CROSS RECURRENCE PLOTS AS A TOOL FOR NON LINEAR DATA ANALYSIS

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Abstract: ***Non linear data analysis has always been an interesting area, because these systems generally show non stationary and very complex behaviours. A non linear system is the one which does not obey the superposition principles. Cross recurrence plots is a new tool for non linear data analysis. The Cross recurrence plots can be used to study the differences between two processes or the matching between two sequences of data series. The advantage of this tool is that, it can be applied to the situations where other tools may fail due to non stationary nature of data.***

1. INTRODUCTION

The distinction between a linear and non linear systems can be expressed in terms of the superposition theorem. Non linear systems are the one which does not obey the superposition theorem. A non linear system is inherently complex and difficult to predict and often expresses the behavioural pattern of chaos systems[1].One of the not so easy phase in the multi variate data analysis is the comparison between different time series data. This is needed to find out the interrelation between the two data series. Generally these data is resourced from non stationary and complexly behaved systems.

There are many methods for the non linear data analysis of which some are listed below.

- Time delay reconstruction [Christian et al., 1998]
- Lyapunov exponents or mutual information [Norbert et al., 2004]
- Surrogate data analysis [Christian et al., 1998]
- Estimation of fractal dimensions [Norbert et al., 2004]
- Channel interrelations
- Principal component analysis [Sirovich et al. 1987]
- Archetypal analysis [Christian et al., 1998]
- Prediction of spatio temporal dynamics [Christian et al., 1998]

Although there are many data analysis methods are there the fundamental problem with them is that they all need very long data to analyze [2].If the data series is not sufficiently long the results may not be accurate or reliable.

In this contrast a new tool was developed known as the recurrence plots. Reccurence is known to a basic property of dissipative dynamical systems. Although slight change in the system input conditions can show an exponential divergence of its state, it is seen that the system will reach a state which is arbitrarily close to the original state. Recurrence plots are designed to visualize such repeated or recurrent behaviour of these dynamical systems. One of the major attraction of these type of tool is its ability to produce satisfactory results from a rather short and non stationary data.

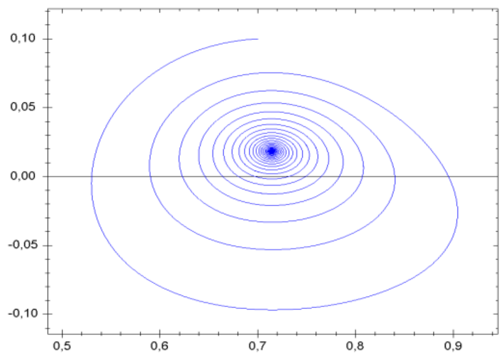
Cross recurrence plots or CRP's are known to be extension of the above tool and is included to analyse the time dependent behaviour of two processes. The fundamental idea is the comparison of phase space trajectories of the two processes. Hence CRP's is used to investigate the similarity between two different phase space trajectories.(The Phase Space is a concept in which each possible state of a system is represented by point. In other words, the phase space of a dynamical system is a space which has a point on it to represent all possible states of that system. For each state there is a corresponding unique point.

2. LITERATURE SURVEY

In the following section a basic idea about the key concepts are given.

2.1 The Phase Space concept

Phase space is a concept where all the parameters of the system is represented by an axis of a multidimensional space. In this the single dimensional system is called the phase line, while a bi dimensional system is known as the phase plane. For all the possible states of the system, or allowed combination of values of the system's parameters, a point is included in the multidimensional space. The system's developing state over time traces a path (known as **phase space trajectory**) through the high-dimensional space. The phase space trajectory represents the set of states compatible with one particular initial condition, positioned in the full phase space that represents the set of states compatible with starting from any initial condition. The shape of the phase space trajectory will give us vague ideas about the system and its nature. That said the phase diagram represents all that the system belong to, and its shape can easily reveal qualities of the system that might not be clear otherwise. A phase space may contain many dimensions. For exammple, a fluid containing many molecules require a separate dimension for each particle's *x*, *y* and *z* positions and momenta. Figure below shows the phase space of dynamic systems with unstable foci.



2.2 Recurrence Plots

Natural processes is having a discrete recurrent behaviour,(like periodicities), and also irregular cyclicities. In addition, the recurrence of states, in the sense that states are randomly in proximity after some time, is a basic property of dynamical systems of deterministic nature and is usual for nonlinear or chaotic systems.

Recurrence Plots or popularly known as the RP is effective and attractive tool for understanding the dynamics of Phase space trajectories. In normal cases the phase space is not having a dimension by the help of which we can picturise it. Eckmann et al. (1987) have introduced a tool by which we can picturise the recurrence of states x_i in a phase space. If we need to picturise the higher dimension phase space, it is needed to be projected in to 2-3 dimensional sub spaces. [3].

Eckmann's tool enables us to inspect the m-dimensional phase space trajectory by using a two-dimensional illustration of its recurrences. Such recurrence of a state at time i at a different time j is plotted within a two-dimensional square matrix with zeros and ones dots (represented as black and white dots in the plot), where both axes are plotted with time . This representation is known as recurrence plot (RP).

RP can be expressed as

$$R_{i,j} = \Theta\,(\,\varepsilon_i - ||\,x_i - x_j||), \quad x_i \in \Re^m, \quad i, j=1\ldots N,$$

where N is the number of considered states x_i, ε_i is a threshold distance, $||\cdot||$ a norm and $\Theta(\cdot)$ the Heaviside function.

Structures in Recurrence Plots

The primary function of RPs is the visual analysis of superior dimensional phase space trajectories. The plot on RPs gives us ideas about the time based nature of these trajectories. The advantage of RPs is the fact that it can also be applied to cases with short and even non-stationary data.

The RPs shows typical large scale and small scale patterns. First patterns were proposed by Eckmann et al. (1987) as *typology* and thesecond one as *texture*. The typology can be categorized as *homogeneous*, *periodic*, *drift* and *disrupted*.

- *Homogeneous* RPs are a case with stationary and autonomous systems in which relaxation times are short in comparison with the time span by the RP.
- Oscillating systems have RPs with diagonal oriented, *periodic* recurrent structures. For near

periodic (*quasi-periodic)* systems, the distances between the diagonal lines are different.

- The *drift* is caused by systems with slowly varying parameters. Such slow (adiabatic) change brightens the RP's upper-left and lower-right corners.
- Rapid and abrupt changes in the dynamics as well as extreme events cause *white areas or bands* in the RP.

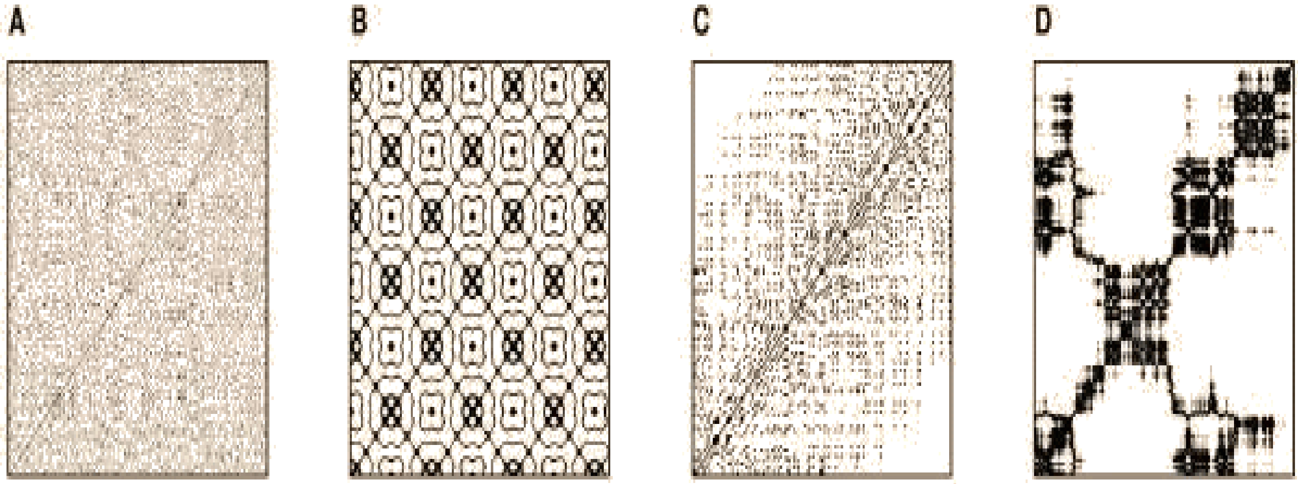


Figure 2: RP structures

A) homogeneous (uniformly distributed noise),
B) periodic (super-positioned harmonic oscillations),
C) drift (logistic map corrupted with a linearly increasing term) and
D) Disrupted

Table 1: RP interpretation

RP patterns	**Interpretation**
Homogeneity	Stationary process
Disruptions (white bands) occur	Non stationarity of the process
Periodic/ near-periodic patterns	Cyclicities in the process; long diagonal lines with different distances to each other reveal a near-periodic process
Single isolated points	heavily fluctuated process; if there is only single isolated points are shown, the process may be an uncorrelated random or anti-correlated process
Fading to the upper left and lower right corners	Non – stationary state ; the process contains a trend or drift
Diagonal lines (parallel to the LOI)	the development of states is alike at different times but with reverse time; (sometimes this is an indication for insufficient embedding
Vertical and horizontal lines/clusters	Laminar states- some states do not have the tendency to change or change is extremely slow.
Long bowed line structures	the dynamics of the system is changing (note that this is not fully valid for short bowed line structures)

2.3 Cross Recurrence Plots

A cross recurrence plot (CRP) can be defined as a graph which shows all those times at which a state in one dynamical system occurs in a second dynamical system at the same time. Simply said , the CRP shows all the times when the phase space trajectory of the first system reaches roughly the same area in the phase space where the phase space trajectory of the second system lies. The data length of both systems can differ, leading to a non-square CRP matrix.

To make the concept clear, consider a trajectory α_i with length X, and a trajectory β_j with length Y. If we add second trajectory to first, the examination between all the points of the former trajectory with that of the latter, will show the Cross Recurrence Plot. If in the latter trajectory a state at time j is near to a state on the first trajectory at time i, a dot will be assigned in the plot. The vectors β and α not necessarily of the same length. That is the matrix representing the plots need not be a square one. This concept of RP was coined forward by Zbilut et al (1998). In order to create a CRP, both trajectories, should represent the same dynamical system. The state variable also need to be equal as they are in the same phase space. If the measurement of different data such as temperature and pressure etc. are to be analysed, the above point should be considered with utmost importance.

And if the measurements of two completely different systems are to be analysed, the concept of intersected RP can be utilised (Romano et al., 2003).

CONCLUSION

Non linear data analysis has always been a sector where there is good scope of research as the nature of non linear systems are not been completely studied. The reason behind this is the lack of a single powerful tool. Non linear dynamical systems are having particular interest as they are highly dynamic in nature. That is they very much depends upon the initial conditions of the system. If we make the slightest error in measuring the state, we will end up in larger error in the approximation of the future. That is the system is highly sensitive to its initial conditions. For such systems the analysis is a painful one. For such systems, we can use the tool CRP with greater accuracy and easiness. Also due to inherent nature of pleasing presentation properties, these plots are easily understandable and presentation friendly.

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