Mathematical model of brain tumor

Khairia El-Said El-Nadi[1], Wagdy G. El-Sayed[2] and Ahmed Khdher Qassem[3]

[1][2][3] *Department of mathematics and computer sciences – faculty of science –Alexandria University- Egypt*

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Abstract - *In this paper, we used thestochastic partialdifferential equationsto solve themodelof the braintumormodels; this model isa generalizedmodelof the formprovidedfor (Sibusiso MOYO and P.G.L. LEACH, 2004). Our aim was tomeasuretumor growthin time and spaceand the use ofin order tounderstandthe behavior ofthe tumorandthe growthmodels tostudythe tumorcellsunder the influence of random disturbances. We used Adomian method decomposition to study thenon-linearmodel.*

Key Words:*stochastic partial differential, brain cancer, Adomian decomposition method.*

AMS Subject Classifications.37C45, 92B05, 65Pxx.

1. INTRODUCTION

Brain tumor is a dynamic system in which cells grow in fast and abnormal manner, and ultimately these abnormal cells kill the healthy cells, by depriving them of leisure, nutritious and oxygen. The tumor occurs in the brain or inside the skull when strange and abnormal cells present in the brain [15].

There are two main types of tumors: malignant tumors (cancer) and benign tumors. Cancerous tumors are divided into a primary tumors that begin in the brain, and secondary tumors that are spread from an organ other than brain and went to the brain [1, 2]. Symptoms of brain tumors may vary depending on the injured part of the brain. These symptoms can include headache, problems in vision, vomiting, and changes in mentality [4].
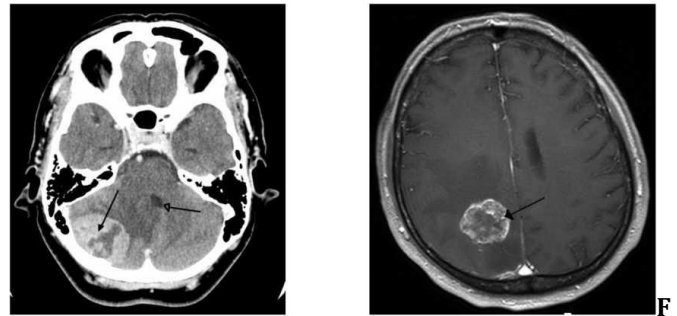

ig -1: illustratesthe spread ofthe tumorinthe right part ofthe braindue tolung cancer

Headaches are very strong in the morning and disappear shortly after vomiting. Other problems may be occur such as, difficulty in walking and talking, or loss of consciousness.

Most causes of brain tumors are unknown. However some specialists stated that the cause of the disease is hereditary under the name of neurofibromatosis, and others expect that exposure to chemicals and radiation is likely to be a cause of brain tumor [5]. While recently been raising the issue of using of mobile phone may be a cause of brain cancer. But so far not confirmed these reasons because the evidence is not clear. The most common types of primary tumors in adults are: meningitides (usually benign), and in astrocytes such as glioblastomas. In children, the most common type is malignant myeloid cancer.

Diagnosis is usually made by a medical examination along with the CT Scan or magnetic resonance imaging (MRI). Then by taking a biopsy, Based on the results, the severity of illness or the tumor degree can be known.

Brain tumor can be likened to brain forest fires, because it spreads on the outer perimeter along all that remains in the center die often because of lack of fuel (oxygen and nutrients from the blood) [15]. Therefore when treatment should be more rapid spread of the tumor to destroy the tumor quickly quite effectively using cancer treatments.

Treatment may include a combination of surgery, radiotherapy and chemical therapy. There may be a need to counter medication during paining (dexamethasone and furosemide) where these drugs are used to reduce the swelling around the tumor.

Some tumors grow gradually .Treatments are being studied to use the immune system of the person ,the outcomes are vary greatly depending on the type of tumor and the extent of its spread in the diagnosed, for example: Glioblastomas usually have bad therapeutic outcomes, while meningitis usually shows good response . Average survival for brain cancer is five years in the United States with rate of 33%.

Secondary or metastatic brain tumors are more common than primary brain tumors, and more secondary brain tumors transmitted come from lung cancer.

Primary brain tumors occur in about 250 000 people a year worldwide, making it the least type of cancers. Children under the age of 15 years have brain tumors and they are the second largest after the acute lymphoblastic leukemia cancer as a cause of cancer [16 - 19].

In Australia the average economic cost of brain cancer is $ 1900000, which is the greatest among other types of cancer. And away from the pessimism, brain tumor is a high deadly disease by a percentage may reach to 100 % within a year of diagnosis. Given that no surgery, nor chemotherapy, or radiation can stop this disease, it would be foolish to expect that mathematics will help in changing this view. But mathematics can help more understanding the tumor.

2. MATHEMATICAL MODEL

The brain-cancer cells is grow very fast, and at any point in time, only a portion of them are replicating and most cancer treatments only kill cells during this active phase. This means that, when determining the net tumor-cell kill rates, models need take this constraint into account.

A small fraction of tumor cells (about one in a thousand) called clonogenic cells are capable of re growing the entire tumor. In other words, if the tumor is not to grow back after treatment all these cells must be killed. A tumor such as glioblastoma multiform has many billions of cells and no single treatment presently available is capable of such a high kill rate [20].

For example a log cell kill rate would kill 90 % of the tumor cells. A two log treatment (such as radiation) would kill 99 % of the tumor cells etc. It is important for the treatment to have the ability to target tumor cells rather than healthy cells and to move through the brain to reach the periphery of the tumor. This does affect how well a particular treatment works. We consider the following model [20].

Let $n(r,t)$ be the concentration of brain tumor cells at a location r and time t. considering now the Burgess equation [3];

$$\frac{\partial n(r,t)}{\partial t} = \frac{D}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2 \partial n(r,t)}{\partial r}\right) + pn(r,t) - k_t n(r,t) \qquad (1)$$

- $n(r, t)$ is the concentration of brain tumor cells at location rat time t,
- D is the diffusion coefficient (estimated at 0.0013 cm2 per day for glioblastoma multiform)
- which measures the invasiveness of the glioblastoma multiform cells,
- p is the proliferation rate of the tumor,
- k_t is the (therapy-dependent) killing rate at time t and
- r measures the distance from the Centre (i.e. the origin of glioblastoma multiform).

Under the rescaling $t = (p - k_t)T$, $R = (|p - kt|D)^2 r$ we may write (1) in the parameter-free form:

$$\frac{\partial n}{\partial t} = \frac{\partial^2 n}{\partial r^2} + \frac{2}{r}\frac{\partial n}{\partial r} + n \qquad (2)$$

in which we have reverted to lower case variables[18]. We can write Equations (2) in the form

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2} + u \qquad (3)$$

The aim of the considered paper is to generalize model (3) to the stochastic case.

$$du = \frac{\partial^2 u}{\partial x^2}dt + udt + \sigma u dW(t) \quad (4)$$

Let $\{W(t), t \geq 0\}$ be a standard Wiener process adapted.

We perturb (4) by a multiplicative noise term and consider the following stochastic integral equation;

$$u(x,t) = \emptyset(x) + \int_0^t \frac{\partial^2 u(x,s)}{\partial x^2}ds + \int_0^t u(x,s)ds + \sigma\int_0^t u(x,s)dW$$

Where σ is a constant it is assumed that $\emptyset(x)$ is a given continuous bounded function on $(-\infty, \infty)$.

Consider the following stochastic integral equation:

$$X_1(t) = 1 + \frac{\sigma^2}{2}\int_0^t X_1(s)\,ds - \sigma\int_0^t X_1(s)\,dW(s) \qquad (6)$$

$$X_2(t) = 1 + \frac{\sigma^2}{2}\int_0^t X_2(s)\,ds - \sigma\int_0^t X_2(s)\,dW(s) \qquad (7)$$

The solutions of these equations are given by:

$$X_1(t) = e^{-\sigma W(t)}$$

$$X_2(t) = e^{\sigma W(t)}$$

Set

$$v(x,t) = X_1(t)u(x,t) \qquad (8)$$

$$dv(x,t) = X_1(t)du + u(x,t)dX_1(t) - \sigma^2 X_1(t)u(x,t)dt$$

$$= X_1(t)\left[\frac{\partial^2 u(x,t)}{\partial x^2}dt + u(x,t)dt + \sigma u(x,t)dW(t)\right]$$

$$+ u\left[\frac{\sigma^2}{2}X_1(t)dt - \sigma X_1(t)dW(t)\right] - \sigma^2 X_1(t)u(x,t)dt$$

$$dv(x,t) = \frac{\partial^2 v(x,t)}{\partial x^2}dt + v(x,t)dt + \sigma v(x,t)dW(t)$$

$$+ \frac{\sigma^2}{2}v(x,t)dt - \sigma v(x,t)dW(t) - \sigma^2 v(x,t)dt$$

$$dv(x,t) = \frac{\partial^2 v(x,t)}{\partial x^2}dt + \frac{2-\sigma^2}{2}v(x,t)dt \qquad (9)$$

$$v(x,t) = \emptyset(x) + \int_0^t \frac{\partial^2 v(x,s)}{\partial x^2}\,ds - \frac{2-\sigma^2}{2}\int_0^t v(x,s)\,ds \qquad (10)$$

To solve equation (10) we use the Adomian decomposition method.

Assume that the unknown function v can be represented by an infinite series of the form:

$$v(x,t) = \sum_{i=0}^{\infty} v_i(x,t) \qquad (11)$$

The function $v_i(x,t)$, $i = 0,1,2, \ldots\ldots\ldots$

Substituting (11) in (10) and identifying the zero component $v_0(x,t)$ by:

$$v_0(x,t) = \phi(x)$$

Then the remaining components can be determined by using the recurrence relation:

$$v_{n+1}(x,t) = \int_0^t \frac{\partial^2 v_n(x,s)}{\partial x^2}\,ds - \frac{2-\sigma^2}{2}\int_0^t v_n(x,s)\,ds \qquad (12)$$

Using the derived Adomian polynomials into equation (10) we obtain:

$$v_0(x,t) = \phi(x)$$

$$v_1(x,t) = \int_0^t \frac{\partial^2 v_0(x,s)}{\partial x^2}\,ds - \frac{2-\sigma^2}{2}\int_0^t v_0(x,s)\,ds$$

$$= \int_0^t \frac{\partial^2 \phi(x)}{\partial x^2}\,ds - \frac{2-\sigma^2}{2}\int_0^t \phi(x)\,ds$$

$$= \frac{\partial^2 \phi(x)}{\partial x^2}t - \frac{2-\sigma^2}{2}\phi(x)t$$

$$v_2(x,t) = \int_0^t \frac{\partial^2 v_1(x,s)}{\partial x^2}\,ds - \frac{2-\sigma^2}{2}\int_0^t v_1(x,s)\,ds$$

$$= \int_0^t \left(\frac{\partial^4 \phi(x)}{\partial x^4} - \frac{2-\sigma^2}{2}\frac{\partial^2 \phi(x)}{\partial x^2}\right)s\,ds$$

$$- \frac{2-\sigma^2}{2}\int_0^t \left(\frac{\partial^2 \phi(x)}{\partial x^2} - \frac{2-\sigma^2}{2}\phi(x)\right)s\,ds$$

$$= \left(\frac{\partial^4 \phi(x)}{\partial x^4} - \frac{2-\sigma^2}{2}\frac{\partial^2 \phi(x)}{\partial x^2}\right)\frac{t^2}{2}$$

$$- \frac{2-\sigma^2}{2}\left(\frac{\partial^2 \phi(x)}{\partial x^2} - \frac{2-\sigma^2}{2}\phi(x)\right)\frac{t^2}{2}$$

And so on ....

Then the solution of equation (5) is:

$$u_0 = X_2 v_0 = \phi(x)e^{\sigma W(t)}$$

Notice that:

$$E[e^{\sigma W(t)}] = \int \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}e^{\sigma\sqrt{t}\,x}\,dx = \int \frac{1}{\sqrt{2\pi}}e^{-\frac{(x-\sigma\sqrt{t})^2}{2}}e^{\frac{\sigma^2 t}{2}}\,dx = e^{\frac{\sigma^2}{2}}$$

Then the expected number $E[u_i(x,t)], i = 0,1,2,\ldots$ at time $t > 0$ and location x is given by:

$$E[u_0] = \phi(x)e^{\frac{\sigma^2 t}{2}}$$

$$u_1 = X_2 v_1 = \left(\frac{\partial^2 \phi(x)}{\partial x^2} t - \frac{2-\sigma^2}{2}\phi(x)t\right)e^{\sigma W(t)}$$

$$E[u_1] = \left(\frac{\partial^2 \phi(x)}{\partial x^2} t - \frac{2-\sigma^2}{2}\phi(x)t\right)e^{\frac{\sigma^2 t}{2}}$$

$$u_2 = X_2 v_2 = \left(\left(\frac{\partial^4 \phi(x)}{\partial x^4} - \frac{2-\sigma^2}{2}\frac{\partial^2 \phi(x)}{\partial x^2}\right)\frac{t^2}{2} - \frac{2-\sigma^2}{2}\left(\frac{\partial^2 \phi(x)}{\partial x^2} - \frac{2-\sigma^2}{2}\phi(x)\right)\frac{t^2}{2}\right)e^{\sigma W(t)}$$

$$E[u_2] = \left(\left(\frac{\partial^4 \phi(x)}{\partial x^4} - \frac{2-\sigma^2}{2}\frac{\partial^2 \phi(x)}{\partial x^2}\right)\frac{t^2}{2} - \frac{2-\sigma^2}{2}\left(\frac{\partial^2 \phi(x)}{\partial x^2} - \frac{2-\sigma^2}{2}\phi(x)\right)\frac{t^2}{2}\right)e^{\frac{\sigma^2 t}{2}}$$

And so on …

If $\phi(x) = \alpha x + \beta$, $where$ α and β are $constants$, then the solution of equation (10) is;

$$v_0(x,t) = \alpha x + \beta$$

$$v_1(x,t) = \frac{\sigma^2 - 2}{2}(\alpha x + \beta)t$$

$$v_2(x,t) = \left(\frac{(\sigma^2 - 2)^2}{4}(\alpha x + \beta)\right)\frac{t^2}{2}$$

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$$v_n(x,t) = \left(\frac{(\sigma^2 - 2)^n}{2^n}(\alpha x + \beta)\right)\frac{t^n}{n!}$$

Then

$$u_n(x,t) = X_2 v_n(x,t) = \left(\frac{(\sigma^2 - 2)^n}{2^n}(\alpha x + \beta)\right)\frac{t^n}{n!}e^{\sigma W(t)}$$

$$E[u_n(x,t)] = \left(\frac{(\sigma^2 - 2)^n}{2^n}(\alpha x + \beta)\right)\frac{t^n}{n!}e^{\frac{\sigma^2 t}{2}}$$

And if $\phi(x) = \mu$, $where$ μ is $constant$, then the solution of equation (10) is;

$$v_0(x,t) = \mu$$

$$v_1(x,t) = \frac{\sigma^2 - 2}{2}(\mu)t$$

$$v_2(x,t) = \left(\frac{(\sigma^2 - 2)^2}{4}(\mu)\right)\frac{t^2}{2}$$

.

.

$$v_n(x,t) = \left(\frac{(\sigma^2 - 2)^n}{2^n}(\mu)\right)\frac{t^n}{n!}$$

Then

$$u_n(x,t) = X_2 v_n(x,t) = \left(\frac{(\sigma^2 - 2)^n}{2^n}(\mu)\right)\frac{t^n}{n!}e^{\sigma W(t)}$$

$$E[u_n(x,t)] = \left(\frac{(\sigma^2 - 2)^n}{2^n}(\mu)\right)\frac{t^n}{n!}e^{\frac{\sigma^2 t}{2}}$$

3. CONCLUSIONS

The solution to any system of cancer helps doctors seeing how much resistance the patient and the disease in order to know when they can increase or decrease the size of the tumor without waiting to see it; and then can gain time to fight the tumor by medical means were either surgical or chemical or Radiation.

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