X\A. Since F $\subset$ X\A,we have A $\subset$ X\F which is a regular semi open set. Therefore $r_I^*$ cl (A) $\subset$ X\F and so F $\subset$ X\ $r_I^*$ cl (A).By hypothesis we have F $\subset$ $r_I^*$ cl (A) and so F = $\varphi$ .Hence $r_I^*$ cl (A)\A contains no non empty regular semi open set.

**Theorem3.12:** Let A be a R*-I closed set in an ideal space X such that A $\subset$ B $\subset$ $r_I^*$ cl (A), then B is also an R*-I closed set.

**Proof:**
Let U be a regular semi open set of X, such that B $\subset$ U. Then A $\subset$ B $\subset$ U. Since A is R*-I closed set, $r_I^*$ cl(A) $\subset$ U. Since A $\subset$ B $\subset$ $r_I^*$ cl(A) $\subset$ U, it implies $r_I^*$ cl (B) $\subset$ $r_I^*$ cl ($r_I^*$ cl (A)). Hence $r_I^*$ cl (B) $\subset$ $r_I^*$ cl (A) $\subset$ U. Hence B is an R*-I closed set.

## 4. I-R*-CLOSED SETS

**Definition 4.1**
The intersection of all I-R closed sets containing A is called the I-R closure and is denoted by $r_I^{**}$ cl (A).

**Definition 4.2**
A subset A of an ideal space (X, $\tau$ , I) is said to be I-R* closed if $r_I^{**}$ cl(A) $\subset$ U whenever A $\subset$ U and U is regular semi open.

**Definition 4.3** A subset A is called I-R*-open if X\A is I-R*-closed.

**Result 4.4** The finite union of two I-R*-closed sets is I-R* closed set.

Proof: Let A and B be two I-R*-closed sets in X. Let U be regular semi open in X. We have $r_I^{**}$ cl(A) $\subset$ U, whenever A $\subset$ U and U is regular semi open and $r_I^{**}$ cl(B) $\subset$ U, whenever B $\subset$ U and U is regular semi open. Let A $\cup$ B $\subset$ U. Hence $r_I^{**}$ cl(A $\cup$ B) $\subset$ U whenever A $\cup$ B $\subset$ U and U is regular semi open. Therefore A $\cup$ B is I-R* closed set.

**Remark 4.5:** The intersection of two I-R*-closed sets need not be I-R* closed set.

**Example 4.6:** Let X = {a, b, c, d} $\tau$ = { X, $\varphi$ ,{b},{d},{b, d}}
I = { $\varphi$ ,{a}}.Then if
A = {b, d} B = {a,c,d}, A $\cap$ B = {d} which is not I-R*-closed.

**Theorem4.7:**In a topological space X, if X and $\varphi$ are the only regular semi open sets, then every subset of X is I-R*-closed set.

Proof: Let X be a topological space and {X, $\varphi$ } be the regular semi open sets. Also let A be a subset of X. Suppose A $\neq$ $\varphi$ , then X is the only the only regular semi open set containing A and so $r_I^{**}$ cl(A) $\subset$ X. Hence A is I-R* closed.

**Remark 4.8:** The converse of the above theorem need not be true as shown in the following example.

**Example 4.9:** Let X={a, b, c, d} $\tau$ = { X, $\varphi$ ,{a},{c},{a, c},{b, c},{a, b, c},{b, c, d}},
I = { $\varphi$ ,{a}}.Then all subsets are I-R*-closed and the regular semi open set is
{X, $\varphi$ ,{a},{b,c,d}.

**Remark 4.10:** Finite intersection of I-R* open sets is I-R* open.

**Theorem 4.11**: Let (X, $\tau$ , I) be an ideal space and A $\subset$ X. If A is I-R* closed, then
$r_I^{**}$ cl(A) \ A does not contain any nonempty regular semi open set.

**Proof:** Suppose A is I-R* closed set in (X, $\tau$ , I).Also let F be a regular semi closed set contained in $r_I^{**}$ cl(A).It implies F $\subset$ $r_I^{**}$ cl(A)\A= $r_I^{**}$ cl(A) $\cap$ X\A. Since F $\subset$ X\A, we have A $\subset$ X\F which is a regular semi open set. Therefore $r_I^{**}$ cl(A) $\subset$ X\F and so
F $\subset$ X\ $r_I^{**}$ cl(A) By hypothesis we have F $\subset$ $r_I^{**}$ cl(A) and so F = $\varphi$ .Hence $r_I^{**}$ cl(A)\ A contains no non empty regular semi open set.

**Remark 4.12**: The converse of the above theorem need not be true from the following example.

**Example 4.13:** Let X={a, b, c, d} $\tau$ = { X, $\varphi$ ,{a},{c},{a, c},{a, b, c},{a, c, d}},
I = { $\varphi$ ,{a}}.Then RSO(X)= { X, $\varphi$ ,{a},{c}{a, b},{a, d},{b, c},{c, d}{b, c, d},{a, b, d}}. Let A = {c}, $r_i^{**}$ cl (A) \A = {b, c, d}\{c} = {b, d}. But A = {c} is not I-R*closed set.

**Theorem 4.14:** Let (X, τ, I) be an ideal topological space and A ⊂ X be an I-R* closed set. Then A ∪ (X \ ($r_I^{**}$ cl(A)) is a I-R* closed set in (X, τ, I ).

**Proof:** Let A be I-R* closed set in (X, τ, I). Suppose that U is a regular semi-open set such that A ∪ (X \ ($r_I^{**}$ cl(A)) ⊂ U. We have

X \ U ⊂ X \ A ∪ (X \ ($r_I^{**}$ cl(A)))

= (X\A ) ∩ $r_I^{**}$ cl(A)

= $r_I^{**}$ cl(A) \ A

Since X\U is regular semi-open set and A is a I-R* closed set, it follows from theorem 4.11 that X \ U =∅. Hence X = U. Thus X is the only regular semi-open set containing A∪(X\ ($r_I^{**}$ cl(X)). Consequently, A ∪ (X \ ($r_I^{**}$ cl(X)) is I-R* closed set in (X, τ, I).

**Theorem 4.15:** Let (X, τ, I) be an ideal topological space and A ⊂ X be a I-R* closed set. Then $r_I^{**}$ cl(A)\A is a I-R* open set in (X, τ, I ).

**Proof:**

Since X \ [ $r_I^{**}$ cl(A)\A ] = X \ [$r_I^{**}$ cl(A) ∩ $A^C$] =

X ∩ [$r_I^{**}$ cl(A) ∩ $A^C$]$^c$

= X ∩ [($r_I^{**}$ cl(A))$^c$ ∪ A ] =

[X ∩ ($r_I^{**}$ cl(A))$^c$ ] ∪ [ X ∩ A]

= [X ∩ ( $r_I^{**}$ cl(A))$^c$ ] ∪ A =

A ∪ [X ∩ ($r_I^{**}$ cl(A))$^C$]

= A ∪ [X \ $r_I^{**}$ cl(A) ]

By the previous theorem, A ∪ [X \ $r_I^{**}$ cl(A) ] is I-R* closed set => X \ [ $r_I^{**}$ cl(A) \ A] is

I- R* closed set => $r_I^{**}$ cl(A)\A is I-R* open set in (X, τ, I ).

**Example 4.16:** Let X = { a,b,c,d }, τ = { X, φ ,{a}, {b}, {a,b}, {a,b,c} } and
I = { φ ,{a} }. Then I-R* closed sets = { X, φ ,{a}, {a,b}, {a,b,c}, {a,b,d}, {b,c,d}},
I-R* open sets = {X, φ ,{a}, {c}, {d}, {c,d}, {b,c,d}} and
$r_I^{**}$ cl(A)\A ∈ I-R* open sets.

**Theorem 4.17:** Let (X, τ, I) be an ideal topological space. The following properties are equivalent: (i) Each subset of (X, τ,I) is a I-R* closed set (ii) A is pre$_I^*$ closed set for each regular semi open set A in X.

**Proof:** (1) => (2) Suppose that each subset of (X, τ, I) is a I-R* closed set. Let A be a regular semi-open set. Since A is I-R* closed set, we have cl* (int A) ⊂ A. Thus A is pre*$_I$ closed set.

(2) => (1) Let A be a subset of (X, τ, I ) and U be a regular semi-open set such that A ⊂ U. By (2), we have $r_I^{**}$ cl(A) ⊂ $r_I^{**}$ cl(U) ⊂ U. Thus A is I-R* closed sets in (X, τ, I).

**Theorem 4.18:** Let (X, τ, I) be an ideal topological space. If A is a I-R* closed set and A ⊂ U ⊂ $r_I^{**}$ cl(A) then U is a I-R* closed set.

**Proof:** Let U ⊂ K and K be a regular semi-open set in X. Since A ⊂ K and A be a I-R* closed set, then $r_I^{**}$ cl(A) ⊂ K. Since U ⊂ $r_I^{**}$ cl(A) , then $r_I^{**}$ cl(U) ⊂ $r_I^{**}$ cl(A) ⊂ K. Thus, $r_I^{**}$ cl(U) ⊂ K and hence U is a I-R* closed set.

**Lemma 4.19:** [6] Let A be an open subset of a topological space (X, τ )
(i) If U is regular semi-open set in X, then so is U ∩ A in the subspace (A, $τ_A$ ).
(ii) If B (⊂ A) is regular semi-open in (A, $τ_A$ ) then there exists a regular semi-open set U in (X, τ) such that B = U ∩ A.

**Theorem 4.20:** Let (X, τ, I) be an ideal topological space and U ⊂ A ⊂ X. If A is an open set in X and U is a I-R* closed set in A, then U is I-R* closed set in X.

**Proof:** Let K be a regular semi-open set in X and U ⊂ K. We have U ⊂ K ∩ A. By lemma 4.19, K ∩ A is a regular semi-open set in A. Since U is an I-R* closed set in A, then $r_I^{**}$ cl $_A$ (U) ⊂ K ∩ A. Also we have,

$r_I^{**}$ cl(U) ⊂ $r_I^{**}$ cl $_A$ (U) ⊂ K ∩ A ⊂ K => $r_I^{**}$ cl(U) ⊂ K whenever
U ⊂ K and K is a regular semi-open set, Thus, U is I-R* closed set in (X, τ, I).

**Theorem 4.21:** Let (X, τ, I) be an ideal topological space and U ⊂ A ⊂ X. If A is a regular semi-open set in X and U is an I-R* closed set in X, then U is I-R* closed set in A.

**Proof:** Let U ⊂ K and K be a regular semi-open set in A. By lemma 4.21 there exist a regular semi-open set L in X such that K = L ∩ A. Since U is a I-R* closed set in X, then $r_I^{**}$ cl(U) ⊂ K. Also we have $r_I^{**}$ cl $_A$ (U) = $r_I^{**}$ cl (U) = $r_I^{**}$ cl (U) ∩ A ⊂ K ∩ A = K.

Thus $r_I^{**}$ cl $_A$ (U) ⊂ K. Hence U is I-R* closed set in A.

## 5. WEAKLY R*-I-CLOSED SETS

**Definition 5.1**
A subset A of an ideal space (X, τ , I) is said to be W-R*-I closed if (intA)*⊂U whenever A⊂U and U is regular semi open set in X.

**Definition 5.2**
A subset A of an ideal space (X, τ , I) is said to be W-R*-I open set if X/A is W-R*-I closed set.

**Theorem 5.3**
Let (X, τ , I) be an Ideal topological space and A⊂X .Then the following properties are equivalent.

1. A is W-R*-I closed set
2. cl*(int(A)) $\subset$U whenever A$\subset$U and U is regular semi open in X.

Proof:1$\Rightarrow$2 Let A be a W-R*-I closed set in (X,$\tau$ ,I).Suppose that A$\subset$U and U is regular semi open in X .We know (int(A))*$\subset$U and that int(A)$\subset$A$\subset$U. Hence we have
( int(A))*$\cup$ int(A)$\subset$U which implies cl*(int(A)) $\subset$U.
2$\Rightarrow$1 Let cl*(int(A)) $\subset$U whenever A$\subset$U and U is regular semi open in X. It implies (int(A))*$\cup$ int(A)$\subset$U .That is (int(A))*$\subset$U whenever A$\subset$U and U is regular semi open. Hence A is W-R*-I closed set.

**Theorem 5.4:**
Let (X,$\tau$ , I) be an ideal space and A$\subset$X. If A is open, regular semi open and W-R*-I closed, then A is * closed.
Proof: Let A be open, regular semi open and W-R*-I closed in (X,$\tau$ ,I).Since A is open. Hence cl*(A) = cl*(int(A)) $\subset$A. Thus A*$\subset$A and hence A is *closed

**Theorem 5.5** Let (X,$\tau$ , I) be an ideal space and A$\subset$X. If A is W- R*-I closed, then (int(A))* \ A contains no nonempty regular semi open set.
Proof: Let A is W- R*-I closed and suppose that F is regular semi open set such that F$\subset$ (int(A))* \ A. Since A is W- R*-I closed, X\F is regular open and A$\subset$X\F, then (int(A))* $\subset$X\F. We have F$\subset$X\(int(A))*.Hence F $\subset$(int(A))*$\cap$X\(int(A))*=$\varphi$ .Thus int(A))*\A contains no nonempty regular semi open set.

**Theorem 5.6:**
Let (X,$\tau$ , I) be an ideal space and A$\subset$X. If A is W-R*-I closed set, then Cl*(Int(A))\A contains no non empty regular semi closed set.
Proof: Suppose U is a regular semi closed set such that U$\subset$Cl*(Int(A))\A. But Cl*(Int(A))\A=
(Int(A))*$\cup$(Int(A)).The result follows from theorem 5.5.

**Remark 5.7:** The converse of the above theorem is not true in general as shown in the following example.

**Example 5.8**:Let X= {a,b,c,d}
$\tau$ ={X, $\varphi$.{a},{b},{a,b},{a,b,c}} I={$\varphi$,{a}}.Let A ={a},then cl*(int(A))\A does not contain any non empty regular semi open set but A is not W-R*-I closed set.

**Theorem 5.9:**
Let A be a W-R*-I closed set in an ideal space X such that A $\subset$B $\subset$cl*(int(A)), then B is also an W-R*-I closed set.
Proof:
Let U be a regular semi open set of X, such that B$\subset$U. Then A$\subset$B$\subset$U. Since A is W-R*-I closed set, cl*(int(A))$\subset$U. Since A $\subset$B $\subset$ cl*(int(A))$\subset$U, it implies cl*(int(B))$\subset$ cl*(int(A)) $\subset$U. Hence B is a W-R*-I closed set.

**Corollary 5.10:** Let (X,$\tau$ , I) be an ideal topological space. If G is a W-R*-I closed set and an open set, then cl*(G) is a W-R*-I closed set.
Proof: Let G be open and W-R*-I closed in (X,$\tau$ ,I).We have G$\subset$cl*(G) $\subset$cl*(int(G)).Hence by theorem 5.9, cl*(G) is a W-R*-I closed set.

**Remark 5.11:** (1) The intersection of two W-R*-I closed sets in an ideal topological space need not be a W-R*-I closed set.
(2) The union of two W-R*-I closed sets in an ideal topological space need not be a W-R*-I closed set.

**Example 5.12:** Let X= {a,b,c,d} $\tau$ ={X, $\varphi$,{a},{c,d},{a,c,d}} I={$\varphi$,{a}}.A={a,c,d} and B= {b,c,d} are W-R*-I closed sets but A$\cap$B={c,d} is not an W-R*-I closed set.

**Example 5.13:** Let X= {a,b,c,d} $\tau$ ={X, $\varphi$, {a},{c,d},{a,c,d}} I={$\varphi$,{a}}.{c} and {d} are W-R*-I closed sets but A$\cup$B={c,d} is not an W-R*-I closed set.

**Theorem 5.14:** Let (X,$\tau$ ,I) be an ideal space and A$\subset$X. If A is nowhere dense in X, Then A is a W-R*-I closed set.
Proof: Let A be a nowhere dense set in X. Since int(A) $\subset$int(cl(A))=$\varphi$ ,then
int (A)= $\varphi$.Hence cl*(int(A)) =$\varphi$.Thus A is a W-R*-I closed set.

**Remark 5.15**: The converse of the theorem need not be true as shown in the following example.

**Example 5.16:** Let X= {a,b,c} $\tau$ ={X, $\varphi$, {a},{b},{a,b}} I={$\varphi$,{a}}.Let A= {b,c}.Then A is W-R*-I closed set but it is not a nowhere dense set.

**Theorem 5.17:** Let (X,$\tau$ , I) be an ideal space and H$\subset$G$\subset$X. If G is an open set in X and H is a W-R*-I closed set in G, then H is a W-R*-I closed set in X.
Proof: Let K be a regular semi open set in X and H$\subset$K. We have H$\subset$K$\cap$G. By Lemma 4.19 K$\cap$G is a regular semi open set in G. Since H is a W-R*-I closed set in G, $Cl_G$*($Int_G$ (H)) $\subset$K$\cap$G. Also, cl*(int(H)) $\subset$$cl_G$*(int(H)) $\subset$$cl_G$*($int_G$ (H)) $\subset$K$\cap$G$\subset$K. Hence Cl*(Int (H))$\subset$K. Thus H is a W-R*-I closed set in X.

**Theorem 5.18** Let (X,$\tau$ ,I) be an ideal space and H$\subset$G$\subset$X. If G is a regular semi open set in X and H is a W-R*-I closed set in G, then H is a W-R*-I closed set in G.
Proof: Let H$\subset$K and K be a regular semi open set in G. By lemma 4.16 there exist a regular semi open set L in X such that K= L $\cap$G. Since H is W-R*-I closed set in X,cl*(int (H)) $\subset$K. Also we have $cl_G$*($int_G$ (H)) = $cl_G$*(int (H)) = cl*(int (H)) $\cap$G $\subset$

$K \cap G = K$. Thus $cl_G*(int_G(H)) \subset K$. Hence H is a W-R*-I closed set in G.

**Theorem 5.19** Let (X, $\tau$, I) be an ideal space and $G \subset X$. If G is a W-R*-I closed set, the following properties are equivalent:

1. G is $pre_I^*$-closed,
2. Cl*(int(G))\G is regular semi closed,
3. (Int(G))*\G is regular semi closed.

Proof:1 $\Rightarrow$ 2: Let G is $pre_I^*$-closed. We have Cl*(int(G)) $\subset$ G. Then Cl*(int(G))\G= $\varphi$ Therefore Cl*(int(G))\G is regular semi closed.
2 $\Rightarrow$ 1: Let Cl*(int(G))\G is regular semi closed. Since G is a W-R*-I closed set,then by theorem 5.6,Cl*(int(G))\G = $\varphi$.Hence we have Cl*(int(G)) $\subset$ G.Thus G is $pre_I^*$-closed.
2 $\Leftrightarrow$ 3:It follows easily since Cl*(int(G))\G = (Int(G))*\G.

**Theorem 5.20**: Let (X, $\tau$, I) be an ideal space and $G \subset X$. Then G is a W-R*-I open set if and only if $H \subset$ int*(cl(G)) whenever $H \subset G$ and H is regular semi closed set.
Proof: Let H is regular semi closed set in X and $H \subset G$. It follows that X\H is regular semi open and X\G $\subset$ X\H. Since X\G is a W-R*-I closed set, cl*(int(X\G)) $\subset$ X\H. We have X\ int*(cl(G)) $\subset$ X\H. Thus $H \subset$ int*(cl(G)).
Conversely, let K be a regular semi open set in X and X\G $\subset$ K. Since X\K is a regular semi closed set such that X\K $\subset$ G, then X\K $\subset$ int*(cl(G)).We have X\int*(cl(G))= cl*(int(X\G)) $\subset$ K. Thus X\G is W-R*-I closed set .Hence G is W-R*-I open set in (X, $\tau$, I).

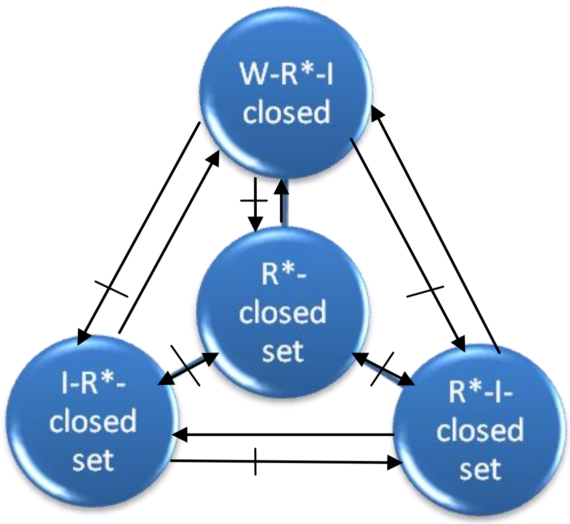
**Theorem 5.21:** Let (X, $\tau$,I) be an ideal space and $G \subset X$. If G is a W-R*-I closed set, then Cl*(Int(G))\G is a W-R*-I open set in (X, $\tau$, I).
Proof: Let G be a W-R*-I closed set in (X, $\tau$, **I)**.Suppose H is a regular semi closed set such that $H \subset$ Cl*(int (G))\G. Since G is W-R*-I closed set, it follows from theorem 5.6 that H= $\varphi$.Thus, we have $H \subset$ Int*(Cl(Cl*(Int(G))\G)).It follows from theorem 5.20 that Cl*(int (G))\G is a W-R*-I open set in (X, $\tau$, I).

**Theorem 5.22:** Let (X, $\tau$,I) be an ideal topological space. If G is W-R*-I open set in (X, $\tau$, I) and int*(cl(G)) $\subset H \subset G$, then H is W-R*-I open set.
Proof; Let G be is W-R*-I open set in (X, $\tau$, I) and Int*(cl(G)) $\subset H \subset G$. Also let K be regular semi closed. Since G is W-R*-I open set, from theorem 5.20 $K \subset$ Int*(cl(G)) $\subset$ Int*(cl(H)).Hence by theorem 5.20 H is W-R*-I open set.

**Corollary 5.23:** Let (X, $\tau$,I) be an ideal topological space and $G \subset X$ . If G is W-R*-I open set in (X, $\tau$, I) and closed set ,then Int*(G) is W-R*-I open set.
Proof: Let G be a W-R*-I open set and closed set in (X, $\tau$, I).Then Int*(Cl(G))=Int*(G) $\subset$ Int*(G) $\subset$ G. Thus by theorem 5.22, Int*(G) is W-R*-I open set in (X, $\tau$, I).

**Theorem 5.24:** Let (X, $\tau$,I) be an ideal topological space .If $G \subset X$ is a W-R*-I open set, then H=X whenever H is regular semi open and Int*(Cl(G)) $\cup$ (X\G) $\subset$ H.
Proof: Let H is regular semi open and Int*(Cl(G)) $\cup$ (X\G) $\subset$ H. We have
X\H $\subset$ X\( Int*(Cl(G)) $\cup$ (X\G))
= (X\ Int*(Cl(G))) $\cap$ G.
= Cl*(Int(X\G))\(X\G).Since X\H is regular semi closed set and X\G is W-R*-I closed set ,it follows from theorem 5.6 that X\H = $\varphi$. Thus we have H =X.

**Figure 5.25:** The above relation between sets is represented below.




**Example 5.26:** Let X= {a,b,c,d}
$\tau$ ={X, $\varphi$,{a},{b},{a,b},{b,c},{a,b,c}} I={ $\varphi$,{a}}.
R*-I closed sets are
{X, $\varphi$,{a,b},{a,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}
I-R*-closed sets are
{X, $\varphi$ a},{a,b},{a,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}
W-R*-I closed sets are
{X, $\varphi$,{a},{c},{d},{a,b},{a,c},{a,d},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}.

R*-closed sets are
{X, $\varphi$,{d},{a,b},{a,d},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}.

W-R*-I closed sets do not imply neither I-R* closed sets nor R*-I closed sets.IR*-closed set does not imply R*-I closed. W-R*-I closed sets does not imply R* closed sets. I-R* closed sets and R*-I closed sets are independent with R*-closed sets.

**Example 5.27:**Let X= {a,b,c,d} $\tau$ ={X, $\varphi$,{b},{d},{b,d}}
I={$\varphi$,{a}}.

W-R*-I closed sets are
{X, $\varphi$,{a},{a,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}.

R*-closed sets are
{X, $\varphi$,{a,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

W-R*-I closed sets does not imply R*-closed sets.

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