Temperature Dependence of Electrical Resistivity of Metals at Very Low Temperatures

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Abstract

In this paper we have discussed on the electrical resistance ratio of metals in respect of temperature dependence, magnitude and constancy in Debye temperatures at very low temperature. Some earlier reports in this context and Bloch Grueneisen law have also been reviewed in the light of present results. Here it has been concluded that the B.G. formula is not justified fully in the metals under the present study in the said temperature zone.

Key Words : Bloch's law, Umklapp process, Phonon drag, B.G. formula, M.R., Debye temperature.

1. **Introduction :** Various theoretical and experimental investigations at low temperatures provide different ideas regarding the temperature dependence of electrical resistivity and its closeness with standard theory in metals. It is worthwhile to give a brief outline of the previous findings in these contexts. Garland and Bower (1968)[1] have reported an AT^2+BT^5 variation of resistance of Indium below 4K and have suggested that the T^2 term is due to electron-electron scattering as discussed by Peierls (1964) [2] and Ziman (1964) [3], Caplin and Rizzuto (1970) [4]. Deviation from the Bloch's expression has been observed by Woods (1956) [5] for sodium and the results, so obtained are as follows :

$\rho_i \propto T^{5.0}$; for $9 \leq T \leq 15$ K

$\rho_i \propto T^{6.0}$; for $4 \leq T \leq 9$ K

Aleksandrov and Dyakov (1963) [6] claimed T^5 variation for Sn, In, Pb, Cd and Zn but found that Al did not obey Bloch's law[7] at temperature below 10K. Bloch's formula[7] for electron-phonon scattering for T $\leq$ 4.2 K is accepted with considerable confidence [Ziman (1964)][3] and therefore, it is assumed that observed departure from T^5 law is due to other effects. Compbell (1971)[8] suggested that the problem could be resolved by assuming a relaxation of conservation of momentum required in electron-phonon scattering. Smith (1971) [9] rejected this and all other previous proposed explanations. Keveh and Wiser (1971)[10] suggested by categorically that Bloch T^5 law[7] does not describe the low temperature electrical resistivity of any metal. They claim that the results of Garland and Bower (1968)[1] as well as those of Woods (1956)[5] can be precisely accounted by a theory which takes into account of Umklapp processes in the scattering of the electron by phonons, the momentum dependence of electron-phonon interaction scattering amplitude and also phonon drag. Dworin (1971)[11] considers Caplin and Rizzuto's result to be consequence of phonon drag.

The electrical resistivity of transition metals clearly exhibits the effect of electron-electron scattering [Rice (1969)][12] although there is no strong evidence of it in simple metals.

In the noble metals, a T^3 term has been found in the resistivity of Ag between 1.4K and 4.2 K [Kos (1972)][13] and more recently in that of copper between 1K and 8.5K [Rumbo (1973)].[14] Wilson (1938)[15] showed that if at low temperature s-d transitions are not prohibited by conservation laws, the resistivity should vary at T^3. However, Dworin (1971) [11] is against the T^3 law. White and Woods (1959) [16] predicted electrical resistivity of metals, viz, Cu, Au and Na except Mn follows a T^5 law developed by Bloch- Grueneisen (1930, 1933) [7,17] down to $\theta/5$, θ being Debye temperature although slight anomalies are shown by V, Cr, Fe, Co and Ni at low temperatures. The resistivity of metals, Viz, Mn, Fe, Co, Ni, Pd, Pt and perhaps in W and Nb at temperature below 10K vary nearly as T^2 whose magnitude are given below.

Mn ($0.15T^2$) ; Nb ($140\times10^{-6}\ T^2$) ; W ($10\times10^{-6}T^2$)

Fe ($13\times10^{-6}T^2$) ; Co ($13\times10^{-6}\ T^2$) ; Ni ($16\times10^{-6}T^{-2}$)

Pd ($33\times10^{-6}T^2$) ; Pt ($14\times10^{-6}\ T^{-2}$)

However, they should not produces any apparent evidence of T^2 dependence in other elements.

Berry (1967)[18] investigated that the resistivity of platinum (Pt) may be represented by T^2 (electron-electron scattering term) plus second term proportional to $T^{4.7\pm0.2}$ in range of temperature 7K-17K and approximately $T^{4.7\pm0.5}$ in the 3K-7K region. De-Hass and De-Boer (1934)[19] observed T^2 dependence in Pt and it was ascribed by Baber (1937)[20] to the effect of electron-electron scattering. Olsen Bar (1958)[21] and also Mendelssohn (1956)[22] have examined ρ_i (electrical resistivity) for elements Fe, Co, Ni, Pt, Pd, and Rh below 25K and the experimental results of Olsen Bar (1958)[21] also suggested approximately T^2 variation at sufficiently low temperatures. White & Woods (1959)[16] did not find T^2 dependence down to about 10K in Zirconium and Rhenium.

These previous results, discussed above, stimulated our interest in the investigation of behaviour of electrical resistivity at very low temperature. In this paper an attempt has been made in this direction.

2. Analysis of data from International Critical Table (1929) in the reference of B.G. formula:

Resistance ratio R_T/R_{273} reported earlier [International Critical Table (1929)][23] with and without impurity contribution in some metals; viz, Cd, K, Zn, Hg, Pt & Au, in temperature interval below about 0.1θ has been analysed. In the process of analysis we have plotted log (R_T/R_{273}) against log T. A linear relationship with probability $\geq$ 99% is found in each case. The slope and intercept for each curve were obtained on the basis of least square method [Yong (1962)][24]. The equations for R_T/R_{273} (with and without

impurity contribution) followed from the logarithmic plots are given under [Where $R_T/R_{273} = W_T$)].

Cd : $W_{iT} = 3.90084 \times 10^{-6}\ T^{2.8539325}$ (1)

R.T. = $13 \leq T \leq 20.42$K ; (Without I.C.)

Cd : $W_T = 1.0326 \times 10^{-5}\ T^{2.5581395}$ (2)

R.T. = $13 \leq T \leq 20.42$K ; (With I.C.)

K : $W_T = 1.6178 \times 10^{-5}\ T^{1.6969696}$ (3)

R.T. = $13 \leq T \leq 20.16$K ; (With I.C.)

K : $W_{iT} = 5.6247 \times 10^{-6}\ T^{2.7227}$ (4)

R.T. = $13 \leq T \leq 20.16$K ; (Without I.C.)

Zn : $W_{iT} = 1.8588 \times 10^{-4}\ T^{2.0754716}$ (5)

R.T. = $13 \leq T \leq 20.4$K ; (Without I.C.)

H_g : $W_T = 1.2598 \times 10^{-4}\ T^{3.5294117}$ (6)

R.T. = $4.06 \leq T \leq 4.24$ K ; (With I.C.)

Pt : $W_T = 3.5188 \times 10^{-7}\ T^{2.7686}$ (7)

R.T. = $4 \leq T \leq 13$K ; (With I.C.)

Au : $W_{iT} = 1.700353 \times 10^{-8}\ T^{4.25}$ (8)

R.T. = $4 \leq T \leq 13$K; (Without I.C.)

(In above equations, the impurity contribution has been eliminated by applying simple M.R. $R_{iT}/R_{273} = W_{iT}$ where impurity contribution has been eliminated)

Further some recent data of resistance ratio W_{iT} [Wilson (1965)[24], Berry (1963)[25], De-Hass, De- Boer & Vanden berg (1934)[26], Mac Donald and Mendelssohn (1950) [27] and Berry (1967)][18] have been analysed on the basis of logarithmic plots, A linear relationship in between log (W_{iT}) and log T with probability $\geq$ 99 % is found in each case. Least square method [Yong (1962)[24], Fisher and Yats

(1948)[28] has been adopted for getting slopes and intercepts, reported in Table-1 in all these cases. The equations describing W_{iT} are summarised below.

Au : $W_{iT} = 4.7438\times10^{-9}\ T^{4.637827}$ (9)

R.T. = 11.1 $\leq T \leq$ 20.4 K ; (Wilson (1965)].

Pt : $W_{iT} = 3.2687\times10^{-8}\ T^{3.8754765}$ (10)

R.T. = 10 $\leq T \leq$ 20 K ; (Berry (1963)].

3. Equations (11) to (17) Corresponding to Data [De- Hass, De- Boer and Vandenberg], Physica, 1, 1115 (1934):

Au_1 : $W_{iT} = 1.1386\times10^{-8}\ T^{4.418538}$ (11)

R.T. = 6.08 $\leq T \leq$ 20.44 K ;

Au_2 : $W_{iT} = 2.0611\times10^{-8}\ T^{4.2019135}$ (12)

R.T. = 6.28 $\leq T \leq$ 20.45 K

Cu_2 : $W_{iT} = 1.1337\times10^{-6}\ T^{2.2113204}$ (13)

R.T. = 14.26 $\leq T \leq$ 20.47 K

Cu_3 : $W_{iT} = 1.1476\times10^{-9}\ T^{4.3673936}$ (14)

R.T. = 14.21 $\leq T \leq$ 20.47 K

Cu_4 : $W_{iT} = 1.2524\times10^{-6}\ T^{2.3675909}$ (15)

R.T. = 14.26 $\leq T \leq$ 20.47 K

Pb_1 : $W = 4.9604\times10^{-6}\ T^{3.2246692}$ (16)

R.T. = 10.71 $\leq T \leq$ 20.4 K

Pb_2 : $W = 4.9151\times10^{-7}\ T^{3.7087602}$ (17)

R.T. = 7.27 $\leq T \leq$ 20.47 K

Na : $W_{iT} = 3.4432\times10^{-8}\ T^{4.528557}$ (18)

R.T. = 8.1 $\leq T \leq$ 20.4 K

[Mac Donald and Mendelssohn (1950)]

Pt : $W_{iT} = 6.7272\times10^{-9}\ T^{4.2956953}$ (19)

R.T. = 6.75 $\leq T \leq$ 11.01 K

Pt : $W_{iT} = 1.0905\times10^{-9}\ T^{5.1191816}$ (20)

R.T. = 5.28 $\leq T \leq$ 11.01 K

Pt : $W_T = 3.61\times10^{-4}\ e^{4.40531\times10^{-3}\ T^2}$ (21)

R.T. = 2.49 $\leq T \leq$ 4.2 K

[Berry (1967)]

In all above equations, R.T. represents the range of temperature in which the given equation is valid, γ [cf. Table- 1] is the correlation factor suggesting the probability of the linear relationship in between log (W_{iT}) and log T [cf. eqns. (9) – (21)]. Details of the data are given in Table (2-5)

4. Discussion of the Results:

Eqn. (21) for the data of resistance ratio for platinum reported by Berry (1967)[18] in the temperature interval 2.49 $\leq T \leq$ 4.2 K may be reduced to,

$W_T = 3.61\times10^{-4}\ (1+4.40531\times10^{-3}\ T^2\ )$ (22)

In which T^2 law is obvious. The value of W_{iT} ideal resistance ratio has been calculated by making use for the equation

$W_{iT} = W_T - 3.61\times10^{-4}\ (1+4.40531\times10^{-3}\ T^2\ )$ (23)

The values of W_{iT}, so obtained, are listed in Table (3). These values are found to fit the equations (19) and (20). Thus W_T in Pt fits the equation.

$W_T = W_o + AT^2 + BT^n$ (24)

The values of the constants $W_o = 3.61\times10^{-4}$, A and B together with n are followed from equations (19) to (21). Further θ has been calculated from eqn.

$W_{iT} = 497.6\ T^5/273\theta^4$ (25)

Making use of the constant 1.0905×10^{-9} in eqn. (20), θ is found to be 202 K against 225K deduced from specific heat and electrical measurements at normal temperatures. Thus the agreement, only in temperature dependence in eqn. (20) with theoretical value, reveals a partial validity of B.G. law in Pt. The T^2 term is in conformity with the

earlier report [White and Woods (1959)][16]. The values of W_{iT} listed in Table- 4 for the specimen Au_1, Au_2, Cu_2, Cu_3, Cu_4 have been obtained from W_T [De-Hass (1934)][19] making use of the eqn.

$$W_{iT} = W_T - W_o \quad (26)$$

Where W_o is the minimum observed value of W_T in the specimen concerned. This process has been based on the simple M.R. and vanishing contribution due to other factors such as scattering of electrons by electrons etc. In case of Pb_1 and Pb_2 the value of resistance ratio W_T has been taken for W_{iT} because the minimum value of it is not available in the literature [De-Hass, De-Boer and Vandenberg (1934)][26]. Subscript 1,2,3 and 4 refer to the different specimen of a metal.

In Pb_1 & Pb_2 [cf. eqn. (17)] the temperature dependence agrees well with the theoretical values. However, the constants in eqns. (16) & (17) deviate appreciably. The present conclusion in Pb_1 is in conformity with the earlier report [De-Hass, De-Boer and Vandenberg (1934)][26] in respect of temperature dependence only. Behaviour of resistivity in Cu_3 [cf. eqn. (14)] is close to the B.G. formula if only the consideration of temperature dependence is taken into account. Below $T = 0.07\theta$, the standard theory requires the constant, pertaining to the equation referring to T^5 law for $\theta = 310K$, to be 1.93655×10^{-10}, however, the present analysis find it to be 1.1476×10^{-9} [cf eqn. (14)] suggesting a large departure. In Cu_2 and Cu_4 [cf. eqn. (13) & (15)] the agreement with the standard theory is not acceptable at all. If only the temperature dependence is taken into account, then the behaviour of resistivity is Au_1 and Au_2 [cf. eqns. (11) (12)] appears to tend towards the theoretical prediction. In Na [cf. eqn. (18)] the agreement in temperature dependence is tolerable through the constant term appearing in eqn. (18), is greater by more than three times [cf. eqn. (5)]. Thus a partial agreement is reached in $8.1 \leq T \leq 20.44$ K. Data for Pt [White and Words (1959)][16] do not obey B.G. formula [cf. eqn. (10)]. A typical set of data for W_{iT} i.e., resistivity ratio [Wilson (1965)][24] for Au is close to the theoretical value in respect of temperature

dependence. However, the constant 4.7438×10^{-9} in eqn. (9) is more than two times the value in the standard theory if θ is taken as 170 K. The behaviour of some data of W_T excluding impurity contribution reported in International critical Tables of numerical data (1929) [23] for Cd, K, Zn, Hg, Pt and Au does not obey B.G. formula [cf. eqn. (1) to (8)] though in these cases also simple M.R. has been applied to obtain the ideal resistance ratio in the present analysis. Thus the observation, reported by White and Woods (1959)[16] in Pt is close to the result in the present analysis in respect of the existence of T^2 term, but a deviation occurs in the magnitude of the constant associated with T^2 [cf. eqn. (21)]. The earlier value is 14×10^{-6} and the present value is 15.903133×10^{-7}. An indication of T^3 variation in Cd, [cf. eqn. (1)] and Pt [cf. eqn. (7)] is found. On the basis of above results, the sound validity of B.G. formula may not be claimed in metals under consideration. This is in conformity with the conclusion reached by Kaveh & Wiser (1971)[10]. The departure from the standard theory at least in noble and alkali metals may not be based on electron-electron scattering or interband conduction phenomena theory. In metals, discussed above, in which temperature dependence is close to the theoretical value, the divergence may be probably in using the same value of the constant A' appearing in the equations for electrical resistivity at low and high temperatures. The other factor which may cause the departure from the standard theory may be the variation of Debye temperature with temperature [Blackmann (1941)][29]. Since T^2 or T^3 variation is expected only at extremely low temperatures the deviation, so obtained, may not be interpreted in terms of these factors also[30-33].

5. Conclusions :

On the basis of above results, it may be concluded that the B.G. formula is not justified fully in the metals under the present study in the said temperature zone. Any regular law to meet the magnitude as well as temperature dependence in consistant

with the experimental findings at low temperature cannot be suggested in the reference of standard theories.

Table-1

Values of slope, intercept and correlation factor corresponding to the logarithmic plots between resistance ratio and temperature [cf. equation (9) –(20)] and between Resistance ratio & T^2 [cf. eqn. (21)].

Metal	Slope	Intercept	correlation factor (γ)	Range of Temp. K
Au	4.637827	-8.323865	0.9849569	11.1-20.4
Pt	3.8754765	-7.4856266	0.9998319	10.0-20.0
Au_1	4.4185358	-7.943398	0.9992781	6.08-20.44
Au_2	4.2019135	-7.6859289	0.999036	6.28-20.45
Cu_2	2.2113204	-5.9455026	-0.9969275	14.26-20.47
Cu_3	4.3673936	-8.9401883	0.9968202	14.26-20.47
Cu_4	2.3675909	-5.9022651	0.97065457	14.26-20.47
Pb_1	3.2246692	-5.3044727	0.9900	10.71-20.44
Pb_2	3.7087602	-6.3084662	0.9948364	7.27-20.47
Na	4.5285527	-7.4630094	0.9981673	8.1-20.4
Pt	4.2956953	-8.2721541	0.9983081	6.75-11.01
Pt	5.119816	-8.9623682	0.9897245	5.28-11.01
Pt	0.0038264	+5.5573855	0.9953818	2.49-4.25

Table-2

Values of Electrical Resistance ratio W_{iT} for Pt Berry (1963), of Eqn. (10)

Temp. K	$W_{iT} \times 10^6$
10	251.6
12	489.7
14	884.5
16	1502.5
18	2408.7
20	3660.7

Table – 3

Values of Electrical resistance ratio W_T& W_{iT} [cf. eqn. (19) – (21)] R.J. Berry, Can. J. Phys., 45, 1963 (1967) for Pt.

Temp. K	$W_T \times 10^6$	$W_{iT} \times 10^6$
2.49	371.24	
3	375.45	
3.36	379.03	
3.5	380.56	
4.25	390.40	
5.28	409.48	4.1446
6.75	452.62	24.95601
7.41	479.86	38.52216
8.51	538.92	62.74938
8.82	559.25	74.53574
9.07	577.00	85.17306
9.96	650.84	132.07838
11.01	764.1	210.3227

$W_{iT} = W_T - 3.61\times10^{-4}\ (1+4.40531\times10^{-3}\ T^2)$

Table- 4

Values of Electrical Resistance ratio (W_{iT}, W_T) [De-Hass, De-Boer and Vandenberg,] Physica, 1, 1115 (1934). [cf. eqns. (11) – (17)].

Metal	Temp. K	$W_{iT}\times10^4$	$W_T\times10^4$
Au_1	6.08	0.30	26.74
	7.28	0.70	27.14
	8.83	1.82	28.26
	9.95	3.15	29.59
	11.84	6.76	33.20
	12.1	7.42	33.86
	15.17	19.2	45.64
	17.03	30.86	57.30
	18.04	39.29	65.73
	20.44	64.04	90.48
Au_2	6.28	0.45	27.02
	8.48	1.63	28.20
	9.76	3.05	29.62
	10.24	3.65	3022
	11.11	5.57	31.73
	12.66	8.85	35.52
	14.25	14.83	41.40
	16.02	24.1	50.67
	18.15	39.85	66.42
	20.45	63.91	90.48

Note : $W_o = 26.57\times 10^{-4}$

Metal	TK	$W_T \times 10^4$	$W_{iT} \times 10^4$
Cu_2	14.26	15.6	4.1
	16.14	16.8	5.3
	17.69	17.9	6.4
	19.1	19.1	7.5
	20.47	20.80	9.3
Cu_3	4.23	11.9	0.4
	6.47	12.0	0.5
	11.78	12.4	0.9
	14.26	12.8	1.3
	16.14	13.5	2.0
	17.69	14.9	3.4
	19.1	16.0	4.5
	20.47	17.6	6.1
Cu_4	14.26	21.0	9.5
	16.14	21.9	10.4
	17.69	22.2	10.7
	19.10	25.1	13.6
	20.47	27.0	15.5

Note : $W_o = 11.5 \times 10^{-4}$ in Cu_2, Cu_3 & Cu_4

1	2	3
Pb_1	7.29	20.8
	9.32	21.2
	10.71	38.5
	14.25	107.72
	16.02	157.3
	18.15	220.7
	20.44	297.1
Pb_2	7.27	6.2
	8.69	15.4
	10.28	33.3
	14.22	109.5
	16.08	159.4
	17.73	208.8
	18.92	251.3
	20.47	300.7

Note : $W_{iT} = W_T$

Table- 5

Values of Electrical resistance ratio W_{iT} [D.K.C. Mac Donald & K. Mendelssohn] Proc. Roy. Soc. 4202, 103 (1950) [cf. eqn. (18)].

Metal	Temperature K	$W_{iT} \times 10^4$
Na	20.4	0.00326
	15.95	0.0098
	14.1	0.00051
	13.1	0.00036
	11.05	0.00017
	9.65	0.0001
	8.1	0.00005

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