Free Vibration Analysis of Cracked and Un-cracked Cantilever Beam

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Abstract: ***Early detection of damage is of special concern for engineering structures. A comparatively recent development for the diagnosis of structural crack location and size by using the finite element method and Fuzzy logics techniques has improved. The traditional methods of damage detection includes visual inspection or instrumental evaluation .A method based on measurement of natural frequencies is presented for detection of the location and size of a crack in a cantilever beam. Numerical and programming in MATLAB is used for solving the Euler equation for un-crack beam to obtain first three natural frequencies of different modes of vibration considering boundary conditions for the beam.***

Here in this, ANSYS software package is used for finite element analysis of both crack and un-crack cantilever beam taking input file as a CAD design developed in CATIA. Experiments is done for total 10 models of crack beam having different cross section, crack location and crack depth and it generates natural frequency for 3 modes of vibration. A part of Matlab introduced is curve fitting toolbox and Fuzzy logic controller. The curve fitting toolbox introduced for crack identification by using simple curve. The Fuzzy controller here used comprises of three input variables (FNF, SNF and TNF) and two output variables (RCD and RCL) are generated with Triangular MF. Fuzzy analysis is done based on some set of fuzzy rules obtained from the information supplemented by Finite Element Analysis and Numerical Analysis. The proposed approach has been verified by comparing results obtained from fuzzy logic technique and finite element analysis.

Key Words: *Cantilever beam, Free vibration, Natural frequency, Crack, ANSYS*

1.Introduction:

With the discovery of musical instruments people started taking interest in vibration. Since then they have applied an investigation and theories to study the vibration phenomenon. Sir Galileo is considered to be the founder of vibration phenomenon. He has performed his experiments on free vibrations of simple pendulum. After sir Galileo, a lot of research is done by Mersenne (Vibrating string), Newton (Laws of motion), Taylor (Taylor series for vibration analysis), D'Alembert (Principles of equilibrium), Euler (Euler's beam theory), Fourier (sine wave analysis),Rayleigh (frequency measurement) and Lagrange's method of vibration analysis. Since then the vibration has covered a brief history from Sir Galileo's work to modern vibration researches.

Any motion which repeats itself after a certain interval of time is called as Vibration. The swing of pendulum is a typical example of vibration. The theory of vibration deals with the study of oscillatory motion of bodies and the forces associated with them. A vibration can caused due to external unbalanced force also. A vibratory system in general includes elastic member for storing potential energy, a mass or inertia member for storing kinetic energy and damper by which gradual loss of energy takes place. A simple pendulum as shown in fig1.1 is an example of vibration system. Pendulum has a string for elastic nature, mass of bob acts as a means for kinetic energy. Like pendulum spring-mass system, vehicle suspension system, simply supported and cantilever beam, lateral vibrating string, vibration due to unbalance reciprocating or rotating force etc. are the examples of vibrating system.

Vibrations with certain amplitude and frequency may be reliable but excessive vibrations causes

a) Resonance and excessive noise.
b) Structural failure of the machine components.
c) Power transmission loss in case of gears.
d) Increases bearing clearances.
e) Reduces the working life of components.
f) Effect on financial growth.

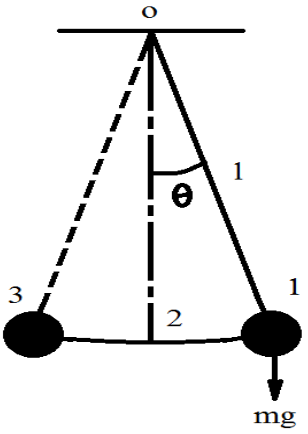

Figure 1.1 Free Vibrations of Simple Pendulum

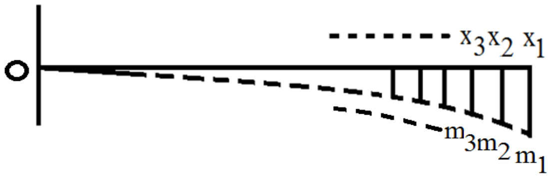

Figure 1.2 Cantilever Beam as Multi Degree of Freedom System

2. Objective:

1. To measure the natural frequencies of various beam models by using Euler's Beam model theory. The theory used in this report is Euler's Beam theory. The natural frequencies and mode shapes are find out by this theory for beams of different crack sizes and cross section.
2. To measure the natural frequencies of various beam models by using Finite element method (ANSYS). The finite element method provides good graphical approach towards the mechanical as well as structural problems. A type of ANSYS, modal analysis is taken for finding the natural frequencies.
3. To compare the natural frequencies of models by above three methods. A comparison is made at the end of report to find the errors in the above methodologies.
4. To identify the crack by using curve fitting toolbox in Matlab. The curve is generated in Matlab to identify the cracks present in model.
5. To introduce Fuzzy logic toolbox in Matlab. The toolbox Fuzzy logic in Matlab software is one of the better methods to identify crack. Here brief introduction of fuzzy logic toolbox is discussed.
6. To identify the crack by using Fuzzy logic toolbox in Matlab. A fuzzy model is developed for proposed theory and finds the numerical solution of the problem.

3. Problem Definition:

Most of the structures and the mechanical systems are failed due to excessive vibrations in their working life. Since most human activities involve vibration in one form or other.

The phenomenon of vibration can be applied to identify the crack size and location.

To develop new techniques for crack identification using Fuzzy logic techniques and curve fitting in Matlab are the alternatives for NDT techniques. These techniques give approximately solution for the problems of cracks. Crack location and depth are analyzed using ANSYS software. The results obtained from theoretical, ANSYS & Fuzzy logic are verified with the experimentation

4.Therotical Formulation

4.1 Crack Theory

4.1.1 Physical parameters affecting Dynamic characteristics of cracked structures

Usually the physical dimensions, boundary conditions, the material properties of the structure play important role for the determination of its dynamic response. Their vibrations cause changes in dynamic characteristics of structures. In addition to this presence of cracking structures modifies its dynamic behaviour. The following aspects of the crack greatly influence the dynamic response of the structure.

1. The position of crack
2. The depth of crack
3. The orientation of crack
4. The number of cracks

4.1.2 Classification of cracks

Based on their geometries, cracks can be broadly classified as follows:

- Cracks perpendicular to the beam axis are known as "**transverse cracks**". These are the most common and most serious as they reduce the cross-section and there by weaken the beam. They introduce a local flexibility in the stiffness of the beam due to strain energy concentration in the vicinity of the crack tip.

• Cracks parallel to the beam axis are known as **"longitudinal cracks".** They are not that common but they pose danger when the tensile load is applied is at right angles to the crack direction i.e. perpendicular to beam axis or the perpendicular to crack.

• **"Slant cracks"** (cracks at an angle to the beam axis) are also encountered, but are not very common. These influence the torsion behaviour of the beam. Their effect on lateral vibrations is less than that of transverse cracks of comparable severity.

• Cracks that open when the affected part of the material is subjected to tensile stresses and close when the stress is reversed are known as **"breathing cracks"**. The stiffness of the component is most influenced when under tension. The breathing of the crack results in non-linearity's in the vibration behaviour of the beam. Cracks breathe when crack sizes are small, running speeds are low and radial forces are large. Most theoretical research efforts are concentrated on "transverse breathing" cracks due to their direct practical relevance.

• Cracks that always remain open are known as **"gaping cracks"** . They are more correctly called "notches" . Gaping cracks are easy to mimic in a laboratory environment and hence most experimental work is focused on this particular crack type.

• Cracks that open on the surface are called **"surface cracks**". They can normally be detected by techniques such as dye-penetrates or visual inspection.

• Cracks that do not show on the surface are called "**subsurface cracks**". Special techniques such as ultrasonic, magnetic particle, radiography or shaft voltage drop are needed to detect them. Surface cracks have a greater effect than sub surface cracks on the vibration behaviour of shafts.

4.2 Governing Equation For Free Vibration Of Beam

The cantilever beam with a transverse edge crack is clamped at left end, free at right end and has same cross section and same length like model in Fig.3.1 and 3.2. The Euler- Bernoulli beam model is assumed for the theoretical formulation. The crack in this particular case is assumed to be an open surface crack and the damping is not being considered in this theory. Both single and double edged crack are considered for the formulation.

The free bending vibration of a beam of a constant rectangular cross section having length l, width b, and depth w is given by the Euler's beam theory as follows:

If the cross sectional dimensions of beam are small compared to its length, the system is known as Euler-Bernoulli beam. Only thin beams are treated in it. The differential equation for transverse vibration of thin uniform beam is obtained with the help of strength of materials. The beam has cross section area A, flexural rigidity EI and density of material ρ.

Consider the small element dx of beam is subjected to shear force Q and bending moment M, as shown in figure 4.3

While deriving mathematical expression for transverse vibration, it is assumed that there are no axial forces acting on the beam and effect of shear deflection is neglected. The deformation of beam is assumed due to moment and shear force.

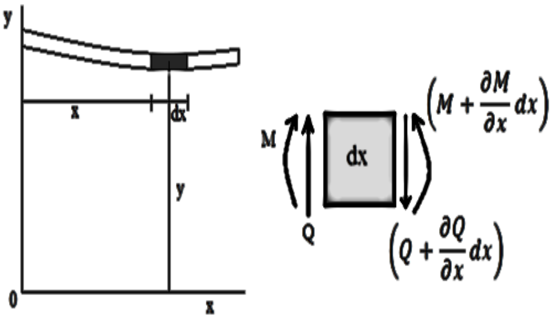

Figure 4.3 Shear Force and Bending Moment acting on beam element

4.3 Theoretical Calculation

The theoretical calculations are carried out by using above theory. Here total 10 models have been studied for natural frequency analysis having different crack depth and location. The table no. 3.1 shows various dimensions for the beam models. The beam models no. 1 to 5 are of rectangular cross section while beam model from 6 to 10 are of square cross section. It is assumed that the natural frequency changes due to the change in moment of inertia only. The beam made up of structural steel material by extrusion process. The Young's modulus for the beam material of length 700 mm is 210×109 N/mm2.

4.3.1 Theoretical calculation for Beam model 1

For Beam model 1,
Length= 700 mm, Cross-section= 32*5 mm, Un-cracked,
ρ= 7850Kg/mm3
A= $32 \times 10^{-3} \times 5 \times 10^{-3}$m2
=1.6×10^{-4}m2
I= (0.032) (0.005)3/12 mm4
= 3.33×10^{-10} m4
Now from equation 6,

$$\omega = C \times \sqrt{\frac{210 \times 10^{9} \times 3.33 \times 10^{-10}}{7850 \times 1.6 \times 10^{-4} \times 0.7^{4}}}$$

$\omega = C \times 15.235$rad/s.
For first three modes we have the value of constant C,
Thus,
$\omega_1 = 0.56 \times 15.235$
$\omega_1 = 8.5316$ rad/s.
Similarly ω2 and ω3 will be
ω2=3.52× 15.235 = 53.628 rad/s
ω3 = 9.82 × 15.235 = 149.61 rad/s.

From the theory it is predicted that presence of crack in structure reduces the natural frequency. The natural frequency is the function of flexural rigidity and inversely proportional to the density of material and length of the beam.

The calculations show the natural frequency and mode shapes for first three beam models.

5. Fuzzy Logic Toolbox

It is observed that the human beings do not need precise, numerical information input to make a decision, but they are able to perform highly adaptive control. Human have are mark able capability to perform a wide variety of physical and mental tasks without any explicit measurements or computations. Examples of everyday tasks are parking a car, driving in city traffic, playing golf, and summarizing a story. In performing such familiar tasks human use perceptions of time, distance, speed, shape, and other attributes of physical and mental objects. Fuzzy logic is a problem-solving control system methodology that lends itself for implementation in systems ranging for simple, small, embedded micro-controllers to large networked workstation-based acquisition and control systems. The theory of fuzzy logic systems is inspired by the remarkable human capability to operate on and reason with perception-based information. The rule-based fuzzy logic provides a scientific formalism for reasoning and decision making with uncertain imprecise information. This methodology can be implemented in hardware, software, or a combination of both. Fuzzy logic approach to control problems mimics how a person would make decisions. Fuzzy systems allow for easier understanding as they are expressed in terms of linguistic variables. Damage detection is one of the key aspects in structural engineering both for safety reasons and because of economic benefits that can result. Many non-destructive testing methods for health monitoring have been proposed and investigated. These methods include modal analysis, strain analysis, photo-elastic techniques, ultrasound and a coustic emissions. A fuzzy logic methodology can be presented for structural fault detection based on Eigen value, and dynamic responses of vibrating structure.

This proposes online crack detection methodology embedded with a new intelligent Fuzzy Interface System. In this approach a fuzzy logic controller is designed and implemented for identifying relative crack depth and location. The designed fuzzy controller has three inputs and two outputs. The inputs for fuzzy controller are three modes of natural frequencies and two outputs are relative crack depth and relative crack location.

5.1 Fuzzy Interface System

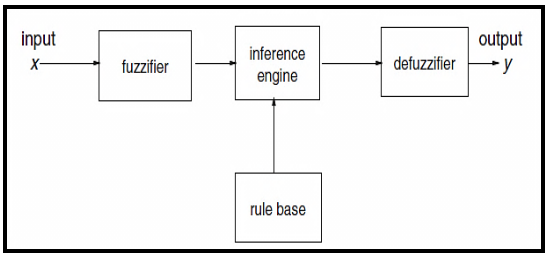

Figure 5.1 Block diagram of Fuzzy Interface System

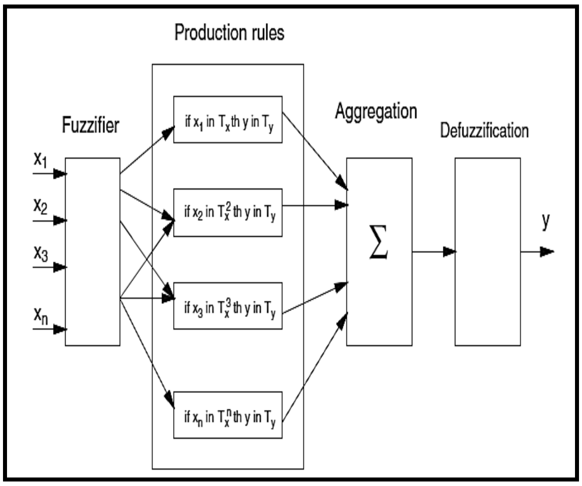

Figure 5.2 Schematic diagram of a Fuzzy Interface System

6. Crack Identification Using MATLAB

6.1 MATLAB: A Tool For Engineering Analysis

MATLAB is high performance language for technical computing. It integrates computation, visualization and programming in an easy to use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses includes following:

- Math and Computation
- Algorithm development
- Data Acquisition
- Modeling, Simulation and Prototyping
- Data Analysis, Exploration and Visualization
- Scientific and Engineering graphics
- Application development including graphical user interface.

MATLAB is interactive system whose basic data element is an array that does not require dimensioning. This allows formulating solutions to many technical computing problems, especially those involving matrix representations, in a fraction of time it would take to write a program in a scalar non interactive language such as C or FORTRAN.

The name MATLAB stands for matrix laboratory. MATLAB was written originally to provide easy access to matrix software developed by the LINPACK (Linear System Package) and EIPACK (Eigen System Package) projects. Today, a MATLAB engine incorporates the LAPACK (Linear Algebra Package) and BLAS (Basic Linear Algebra Subprogram) libraries, constituting the state of the art in software computation. In university environment, MATLAB is the standard computational tool for introductory and advanced courses in mathematics, engineering, and science. In industry, it is the computational tool of choice for research and development, and analysis. It is complemented by a family of application specific solutions called toolboxes. The image processing toolbox is a collection of MATLAB environment for the solution of digital image processing problems. Other toolboxes that sometimes complement IPT are Signal Processing, Neural Network, Fuzzy Logic and Wavelet Toolboxes.

In this report, two types of toolboxes namely CF toolbox and Fuzzy logic toolbox are studied. The CF toolbox commonly called as Curve Fitting toolbox used to fit a curve by analysing suitable data matrix. The curve fitting is required when we have available only one of the natural frequency instead of different modes. Here curve is fitted by knowing the first natural frequency.

7.1 ANSYS:

The calculation of stresses condition as well as factor of safety for complex geometries having complex boundary conditions is very tedious task by means of analytical calculation. Even it is unpredictable to satisfy the governing differential equation. ANSYS is general-purpose finite element analysis (FEA) software package. Finite Element Analysis is a numerical method of deconstructing a complex system into very small pieces (of user-designated size) called elements. The software implements equations that govern the behaviour of these elements and solves them all; creating a comprehensive explanation of how the system acts as a whole. These results then can be presented in tabulated or graphical forms. This type of analysis is typically used for the design and optimisation of a system far too complex to analyse by hand. Systems that may fit into this category are too complex due to their geometry, scale, or governing equations.

7.2 Generic steps to solving any problem in ANSYS

1. Build Geometry.
2. Define Material Properties.
3. Generate Mesh.

4. Apply Loads.
5. Obtain Solution.
6. Present the Results

8.CONCLUSION:

- The present investigation based on the theoretical Analysis, FEA Analysis and experimental analysis draws the following conclusions.
- Inputs for FEA and FFT are crack location and crack depth and outputs are natural frequency for different modes of vibration.
- The results show that the values of natural frequencies by theory, ANSYS and FFT are `close to the agreement.
- The fuzzy controller is developed with triangular membership function for input and output and results show that the triangular MF predicts the output parameters of crack.
- Crack depth and crack location of a beam can be predicted by fuzzy controller is within nanoseconds. Hence it saves considerable amount of computation time.
- Significant changes in natural frequency observed at the vicinity of crack location.
- When the crack location is constant the crack depth increases and the natural frequency of the beam decreases.
- When the crack depth is constant and crack location from the cantilever end varied, Natural frequencies of first, second and third modes are also increased.
- By Comparing the Fuzzy results with the theoretical results it is observed that the developed Fuzzy Controller can predict the relative crack depth and relative crack location in a very accurate manner.
- Certain precision and skilled operating is required to develop Fuzzy controller.
- Results based on fuzzy techniques are not much accurate as it depends on some training pattern of fuzzy controller, whereas in ANSYS, it is much accurate as it is based on finite elements But it is practically suitable as natural frequency can be obtained but crack location and crack depth are not possible as they have very small values.
- both ANSYS and Fuzzy, in which natural frequency obtained in ANSYS can be used as input for fuzzy controller for determination of accurate value of crack depth and crack location.
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