THE TORROIDAL PLASMA CURRENT PROFILE CONTROL IN DIII-D TOKAMAK

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Abstract-***Tokamak is a fusion reactor which actually is a transformer, the power production through nuclear fusion can be realized through this massive device having primary windings and the fuel kept inside it in the plasma state as secondary windings. Since the performance of the device is characterized by good plasma confinement and high magnetohydrodynamic stability, for advanced operating scenarios, control of the spatial profile of plasma current will be essential. A control-oriented model of the current profile evolution in (L-mode) discharges in the DIII-D tokamak is employed for regulating the current profile evolution around desired trajectories. For addressing the current profile evolution we use the well-known magnetic diffusion equation, motivating the design of a boundary feedback control law for better tracking of plasma current, power and density profiles. The proposed scheme uses a backstepping control design technique to obtain a boundary feedback control law through the transformation of a spatially discretized version of the original system into an asymptotically stable target system. A simulated annealing algorithm optimizes the controller constants which improves the system performance further. To analyze the performance of the proposed approach, computer simulations are carried out to illustrate the ability of the controller to track the reference profiles and to improve the system performance.***

Key Words*- MHD stability, backstepping boundary control, plasma, Tokamaks.*

1. INTRODUCTION

Nuclear fusion is, in a sense, the opposite of nuclear fission which produces energy through the splitting of heavy atoms like uranium in controlled chain reactions. Unfortunately the by-products of fission are highly radioactive and long lasting. In contrast, fusion is the process by which the nuclei of two light atoms such as hydrogen are fused together to form a heavier nucleus(helium), with huge amount of energy produced as a by-product unlike nuclear fission, there is no risk of a runaway nuclear reaction, and no generation of high-level nuclear waste. The amount of released energy is given by Einstein's equation, E = $(M_r - M_p)c^2$ where E is the energy, M_r the mass of the reactant nuclei, Mp the mass of the product nuclei and c the speed of light. Out of the energy released twenty percent will be used for sustaining fusion and the rest will be available for the generation of electricity. Of these fuels deuterium can be extracted from sea water and tritium can be bred from lithium.Tokamaks are magnetic confinement devices, which means that magnetic fields produced by currents in large coils are used to confine the plasma kept inside the device. The device is having D shaped torroidal field coils which generates torroidal component of magnetic field and the poloidal field coils generates poloidal magnetic field, the resultant of these two magnetic fields creates a helical magnetic field. The poloidal field serves the function of shaping and positioning of plasma as shown in fig (1).

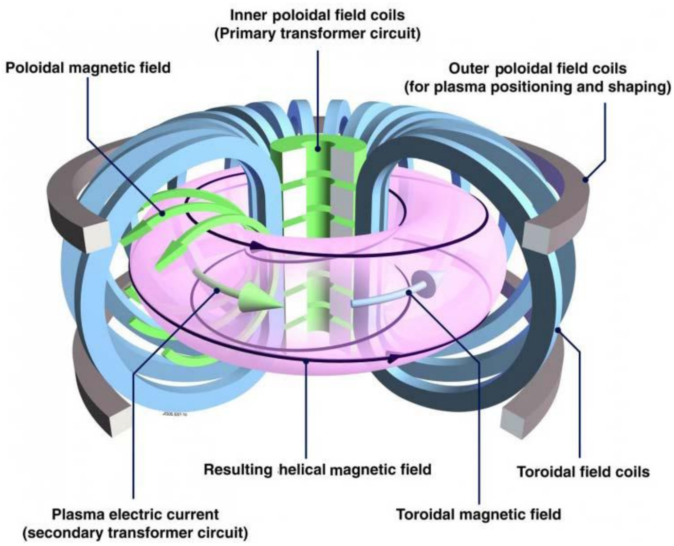

Fig-1: Internal diagram of DIII-D tokamak

Tokamak being a huge transformer having plasma as its secondary circuit, most of the plasma current needed for confinement is achieved through inductive means but plasma current generated through induction cannot be sustained for extended periods of time and steady-state tokamak operation will require the plasma current to be injected to plasma by noninductive methods.Since plasmaand its associated current is operating under extreme temperatures and is subjected to high non-linearities and instabilities it is taken as the field of study, and one of the major challenges is to sustain fusion reaction under long plasma discharges. The evolution in time of the current profile is related to the evolution of the spatial derivative of the poloidal flux profile using the well-known magnetic diffusion equation which is a (PDE) defined in normalized cylindrical coordinates. The poloidal flux ψ at a point P is the total flux through the surface S bounded by the toroidal ring passing through P, i.e., $\psi = \int B_{p01}\, dS$ as shown in fig (2). Where B_{p01} is the poloidal component of the magnetic field.

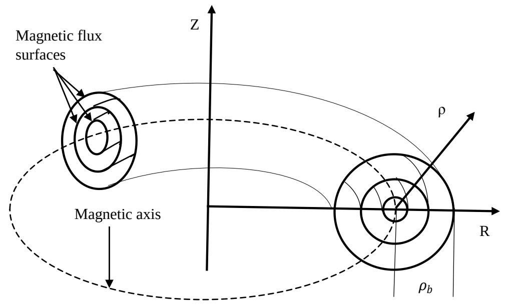

Fig-2: Plasma coordinate system

We can control the current profile by controlling the poloidal flux profile, the dynamics of the plasma poloidal flux profile can be modified by three actuators: the total plasma current, NBI power, and average plasma density.

2. PLASMA CURRENT PROFILE EVOLUTION MODEL

Let 'ρ' be an arbitrary coordinate which represents the magnetic surfaces. The spatial index 'ρ' is later to be replaced with the normalized variable $\hat{\rho} = \rho / \rho_b$, where ρ_b is the minor radius. The torroidal magnetic flux can be mathematically expressed as in equations (1):

$$\pi B_{\Phi,0}\rho^2 = \Phi \qquad (1)$$

Where 'Φ' is the torroidal magnetic flux and $B_{\Phi,0}$ is the reference magnetic field at the geometric major radius R_0 of the tokamak. Another important parameter is its safety factor:

$$q(\hat{\rho}, t) = -\frac{B_{\phi,0}\rho_b^2\hat{\rho}}{\partial\psi/\partial\hat{\rho}} \qquad (2)$$

Now define poloidal flux gradient $\theta(\hat{\rho}, t)$ as:

$$\theta(\hat{\rho}, t) = \frac{\partial\psi}{\partial\hat{\rho}}(\hat{\rho}, t) \qquad (3)$$

The poloidal magnetic flux evolution can be defined using the magnetic diffusion equation defined as:

$$\frac{d\psi}{dt} = \frac{\eta(T_e)}{\mu_0\rho_b^2\hat{F}^2}\frac{1}{\hat{\rho}}\frac{\partial}{\partial\hat{\rho}}\left(\hat{\rho}\hat{F}\,\hat{G}\hat{H}\frac{\partial\psi}{\partial\hat{\rho}}\right) + R_0\hat{H}\,\eta(T_e)\frac{\langle \bar{J}_{NI}.\bar{B}\rangle}{B_{\phi,0}} \qquad (4)$$

Where ψ represents the poloidal magnetic flux, 't' is time, and $\eta$is the plasma resistivity dependent on the electron temperature,T_e, μ_0 is the vacuum permeability, $\overline{J_{NI}}$ is the noninductive current density from neutral beam injection (NBI), $\bar{B}$is the toroidal magnetic field, and <> denotes the flux-surface average of a quantity. $\hat{F}$, $\hat{G}$ and$\hat{H}$are spatially varying geometric factors of the DIII-D tokamak which are considered constant in this model. The boundary conditions for magnetic diffusion equation is given as:

$$\frac{\partial\psi}{\partial\hat{\rho}}\Big|_{\hat{\rho}=0} = 0,\ \frac{\partial\psi}{\partial\hat{\rho}}\Big|_{\hat{\rho}=1} = -\frac{\mu_0}{2\pi}\frac{R_0}{\hat{G}|_{\hat{\rho}=1}\hat{H}|_{\hat{\rho}=1}}I(t) \qquad (5)$$

The model for the noninductive toroidal current density is:

$$\frac{\langle \bar{J}_{NI}.\bar{B}\rangle}{B_{\phi,0}} = k_{NI}\,j_{NI}^{profile}(\hat{\rho})\frac{I(t)^{1/2}P_{tot}(t)^{5/4}}{\bar{n}(t)^{3/2}} \qquad (6)$$

Where k_{NI} is a constant and $j_{NI}^{profile}(\hat{\rho})$is a reference profile for the noninductive current deposition. The model for the electron temperature is given by

$$T_e(\hat{\rho}, t) = k_{T_e}T_e^{profile}(\hat{\rho})\frac{I(t)\sqrt{P_{tot}(t)}}{\bar{n}(t)} \qquad (7)$$

Where $k_{T_e}$is a constant, $T_e^{profile}(\hat{\rho})$is a reference profile, $P_{tot}(t)$ is the total average NBI power, and $\bar{n}(t)$is the line averaged plasma density. The plasma resistivity $\eta\ (T_e)$ is given by:

$$\eta(\hat{\rho}, t) = \frac{k_{eff}Z_{eff}}{T_e^{3/2}(\hat{\rho}, t)} \qquad (8)$$

Where, k_{eff} is a constant. The effective atomic number of the plasma, Z_{eff} , is considered to be constant in this model. Equation (4) can be modified using models (6) to (8) as:

$$\frac{\partial\psi}{\partial t} = f_1(\hat{\rho})g_1(t)\frac{1}{\hat{\rho}}\frac{\partial}{\partial\hat{\rho}}\left(\hat{\rho}f_4(\hat{\rho})\frac{\partial\psi}{\partial\hat{\rho}}\right)f_2(\hat{\rho})g_2(t) \qquad (9)$$

With boundary conditions given by

$$\frac{\partial\psi}{\partial\hat{\rho}}\Big|_{\hat{\rho}=0} = 0,\ \frac{\partial\psi}{\partial\hat{\rho}}\Big|_{\hat{\rho}=1} = -k_3u(t) \qquad (10)$$

Where

$$f_1(\hat{\rho}) = \frac{k_{eff}Z_{eff}}{k_{T_e}^{3/2}\mu_0\rho_b^2\hat{F}^2(\hat{\rho})(T_e^{profile}(\hat{\rho}))^{3/2}} \qquad (11)$$

$$f_2(\hat{\rho}) = \frac{k_{eff} Z_{eff} R_0 k_{NI} \bar{H}(\hat{\rho}) j_{NI}^{profile}(\hat{\rho})}{k_{T_e}^{3/2} (T_e^{profile}(\hat{\rho}))^{3/2}} \quad (12)$$

$$f_4(\hat{\rho}) = \hat{F}(\hat{\rho})\hat{G}(\hat{\rho})\hat{H}(\hat{\rho}),\ k_3 = \frac{\mu_0}{2\pi}\frac{R_0}{\hat{G}|_{\hat{\rho}=1}\hat{H}|_{\hat{\rho}=1}} \quad (13)$$

$$g_1(t) = \left(\frac{\bar{n}(t)}{I(t)\sqrt{P_{tot}(t)}}\right)^{3/2},\ g_2(t) = \frac{\sqrt{P_{tot}(t)}}{I(t)},\ u(t) = I(t) \quad (14)$$

Boundary actuators g_1, g_2 and u represents non-linear combination of physical actuators: $I(t)$,$P_{tot}(t)$, and $\bar{n}(t)$.

Apply chain rule to (9) which results in:

$$\frac{\partial \psi}{\partial t} = f_1 g_1(t) \frac{1}{\hat{\rho}}\left(\hat{\rho}\frac{\partial \psi}{\partial \hat{\rho}}\frac{\partial f_4}{\partial \hat{\rho}} + f_4 \frac{\partial \psi}{\partial \hat{\rho}} + \hat{\rho} f_4 \frac{\partial^2 \psi}{\partial \hat{\rho}^2}\right) + f_2 g_2(t) \quad (15)$$

Substituting (3) into (15) resulting in:

$$\frac{\partial \psi}{\partial t} = f_1 g_1 \frac{1}{\hat{\rho}}(\hat{\rho}\theta f_4' + f_4\theta + \hat{\rho} f_4 \theta') + f_2 g_2 \quad (16)$$

Differentiating (16) with respect to $\hat{\rho}$, the PDE for the evolution of $\theta(\hat{\rho}, t)$ will become:

$$\frac{\partial \theta}{\partial t} = h_0 g_1 \theta'' + h_1 g_1 \theta' + h_2 g_1 \theta + h_3 g_2 \quad (17)$$

With its boundary conditions modified as:

$$\theta|_{\hat{\rho}=1} = 0,\quad \theta|_{\hat{\rho}=1} = -k_3 u \quad (18)$$

Where spatially varying functions h_0,h_1, h_2 and h_3 are given by

$$h_0 = f_1 f_4 \quad (19)$$

$$h_1 = f_1' f_4 + f_1 f_4 \frac{1}{\hat{\rho}} + 2 f_1 f_4' \quad (20)$$

$$h_2 = f_1' f_4' + f_1' f_4 \frac{1}{\hat{\rho}} + f_1 f_4' \frac{1}{\hat{\rho}} - f_1 f_4 \frac{1}{\hat{\rho}^2} + f_1 f_4'' \quad (21)$$

$$h_3 = f_2' \quad (22)$$

3. OBJECTIVE

Let $u_{ff}(t)$ be a set of feedforward control input trajectories and θ_{ff} be the associated poloidal flux gradient profile evolution for a nominal initial condition $\theta_{ff}(\hat{\rho}, 0)$, correspondingly (17), (18) can be modified as:

$$\frac{\partial \theta_{ff}}{\partial t} = g_{1ff}(h_0\theta_{ff}'' + h_1\theta_{ff}' + h_2\theta_{ff}) + h_3 g_{2ff} \quad (23)$$

$$\theta_{ff}|_{\hat{\rho}=0} = 0,\quad \theta_{ff}|_{\hat{\rho}=1} = -k_3 u_{ff} \quad (24)$$

Errors in initial conditions or other disturbances deviates the actual state from the desired, considering errors also we can write:

$$\frac{\partial(\theta_{ff} + \tilde{\theta})}{\partial t} = g_{1ff}[h_0(\theta_{ff}'' + \tilde{\theta}'') + h_1(\theta_{ff}' + \tilde{\theta}') + h_2(\theta_{ff} + \tilde{\theta})] + h_3 g_{2ff} \quad (25)$$

$$(\theta_{ff} + \tilde{\theta})|_{\hat{\rho}=0} = 0,\ (\theta_{ff} + \tilde{\theta})|_{\hat{\rho}=1} = -k_3(u_{ff} + u_{fb}) \quad (26)$$

The feedforward inputs are calculated offline so they can't compensate for these errors and so we design a boundary feedback control law u_{fb}, noting (23) and (24), (25) and (26) can be reduced to:

$$\frac{\partial \tilde{\theta}}{\partial t} = h_0 g_{1ff}\tilde{\theta}'' + h_1 g_{1ff}\tilde{\theta}' + h_2 g_{1ff}\tilde{\theta} \quad (27)$$

$$\tilde{\theta}|_{\hat{\rho}=0} = 0,\quad \tilde{\theta}|_{\hat{\rho}=1} = -k_3 u_{fb} \quad (28)$$

The control objective is then to force $\tilde{\theta}$ to zero.

4. DESIGN OF CONTROLLER.

4.1. Backstepping Boundary Controller

Let $h = 1/N$, where N is an integer, and using the notation $x^i(t) = x(ih, t)$, equation (27) can be written as:

$$\dot{\tilde{\theta}} = h_0^i g_{1ff}\frac{\tilde{\theta}^{i+1} - 2\tilde{\theta}^i + \tilde{\theta}^{i-1}}{h^2} + h_1^i g_{1ff}\frac{\tilde{\theta}^{i+1} - \tilde{\theta}^{i-1}}{2h} + h_2^i g_{1ff}\tilde{\theta}^i \quad (29)$$

With boundary conditions:

$$\tilde{\theta}^0 = 0,\quad \tilde{\theta}^N = -k_3 u_{fb} \quad (30)$$

Choose the asymptotically stable target system as:

$$\frac{\partial \tilde{w}}{\partial t} = h_0 g_{1ff}\tilde{w}'' + h_1 g_{1ff}\tilde{w}' + h_2 g_{1ff}\tilde{w} - C_w(\hat{\rho}) g_{1ff}\tilde{w} \quad (31)$$

With boundary conditions

$$\tilde{w}|_{\hat{\rho}=0} = 0,\quad \tilde{w}|_{\hat{\rho}=1} = 0 \quad (32)$$

Where the term $C_w(\hat{\rho})>0$ is chosen such a way to limit the system performance as well as for scaling the boundary actuator limits. The target system (31) & its boundary conditions (32) can be spatially discretized as:

$$\dot{\tilde{w}} = h_0^i g_{1ff} \frac{\tilde{w}^{i+1} - 2\tilde{w}^i + \tilde{w}^{i-1}}{h^2} + h_1^i g_{1ff} \times \frac{\tilde{w}^{i+1} - \tilde{w}^{i-1}}{2h} + h_2^i g_{1ff} \tilde{w}^i - C_w^i g_{1ff} \tilde{w}^i \quad (33)$$

$$\tilde{w}^0 = 0, \quad \tilde{w}^N = 0 \quad (34)$$

Next a backstepping transformation is sought in the form:

$$\tilde{w}^i = \tilde{\theta}^i - \tilde{\alpha}^{i-1}(\tilde{\theta}^0, \ldots, \tilde{\theta}^{i-1}) \quad (35)$$

Subtracting (33) from (29), the expression

$\dot{\alpha}^{i-1} = \dot{\tilde{\theta}}^i - \dot{\tilde{w}}^i$ is obtained in terms of $\alpha^{k-1} = \tilde{\theta}^k - \tilde{w}^k, for\ k = i-1, i, i+1.$

$$\dot{\alpha}^{i-1} = h_0^i g_{1ff} \frac{\tilde{\alpha}^i - 2\alpha^{i-1} + \tilde{\alpha}^{i-2}}{h^2} + h_1^i g_{1ff} \frac{\tilde{\alpha}^i - \tilde{\alpha}^{i-2}}{2h} + h_2^i g_{1ff} \tilde{\alpha}^{i-1} + C_w^i g_{1ff} \tilde{\theta}^i - C_w^i g_{1ff} \alpha^{i-1} \quad (36)$$

Equation (36) can be solved to get α^i as:

$$\alpha^i = -\left[\frac{1}{\frac{h_0^i}{h^2} + \frac{h_1^i}{2h}}\right]\left[\left(\frac{-2h_0^i}{h^2} + h_2^i - C_w^i\right)\alpha^{i-1} + \left(\frac{h_0^i}{h^2} - \frac{h_1^i}{2h}\right)\alpha^{i-2} - \frac{1}{u_{1ff}}\dot{\alpha}^{i-1} + C_w^i \tilde{\theta}^i\right] \quad (37)$$

Where $\alpha^0 = 0$ and $\dot{\alpha}^{i-1}$ is of the form:

$$\dot{\alpha}^{i-1} = \sum_{k=1}^{i-1} \frac{\partial \alpha^{i-1}}{\partial \tilde{\theta}^k} \dot{\tilde{\theta}}^k \quad (38)$$

Now for obtaining backstepping boundary control law subtract (34) from (30) to get:

$$u_{fb} = -\frac{1}{k_3} \alpha^{N-1} \quad (39)$$

Eqn (39) will be a linear combination of N-1 measurements from plasma. For calculating coefficients of this linear combination let us define $\Phi \in \mathrm{R}^{N-1 \times N}$ whose (i+1)th column is coefficients of $\tilde{\theta}$ measurements used for obtaining α^i

$$\alpha^i = \sum_{j=1}^{i} \Phi_{j,i+1} \tilde{\theta}^j \quad (40)$$

Also,

$$\dot{\alpha}^i = \sum_{j=1}^{i} \Phi_{j,i+1} \dot{\tilde{\theta}}^j \quad (41)$$

Substituting for $\dot{\tilde{\theta}}$ in (41) we'll get:

$$\frac{\dot{\alpha}^i}{g_{1ff}} = \sum_{j=1}^{i} \Phi_{j,i+1}\left[h_0^j \frac{\tilde{\theta}^{j+1} - 2\tilde{\theta}^j + \tilde{\theta}^{j-1}}{h^2} + h_1^j \frac{\tilde{\theta}^{j+1} - \tilde{\theta}^{j-1}}{2h} + h_2^j \tilde{\theta}^j\right] \quad (42)$$

This term is also a time invariant linear combination of measurements, so we define $\Psi \in \mathrm{R}^{N-1 \times N}$ for which (i+1)th column is coefficients of $\tilde{\theta}$ measurements used for obtaining $\dot{\alpha}^i / g_{1ff}$, so:

$$\frac{\dot{\alpha}^i}{g_{1ff}} = \sum_{j=1}^{i+1} \Psi_{j,i+1} \tilde{\theta}^j \quad (43)$$

Now we can write (37) as:

$$\alpha^i = \sum_{j=1}^{i} \Phi_{j,i} \tilde{\theta}^j$$

$$= -\left[\frac{1}{\frac{h_0^i}{h^2} + \frac{h_1^i}{2h}}\right] \times \left[\left(\frac{-2h_0^i}{h^2} + h_2^i - C_w^i\right)\sum_{j=1}^{i-1} \Phi_{j,i-1}\tilde{\theta}^j + \left(\frac{h_0^i}{h^2} - \frac{h_1^i}{2h}\right) \times \sum_{j=1}^{i-2} \Phi_{j,i-2}\tilde{\theta}^j - \sum_{j=1}^{i} \Psi_{j,i}\tilde{\theta}^j + C_w^i \tilde{\theta}^i\right] \quad (44)$$

Equations (43) and (44) can be used recursively to fill the columns of Ψ and Φ. The control law (39) can now be written as:

$$u_{fb} = -\frac{1}{k_3} \sum_{j=1}^{N-1} \Phi_{j,N} \tilde{\theta}^j \quad (45)$$

This control law is added up with u_{ff}, which helps us to calculate physical actuators:

$$I_p = u \quad (46)$$

$$P_{tot} = u^2 g^2{}_{2ff} \quad (47)$$

$$\bar{n} = g_{1ff}^{2/3} u^2 g_{2ff} \quad (48)$$

The set of ODEs describing the target system can be expressed as:

$$\dot{\beta}(t) = (M - C_w)\beta(t) g_{1ff}(t) \quad (49)$$

Where $\beta = [\tilde{w}^1, \ldots, \tilde{w}^{N-1}]^T$ and $M \in \mathbb{R}^{N-1 \times N-1}$ is the system matrix. Taking $V = \frac{1}{2}\beta^T \Gamma \beta$ as a Lyapunov functional, where Γ is a positivedefinite matrix. The time derivative will be:

$$\dot{V} = \beta^T \Gamma \dot{\beta} = \beta^T \Gamma (M - C_w) g_{1ff}(t) \beta \quad (50)$$

Since $g_{1ff}(t) > 0$ and Γ is positive definite, we have that $(M - C_w)$ must be negative definite to ensure that $\dot{V}$ is negative definite for $\beta \neq 0$. For the discharges considered M is designed as a negative to make sure the negative definiteness of (50) since $C_w > 0$. And so the system is asymptotically stable.

4.2 Simulated Annealing Algorithm.

This is an optimization technique used for the optimization of discrete time systems and rarely for continuous time systems. The statement of algorithm is as follows:

Statement of algorithm

Select an initial soln ω∈Ω = u_{fb}

Select temperature change counter k = 0
Select a temp. cooling schedule t_k = 5
Set initial temp (t_0 ≥ 0) = 40
Set repetition schedule M_k = 10
Repeat
Set repetition counter m=0
Repeat
Generate a solution ω′∈N(ω)
Calculate $\Delta_{\omega,\omega'}$ = f(ω′)-f(ω)
If $\Delta_{\omega,\omega'} \leq 0$, then ω←ω′
If $\Delta_{\omega,\omega'} > 0$, then ω←ω′ with prob. Exp $\left(-\frac{\Delta_{\omega,\omega'}}{t_k}\right)$
m ←m+1 until m=$M_k = 10$
k ←k+1 until stopping criterion is met

5. SIMULATION RESULTS

Our main aim is to check whether the current, power and density profiles are perfectly tracking the reference profiles. The simulation parameters that we use are, initially we select the grid size N = 10, then the design parameter $C_w^i = 3.75 \times 10^{-16}$ for $1 \leq i \leq 5$ and $C_w^i = 7.5 \times 10^{-16}$ for $6 \leq i \leq N$. during the designing and tuning process largest weight is placed on profile errors around j= 4, corresponding to $\hat{\rho} = 0.4$.

The steady-state error could be made smaller by increasing the gain of the controller (through the parameters C_w^i), however, this would increase the sensitivity of the closed-loop system to measurement noise and therefore introduced the Simulated Annealing Algorithm.

Figures (3), (4) and (5) shows the tracking ability of the current, power and density profiles to the linear reference inputs. These simulation results obtained are having satisfactory results provided with the provision for comparing the backstepping controller performance with and without the optimization using the Simulated Annealing Algorithm.

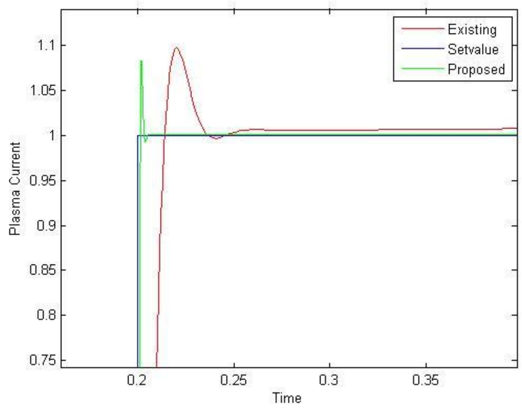

Fig-3: tracking performance of plasma current for linear reference input.

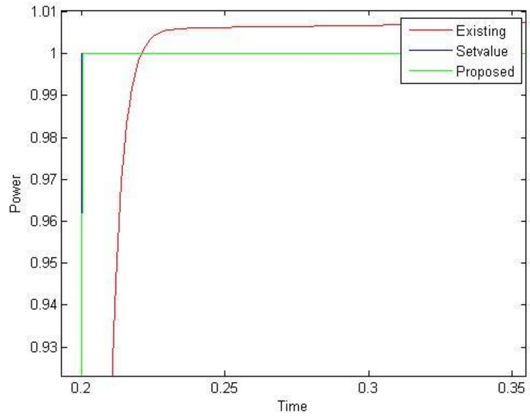

Fig-4: Tracking performance of power for linear reference input

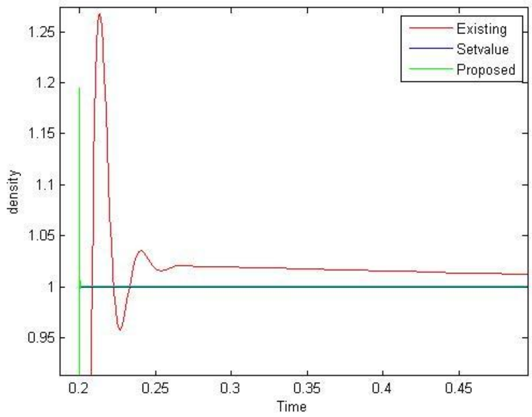

Fig-5: Tracking performance of power for linear reference input.

6. CONCLUSIONS

A feedback controller based on PDE backstepping that achieves asymptotic stabilization of the system, plasma current, Neutral Beam Injection (NBI) power and plasma density profiles in a plasma has been designed. In addition to it the simulated annealing algorithm optimizes the controller to improve the system performance further. In this work only L-mode discharges are considered and addressing the H-mode discharges also using the appropriate modern controllers could be suggested as a future scope of this work.

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