On Solving Fuzzy Solid Assignment Problems

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Abstract -***This paper presents solid assignment problem with imprecise costs. Robust's ranking method is adopted for ranking the imprecise data. The fuzzy solid assignment problem has been transformed into crisp one and solved by plane point method. Numerical example is provided to illustrate the approach.***

Key Words:*Fuzzy Solid assignment problem (FSAP), Ranking method, Plane-point method*

1.INTRODUCTION

The solid assignment problem (SAP) is an important augmentation of the assignment problem (AP). SAP has wide applications in, multi-passive-sensor, capital investment, dynamic facility location, satellite launching, time tabling problems and so on. SAP was first proposed by Pierskalla [10]. An algorithm for solving SAP with application to scheduling in a teaching practice was investigated by Frieze and Yadegar [4]. Balas [1] studied an algorithm for the three-index AP. Crama and Spieksma [2] developed approximation algorithms for three dimensional AP with triangle inequalities. Poore [11] discussed the application of multidimensional AP. Magos and Miliotis [7] have introduced a branch and bound procedure for obtaining an optimal solution of planar 3-index assignment problems. Magos [8] introduced a tabu search for the planar three-index AP. Poore and Robertson [12] discussed a new Lagrangean relaxation based algorithm for a class of multidimensional APs. Storms and Spieksma [14] obtained a solution procedure for geometric three-dimensional APs. Kuroki and Matsui [16] discussed anapproximation algorithm for multidimensional AP. Federico [3] discussed an application of multidimensional AP. Pandian [9] proposed a new algorithm for solving SAP.In real life, we frequently deal with vague orimprecise information. Vagueness is usually expressed by intervals or fuzzy numbers. FSAP can arise when uncertainty exists in data problem and decision makers are more comfortable expressing it as fuzzy numbers.In fuzzy decision making, the ranking of fuzzy number plays a vital role. Ranking of fuzzy numbers was first proposed by Jain [6]. Dominance of fuzzy numbers can be explained by many ranking methods of these, Robust's ranking method [15] proposed four indices which may be employed for the purpose of ordering fuzzy quantities in [0,1]. Srinivasan and Geetharamani [13] proposed a new method for solving FAP by using Robust's ranking function.Jahir and Jayaraman [5] proposed an algorithm based on the ranking method for solving fuzzy AP.

In this paper, algorithm for finding an optimum assignment schedule forfuzzy solid assignment problem is proposed and the same is illustrated with the help of numerical example. The proposed algorithm enables the decision makers to evaluate the economical activities and make self-satisfied managerial decisions.

2. PRELIMINARIES

We need the following definitions of fuzzy set, fuzzy number and membership function which can be found in [17].

2.1Definition: Let A be a classical set and $\mu_A(x)$ be a membership function from A to [0,1]. A fuzzy set $\tilde{A}$ with the membership function $\mu_A(x)$ is defined by $\tilde{A}=\{ (x,\mu_A(x)): x\in A \text{ and } \mu_A(x)\in[0,1] \}$.

2.2 Definition: A Fuzzy set $\tilde{A}$ is called positive if its membership function is such that $\mu_{\tilde{A}}(x)=0$ for all $x\le 0$.

2.3 Definition: A Fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number of its member ship function has the following conditions:

(i) $\mu_{\tilde{A}}(x): R\to[0,1]$ is continuous.

(ii) $\mu_{\tilde{A}}(x)=0$ for all $(-\infty,a]\cup[c,\infty)$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on [a,b] and strictly decreasing on [b,c].

(iv) $\mu_{\tilde{A}}(x)=1$ for all $x\in b$ where $a\le b\le c$.

2.4 Definition: A fuzzy number $\tilde{A}$ is denoted as a triangular fuzzy number by (a_1,a_2,a_3) and its membership function $\mu_{\tilde{A}}(x)$ is given as:

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & \text{if} \quad a_1 \le x \le a_2 \\ \dfrac{x-a_3}{a_2-a_3} & \text{if} \quad a_2 \le x \le a_3 \\ 0 & \text{otherwise}\end{cases}$$

2.5 Definition: The α-cut of a fuzzy number $A(x)$ is defined as

$$A(\alpha)=\{x/\mu(x)\ge\alpha, \alpha\in[0,1]\}$$

2.6 Robust's ranking method: The Robust's ranking is defined as $R(\tilde{c})=\int_0^1 0.5(c_\alpha^L, c_\alpha^U)d\alpha$, where (c_α^L, c_α^U) is the α-level cut of the fuzzy number $\tilde{c}$.

Robust's ranking technique satisfies compensation, linearity and additive property which provides results that are consistent with human intuition.

3 FUZZY SOLID ASSIGNMENT PROBLEM

Mathematicallyfuzzy solid assignment problem (FSAP) can be stated as

(P) Minimize $z=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\tilde{c}_{ijk}x_{ijk}$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk}=1,\ i=1,2,\ldots,m \qquad (1)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk}=1,\ j=1,2,\ldots,n \qquad (2)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}=1,\ k=1,2,\ldots,l \qquad (3)$$

$$x_{ijk}=0 \text{ (or) } 1, \text{ for all } i,\ j \text{ and } k \qquad (4)$$

where $\tilde{c}_{ijk}$ is the cost of assigning job j to be performed by men$i$ in factory$k$.

The proposed algorithm for fuzzy solid assignment problem proceeds as follows:

Step 1: Compute the Robust's ranking index for each fuzzy cost $\tilde{C}_{ijk}$ of the given problem (P) by the formula

$$R(\tilde{C})=\int_0^1 (0.5)(c_\alpha^L, c_\alpha^U)d\alpha$$

Step 2: Replace the fuzzy cost $\tilde{C}_{ijk}$ by their respective ranking indices obtained from Step 1.

Step 3: Solve the reduced table obtained from Step 2 using the plane point method[9] to find the optimal assignment schedule.

The solution procedure of obtaining an optimal solution to a FSAP using the proposed algorithm is illustrated by the following example.

Example 3.1 Suppose that there are three men denoted by M_1, M_2 and M_3, three factories denoted by F_1, F_2 and F_3, and three jobs denoted by J_1, J_2 and J_3. Besides, three men, three factories and three jobs can be associated with only one of the others, that is, only one men with only one factory with only one job. The basic aim is to find out an optimum assignment schedule. The following table 1exhibits the cost $\tilde{c}_{ijk}$ which is in the form of fuzzy numbers.

Factories	F_1			F_1			F_1		
		F_2			F_2			F_2	
			F_3			F_3			F_3
Mens ↓/ *Jobs* →	J_1			J_2			J_3		
M_1	$\tilde{c}_{111}$	$\tilde{c}_{112}$	$\tilde{c}_{113}$	$\tilde{c}_{121}$	$\tilde{c}_{122}$	$\tilde{c}_{123}$	$\tilde{c}_{131}$	$\tilde{c}_{132}$	$\tilde{c}_{133}$
M_2	$\tilde{c}_{211}$	$\tilde{c}_{212}$	$\tilde{c}_{213}$	c_{221}	$\tilde{c}_{222}$	$\tilde{c}_{223}$	$\tilde{c}_{231}$	$\tilde{c}_{232}$	$\tilde{c}_{233}$
M_3	$\tilde{c}_{311}$	$\tilde{c}_{312}$	$\tilde{c}_{313}$	$\tilde{c}_{321}$	$\tilde{c}_{322}$	$\tilde{c}_{323}$	$\tilde{c}_{331}$	$\tilde{c}_{332}$	$\tilde{c}_{333}$

Table -1: Assignment cost $\tilde{c}_{ijk}$

Where
$\tilde{C}_{111}=(10,12,14)$; $\tilde{C}_{112}$=(6,10,14);
$\tilde{C}_{113}$=(12,14,16); $\tilde{C}_{121}$=(10,11,12);
$\tilde{C}_{122}$=(10,12,14); $\tilde{C}_{122}$=(27,29,31);
$\tilde{C}_{131}$=(15,17,19); $\tilde{C}_{132}$=(10,12,14);
$\tilde{C}_{133}$=(14,15,16); $\tilde{C}_{211}$=(8,10,12);
$\tilde{C}_{211}$=(4,8,12); $\tilde{C}_{213}$=(7,9,11); $\tilde{C}_{221}$=(9,11,13);
$\tilde{C}_{222}$=(6,8,10); $\tilde{C}_{223}$=(12,14,16); $\tilde{C}_{231}$=(8,9,10);
$\tilde{C}_{231}$=(8,9,10); $\tilde{C}_{232}$=(11,13,15); $\tilde{C}_{233}$=(13,14,15); $\tilde{C}_{311}$=(9, 11,13); $\tilde{C}_{312}$=(7,9,11); $\tilde{C}_{313}$=(6,8,10);
$\tilde{C}_{321}$=(8,12,16); $\tilde{C}_{322}$=(7,9,11); $\tilde{C}_{323}$=(12,14,16); $\tilde{C}_{331}$=(9,1 0,11); $\tilde{C}_{332}$=(7,8,9); $\tilde{C}_{333}$=(8,10,12)

Now, using Step 1, the following ranking indices for the costs $\tilde{C}_{ijk}$ is obtained:

Table -2: Reduction table after ranking
$R(\tilde{C}_{111})=12$, $R(\tilde{C}_{112})=10$, $R(\tilde{C}_{113})=14$,
$R(\tilde{C}_{121})$ = 11, $R(\tilde{C}_{122})$ = 12, $R(\tilde{C}_{123})$ = 29,
$R(\tilde{C}_{131})$ = 17, $R(\tilde{C}_{132})$ = 12, $R(\tilde{C}_{133})$ = 15,
$R(\tilde{C}_{211})$ = 10, $R(\tilde{C}_{212})$ = 8, $R(\tilde{C}_{213})$ = 9,
$R(\tilde{C}_{221})$ = 11, $R(\tilde{C}_{222})$ = 8, $R(\tilde{C}_{223})$ = 14,
$R(\tilde{C}_{231})$ =9, $R(\tilde{C}_{232})$ = 13, $R(\tilde{C}_{233})$ = 14,
$R(\tilde{C}_{311})$ =11, $R(\tilde{C}_{312})$ =9, $R(\tilde{C}_{313})$ =8,
$R(\tilde{C}_{321})$ = 12, $R(\tilde{C}_{322})$ = 9, $R(\tilde{C}_{323})$ = 14,
$R(\tilde{C}_{331})$ =10, $R(\tilde{C}_{332})$ =8, $R(\tilde{C}_{333})$ = 10

Now, using Step 2 the Robust's indices for the costs $\tilde{C}_{ijk}$ corresponding to the given fuzzy solid assignment problem is given in table 2:

Factories	F_1			F_1			F_1		
		F_2			F_2			F_2	
			F_3			F_3			F_3
$Mens \downarrow / Jobs \rightarrow$	J_1			J_2			J_3		
M_1	12	10	14	11	12	29	17	12	15
M_2	10	8	9	11	8	14	9	13	14
M_3	11	9	8	12	9	14	10	8	10

Now, using the plane point method [9], the optimal solution to the above reduced problem is $x_{121}=x_{213}=x_{332}=1, x_{ijk}=0$ which produces an objective function value equal to 28.

Therefore an optimal solution for the given fuzzy SAP is $\tilde{C}_{121}$ = (10,11,12), $\tilde{C}_{213}$ = (7,9,11) , $\tilde{C}_{332}$ = (7,8,9) with the fuzzy objective value equal to (24,28,32).

Thus optimal associations to the fuzzy solid assignment problem are:

- Men 1, Job 2 and Factory 1.
- Men 2, Job 1 and Factory 3.
- Men 3, Job 3 and Factory 2.

4. CONCLUSION

In this paper, we consider the three dimensional assignment problems with uncertain data. The solution procedure of the proposed algorithm is illustrated with help of a real life example. This method helps the decision-makers to choose an appropriate decision while handling various types of three dimensional assignment problems in real life situations.

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