An EPQ model with Quality, Reliability, Flexibility and Package cost in a Fuzzy environment

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Abstract

Today the world is at the risk of environmental disarray. The main reason is population explosion. The result of the uncontrollable population forces us to maintain the inventory of all products at a high level. Also the manufacturers handle various tactics to produce products of high quality and to make it fit for the taste of the customers. To make the rate of production fast, the machines are subjected to continuous wear and tear which results in depreciation. To retain the confidence of the buyer the manufacturers have to maintain high degree of reliability and flexibility and the suppliers seek logistics and packaging for the delivery of the goods to the customers at the right time and in the right manner respectively. Fuzzy concepts are applied to formulate the Economic Production Quantity (EPQ) model which facilitates the manufacturers to face risk present in the production sector. packagingcost is also considered here.

Keywords

Fuzzy triangular number, Quality, Reliability, Flexibility, Inventory, Packaging, Social cost

1.INTRODUCTION

The Economic Production Quantity (EPQ) model has been extensively used in inventory management for a long time. In these days, the manufacturers have started to give significance to quality management. The manufacturers handle various tactics to produce products of high quality and to make it fit for the taste of the customers. The manufacturers face internal and external challenges and they require efficient machinery and marketing. The ultimate target of the manufacturer is promoting the production of the product. Many techniques are available which are helpful to the manufacturers to make their product reach the customers in desirable manner. It is commonly observed that the manufacturers have started to give significance to quality management. Promoting the production of the product serves as the ultimate target of. To make the rate of production fast, the machines are subjected to continuous wear and tear which results in depreciation. To withstand these circumstances the manufacturers have to maintain high degree of reliability and flexibility. This paper elucidates about packaging, comprises the formulation of an EPQ model with the inclusion of package costand discusses the way and means of reducing the social cost from packaging.

1.1 Packaging

Packaging is the science, art, technology and a coordinated system of enclosing or protecting products for distribution. The benefits of packing are as follows.

- to provide protection from physical damage, contamination and deterioration;
- to give sales appeal;
- to ensure the product identity in a easily recognizable manner;
- to give information about the product;
- to optimize distribution and storage costs;
- to provide consumer convenience and safety.

Thus packaging preserves the products, reserves the decorum of the producers and sustains the consumption of the customers. An extensive range of materials are used for packaging, that includes metal, glass, wood, paper or pulp-based materials, plastics or combination of more than one material as composites.
There are three broad categories of packaging:
-Primary packaging, which is normally in contact with the goods and taken home by consumers.
-Secondary packaging, which covers the larger packaging such as boxes, used to carry quantities of primary packaged goods.
-Tertiary packaging, which refers to the packaging that is used to assist transport of large quantities of goods, such as wooden pallets and plastic wrapping.

Therefore tertiary packaging is employed at times, where the items are carried between the vendor and the buyer. As technology develops, the packaging industry also makes use of modern techniques. Fuzzy EPQ model is developed by including transportation cost and packaging cost along with various costs in fuzzy nature as developed in Fuzzy Quality control with Reliability and Flexibility by P.Parvathi and P.Anitha[10]. We have taken demand, production quantity, ordering cost, holding cost, labour cost and material cost as triangular fuzzy number. We defuzzifythe above cost using signed distance method.

2. LITERATURE REVIEW

The generalized form of the Economic Order Quantity by Morteza.et.al.,[1] isthe oldest and most useful in production and inventory management. The EPQ model has been the basis for building more complex and realistic production models by Ali Yassine.et.al., [2]. The production inventory problem has been investigated in recent year, which increasing number of researchers to analyze machine breakdown effect in production inventory problem. The effects of machine break down and corrective maintenance on the economic lot sizing were studied by Gedeal.et.al.,[3]. The Mathematical model with random machine break downs, preventive maintenance and safety stock were developed by Cheung and Hausman, [4].Many scholars have modified the classical EPQ models by introducing new costs like shortage cost and backorder cost. In addition to the purchasing cost, setup cost, holding costs which are the components of the classical EPQ model.EQP model with the inclusion of shortage cost was developed by Cardenas-Barron, 2001 [5] and then Ronald, 2004 [6] derived shortage model with and without derivatives by Jason Changa, 2005[7] examined the concept of backorders.Teunter and Dekker, 2008 [8] put forward the EPQ model with backorders in a simple way. The items that are transported, must reach the buyers in good condition, for which packaging is essential. The cost of packaging is included along with the Enviro-EQP model byJaber, 2011[9].

3. PRELIMINARY CONCEPTS

3.1Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function μ_A satisfied the following conditions, is a generalized fuzzy number $\tilde{A}$.

(i) μ_A is a continuous mapping from R to the closed interval [0, 1],

(ii) $\mu_A = 0$, $-\infty < x \le a_1$

(iii) $\mu_A = L(x)$ is strictly increasing on $[a_1, a_2]$

(iv) $\mu_A = w_A$, $a_2 \le x \le a_3$

(v) $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$

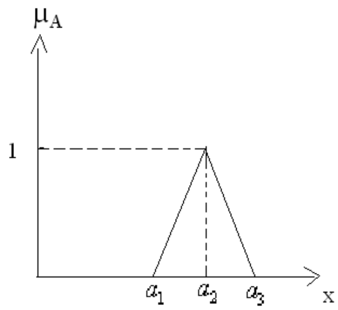
(vi) $\mu_A = 0$, $a_4 \le x \le \infty$

Where $0 < w_A \le 1$ and a_1, a_2, a_3 and a_4 are real numbers. Also this type of generalized fuzzy number be denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$; when $w_A = 1$, it can be simplified as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)_{LR}$.

3.2 Triangular fuzzy number

The fuzzy set $\tilde{A} = (a_1, a_2, a_3)$ where $a_1 < a_2 < a_3$ and defined on R, is called the triangular fuzzy number, if the membership function of $\tilde{A}$ is given by

$$\mu_A = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$$



3.3 The Function Principle

The function principle was introduced by Chen [6] to treat fuzzy arithmetical operations. This principle is used for addition, subtraction, multiplication and division of fuzzy numbers.

Suppose $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers. Then

(i) The addition of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(ii) The multiplication of $\tilde{A}$ and $\tilde{B}$ is $\tilde{A} \times \tilde{B} = (c_1, c_2, c_3)$

where $T = (a_1b_1, a_1b_3, a_3b_1, a_3b_3), c_1 = \min T, c_2 = a_2b_2, c_3 = \max T$ if a_1, a_2, a_3, b_1, b_2, b_3 are all non zero positive real numbers, then $\tilde{A} \times \tilde{B} = (a_1b_1, a_2b_2, a_3b_3)$.

(iii) $-\tilde{B} = (-b_3, -b_2, -b_1)$ then the subtraction of $\tilde{B}$ from $\tilde{A}$ is $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$ where a_1, a_2, a_3, b_1, b_2, b_3 are any real numbers.

(iv) $\frac{1}{\tilde{B}} = \tilde{B}^{-1} = (\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1})$ where b_1, b_2, b_3 are all non-zero positive real numbers, then the division of $\tilde{A}$ and $\tilde{B}$ is $\frac{\tilde{A}}{\tilde{B}} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$

(v) For any real number K, $\begin{matrix} K\tilde{A} = (Ka_1, Ka_2, Ka_3) if K > 0 \\ K\tilde{A} = (Ka_3, Ka_2, Ka_1) if K < 0 \end{matrix}$

3.4. Signed Distance Method

Defuzzification of $\tilde{A}$ can be found by signed distance method. If $\tilde{A}$ is a triangular fuzzy number and is fullydetermined by (a_1, a_2, a_3), the signed distance from $\tilde{A}$ to 0 is defined as

$$d(\tilde{A},\tilde{0}) = \int_0^1 d\left([A_L(\alpha), A_R(\alpha)],\tilde{0}\right)d\alpha = \frac{(a_1 + 4a_2 + a_3)}{4}$$

4. NOTATIONS AND ASSUMPTIONS

The following notations and assumptions have been used to develop the fuzzy mathematical model.

4.1 Notations

$\tilde{D}$	Fuzzy demand per unit of time
P	Production per unit of time
β	The fraction of time the production process spends
1- β	The fraction of time the production process spends actually idling
$\tilde{Q}$	Fuzzy economic production Quantity
Q	Optimal production Quantity
$\tilde{A}$	Fuzzy fixed ordering cost per production run
$\tilde{h}$	Fuzzy holding cost per unit per unit of time in rupees.
C	Unit production cost
p_i	Depreciationcost of machine
	m_i =(i=1,....n) in percentage
R_i	Cost of the machine m_i=(i=1,....n) in rupees
f_i	Maintenance cost of machine m_i(i=1,....n) in rupees
M_i	Volume of investments in marketing
T	Cycle length
$\tilde{L}$	Fuzzy labour cost for packing per parcel in rupees
$\tilde{K}$	Fuzzy material cost used for packing per parcel in rupees
a	Fixed cost per trip
b	Variable cost per unit transported per distance travelled

d	Distance travelled
α	Proportion of demand returned 0<α<1
μ	Social cost from vehicle emission
v	Average Velocity(km/hr)
γ	Cost to dispose waste to the environment
θ	Proportion of waste produced per lot Q
r	Number of parcels

4.2 Assumptions

1. Production rate is constant.
2. Replenishment is instantaneous.
3. No excess stock is carried and no shortage is allowed. Production is always greater than demand.
4. Depreciation is the function of use.
5. The units that are transported are finally packed in parcels, packaging cost is incurred for each parcel.
6. The packaging cost per parcel includes the labour cost and the material cost used for packaging.
7. The demand, ordering cost, proportion of the demand returned are constant per cycle.
8. The proportion of waste produced per lot Q incorporates the waste produced due to the disposal of the material used for packaging.

5.MODEL DEVELOPMENT

The EPQ cost per unit of time

$$C(\tilde{Q}) = A\frac{\tilde{D}}{\tilde{Q}} + \frac{\tilde{h}\tilde{Q}(1-\beta)}{2}$$

The Production cost per cycle

$$C_p(\tilde{Q}) = C\tilde{Q}$$

5.1 The Depreciation cost per cycle

Suppose that the manufacturers owns machine $m_i(i=1,....n)$ to execute sequentially the various tasks involved in the production process. It is quite naturals for the machine to get degraded due to continuous wear and tear, so the value of the machines gets depreciated .The proportion (p_i) of costs of machine $m_i(i=1,....n)$ depreciated is determined by various methods and it varies from one machine to another. Therefore the depreciated cost per cycle is as follows.

$$C_{\tilde{D}}(\tilde{Q}) = \sum_{i=1}^{n} p_i R_i = E \ \text{(say)}$$

5.2 The Maintenance cost per cycle

To maintain the degree of reliability and to prevent machine break down the machine have to be properly taken care. Since each machine serves different purposes the pattern of its maintenance also differs. Therefore the maintenance per cycle is as follows.

$$C_M(\tilde{Q}) = \sum_{i=1}^{n} f_i = F \ \text{(say)}$$

5.3 The Marketing cost per cycle

To propagate the product in the marker rapidly the manufacturers may prefer one or more marketing strategies, (i.e) for example the manufacturer may advertise his products via two modes viz. Newspaper and Television for which separate cost are incurred. Therefore the marketing cost per cycle is as follows

$$C_A(\tilde{Q}) = \sum_{i=1}^{k} M_i = S \ \text{(say)}$$

Transportation cost per cycle (delivery and collection of returned items)

$$C_t(\tilde{Q}) = 2a + bd\tilde{Q} + bd\alpha\tilde{Q}$$

Emission cost from transportation and package per cycle

$$C_e(\tilde{Q}) = 2\mu\frac{d}{v}$$

Cost incurred due to the waste produced by the inventory system per cycle

$$C_w(\tilde{Q}) = \gamma_0 + \gamma\tilde{Q}(\theta+\alpha)$$

Packaging cost for per parcel includes both the labour costs and the material costs. Packaging cost per parcel = $\tilde{L}+\tilde{K}$

The total cost of packaging per cycle

$$C_p(\tilde{Q})=(\tilde{L}+\tilde{K})r$$

Total cost per unit of time $\psi(\tilde{Q}) = C(\tilde{Q}) + \dfrac{C_p(\tilde{Q})+C_D(\tilde{Q})+C_M(\tilde{Q})+C_A(\tilde{Q})+C_t(\tilde{Q})+C_e(\tilde{Q})+C_w(\tilde{Q})+C_p(\tilde{Q})}{T}$

$$= \tilde{A}\frac{\tilde{D}}{\tilde{Q}} + \frac{\tilde{h}\tilde{Q}(1-\beta)}{2} + C\tilde{D} + \frac{\tilde{D}}{\tilde{Q}}(E+F+S) + 2a\frac{\tilde{D}}{\tilde{Q}} + bd\tilde{D} + bd\alpha\tilde{D} + 2\frac{\mu d\tilde{D}}{v\tilde{Q}} +$$
$$\gamma_0\frac{\tilde{D}}{\tilde{Q}} + \gamma\tilde{D}(\theta+\alpha) + (\tilde{L}+\tilde{K})\frac{r\tilde{D}}{\tilde{Q}}$$

$$\frac{\partial\psi}{\partial\tilde{Q}} = \tilde{A}(\frac{-\tilde{D}}{\tilde{Q}^2}) + \frac{\tilde{h}(1-\beta)}{2} + 0 - \frac{\tilde{D}}{\tilde{Q}^2}(E+F+S) - 2a\frac{\tilde{D}}{\tilde{Q}^2} +$$
$$0 + 0 - 2\frac{\mu d\tilde{D}}{v\tilde{Q}^2} - \frac{\gamma_0\tilde{D}}{\tilde{Q}^2} + 0 - \frac{(\tilde{L}+\tilde{K})r\tilde{D}}{\tilde{Q}^2}$$

$$\frac{\partial^2\psi}{\partial\tilde{Q}^2} = \frac{2\tilde{A}\tilde{D}}{\tilde{Q}^3} + 0 + 0 + \frac{2\tilde{D}}{\tilde{Q}^3}(E+F+S) + \frac{4a\tilde{D}}{\tilde{Q}^3} + \frac{4\mu d\tilde{D}}{v\tilde{Q}^3} + \frac{2\gamma_0\tilde{D}}{\tilde{Q}^3} + \frac{2(\tilde{L}+\tilde{K})r\tilde{D}}{\tilde{Q}^3} > 0$$

$\therefore \psi(\tilde{Q})$ is convex with respect to $\tilde{Q}$

By equating $\dfrac{\partial\psi}{\partial\tilde{Q}}$ to zero we find the optimal value of $\tilde{Q}$ as

$$\tilde{Q} = \frac{2D[A+E+F+S+2a+\frac{[illegible]}{v}+\gamma_0+(\tilde{L}+\tilde{K})r]}{\tilde{h}(1-\beta)}$$

6. NUMERICAL ILLUSTRATIONS

Example1: To illustrate the model a particular EPQ problem is taken,

n=3,k=2,R_1 = Rs.10, 000, R_2=Rs.20,000, R_3=Rs.30,000, p_1=1%, p_2=2%,p_3=1%,

f_1=Rs.500/yr, f_2=Rs. 1000/yr, f_3=Rs.1500/yr,M_1=Rs.200/yr , M_2= Rs.2000/yr,

$\tilde{A}$= (200,300,400), $\tilde{h}$=(1,1.50,2), $\tilde{D}$= (500,1000,1500),

a=Rs.50, v=180 ,d=250 Km, μ=0.5,γ_0=0.5,$\tilde{L}$=(Rs10, Rs.15,Rs.20), $\tilde{K}$=(10,20,30)

r=1,β=0.1 . The optimal value of Q =3128.

7 .SENSITIVITY ANALYSIS

A sensitivity analysis is performed to study the effects of βin $Q$by keeping all the values fixed as in Example1 except β. By varying the values $of\ \beta$ we can see that the production quantity increases. The results are shown in Table 1. So by maintaining the machines in good condition we can produce more quantity.

Table 1 : Variation in Q with β

β	Q
0.1	3128
0.2	3318
0.3	3547
0.4	3831
0.5	4197
0.6	4692
0.7	5418
0.8	6636
0.9	9384

We can also represent this situation pictorially (Fig.1).

Q

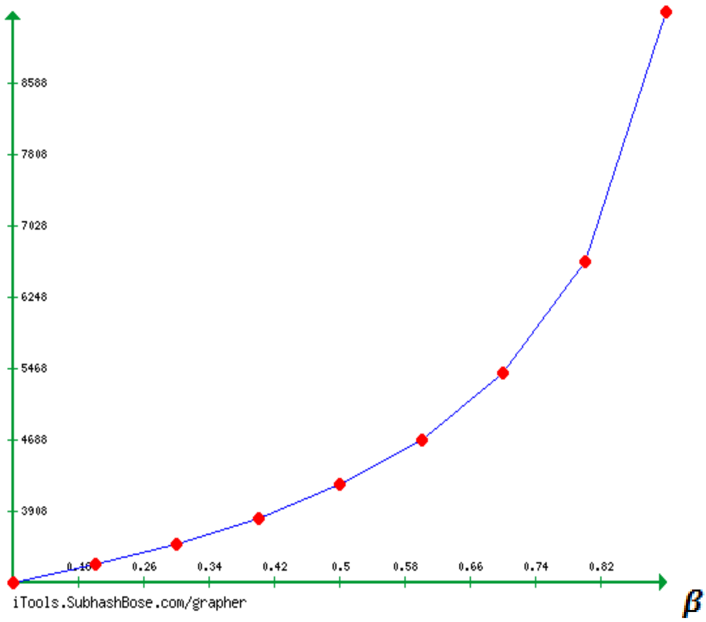


Fig.1

Table 2 is obtained by keeping all the values fixed as in Example 1, except p_1, we can find the changes in Q. Here the depreciation of the Machine 1 Varies. Increase in $p_1$means that the machine 1 works more time than in earlier case and hence the quantity produced is more.

Table 2: Variation in Q with p_1

p_1 (%)	p_2 (%)	p_3 (%)	Q
1	2	1	3128
2	2	1	3152
3	2	1	3175
4	2	1	3198
5	2	1	3221
6	2	1	3244

Table 3 is obtained by keeping all the values fixed as in Example 1, except p_2, we can find the changes in Q. Here the depreciation of the Machine 2 Varies. Increase in $p_2$means that the machine 2 works more time than in earlier case and hence the quantity produced is more.

Table 3: Variation in $Q$with p_2

p_1 (%)	p_2 (%)	p_3 (%)	Q
1	2	1	3128
1	3	1	3175
1	4	1	3221
1	5	1	3267
1	6	1	3312
1	7	1	3357

Table 4 is obtained by keeping all the values fixed as in Example 1, exceptp_3, we can find the changes in Q. Here the depreciation of the Machine 3 Varies.Increase in $p_3$means that the machine 3 works more time than in earlier case and hence the quantity produced is more.

Table 4: Variation in Q with p_3

p_1 (%)	p_2 (%)	p_3 (%)	Q
1	2	1	3128
1	2	2	3152
1	2	3	3175
1	2	4	3198
1	2	5	3221
1	2	6	3244
1	2	7	3267

8. CONCLUSIONS

This model is highly beneficial to the manufactures as the EPQ model is focused on quality, reliability and flexibility.Increase in depreciation of a machine means that the machine works more time than in earlier case and hence the quantity produced is more. If the working time of the machine is larger, then the quantity produced will also be higher. So the manufacturers have to maintain high degree of reliability and flexibility of the machines .The suppliers seek logistics and packaging for the delivery of the goods to the customers at the right time and in the right manner to retain the confidence of the buyer. For this reason we include the cost of packaging in the proposed model. The optimal ordering quantity depends on the social cost which is the output of package waste. This model can be extended by increasing the modes of advertisement.

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