Slip Flow of a Casson Fluid Flow over an Exponentially Stretching Surface and Heat and Mass Transfer

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ABSTRACT

Numerical solutions for a class of nonlinear differential equations arising in heat and mass transfer of a non-Newtonian fluid towards an exponentially stretching surface are obtained. Casson fluid model is used to characterize the non-Newtonian fluid behaviour. Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. These differential equations are highly nonlinear which cannot be solved analytically. Therefore, bvp4c MATLAB solver has been used for solving it. The flow fields and heat and mass transfer are significantly affected by the physical parameters. The effect of increasing values of the Casson parameter and momentum slip parameter is to suppress the velocity field. But the temperature and concentration is enhanced with increasing Casson parameter and momentum slip parameter.

Keywords: Exponentially Stretching, Casson Fluid, Heat and Mass Transfer, Slip Flow, Chemical Reaction.

1. Introduction

The fluid flow behavior of non-Newtonian fluid has attracted special interest in recent years due to the wide application of these fluids in the chemical, pharmaceutical, petrochemical, food industries and electronic industries. A large number of fluids that are used extensively in industrial application are non-Newtonian fluids exhibiting a yield stress τ_y, stress that has to be exceeded before the fluid moves. As a result the fluid cannot sustain a velocity gradient unless the magnitude of the local shear stress is higher than this yield stress. Fluids that belong to this category include cement, drilling mud, sludge, granular suspensions, aqueores foams, slurries, paints, plastics, paper pulp and food products. Due to the increasing importance of non-Newtonian fluids in industry, the stretching sheet concept has more recently been extended to fluids obeying non-Newtonian constitutive equations (Prasad et al. [1]). Khan [2] and Sanjayanand and Khan [3] studied the viscous-elastic boundary layer flow and heat transfer due to an exponentially stretching sheet. Sayed Ahmed et al. [4] numerically investigated the laminar boundary layer flow of Herschel-Bulkley fluids through a rectangular duct and heat transfer analysis using finite element method. Swati Mukhopadhyay [5] presented the non-Newtonian Casson fluid flow and heat transfer over a nonlinearly stretching surface. Mustafa et al.[6]

investigated the unsteady flow and heat transfer for a Casson fluid over a moving flat plate with a parallel free stream using homotopic method and also conclude that the surface shear stress and surface heat transfer are enhanced by increasing Casson parameter. Eldabe and Elmohands [7] have studied the Casson fluid for the flow between two rotating cylinders and Boyd et al. [8] investigated the Casson fluid flow for the steady and oscillatory blood flow. Pramaik [9] investigated the Casson fluid flow of an exponentially porous stretching surface in the presence of thermal radiation and conclude that velocity decreases with increasing the Casson parameter, but temperature is enhanced with raising the Casson parameter and skin friction coefficient increases with increasing the suction parameter.

Suction/injection (blowing) of a fluid through the bounding surface can significantly change the flow field. In general, suction tends to increase the skin friction, whereas injection acts in the opposite manner. The process of suction/blowing has also its importance in many engineering activities such as in the design of thrust bearing and radial diffusers, and thermal oil recovery [10]. Suction is applied to chemical processes to remove reactants [11]. Iswar Chandra Mandal and Mukhopadhyay [12] studied the exponentially stretching porous sheet embedded in a porous medium with variable surface heat flux and also conclude that momentum and thermal boundary layer thickness decrease with increasing exponential parameter. Mukhopadyay [13] investigated the MHD boundary layer flow of an exponential stretching sheet embedded in a thermally stratified medium and also conclude that fluid velocity and temperature decreases significantly with increasing suction parameter.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries, For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal

occurrence in many branches of science and engineering. Chambre and Young [14] have presented a first order chemical reaction in the neighborhood of a horizontal plate. Akyildiz et al. [15] analyzed the transport and diffusion of chemically reactive species over a stretching sheet in a non-Newtonian fluid (fluid of differential type, second grade). Muthcumaraswamy and Ganesan [16] studied effect of the chemical reaction and injection on the flow characteristics in an unsteady upward motion of an isothermal plate. Chamkha [17] studied the MHD flow of a numerical of uniform stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. Seddeek *et al.*[18] analyzed the effects of chemical reaction, radiation and variable viscosity on hydromagnetic mixed convection heat and mass transfer Hiemenz flow through porous media. Gangadhar [19] studied the heat and mass transfer over a vertical plate with a convective surface boundary condition and chemical reaction and also conclude that the spices boundary layer thickness decreases with increasing the chemical reaction parameter. Gangadhar and Bhaskar Reddy [20] studied the MHD boundary layer flow of heat and mass transfer over a moving vertical plate in a porous medium with suction. Gangadhar et al. [21] investigated the hydro magnetic flow of heat and mass transfer over a vertical plate with a convective boundary condition and chemical reaction and also conclude that concentration of the fluid decreases with an increasing the chemical reaction parameter.

The present study investigates the numerical solutions for a class of nonlinear differential equations arising in heat and mass transfer of a non-Newtonian fluid towards an exponentially stretching surface is obtained. Casson fluid model is used to characterize the non-Newtonian fluid behaviour. Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. These differential equations are highly nonlinear which cannot be solved analytically. Therefore, bvp4c MATLAB solver has been used for solving it. The flow fields and heat and mass transfer are significantly affected by the physical parameters.

2. MATHEMATICAL FORMULATION

Let us consider the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane $y = 0$. The fluid flow is confined to $y > 0$. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed (see Fig.A). The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_y / \sqrt{2\pi}\right)e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + p_y / \sqrt{2\pi_c}\right)e_{ij}, \pi < \pi_c \end{cases}$$

Here $\pi = e_{ij}e_{ij}$ and e_{ij} is the (i,j)-th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non- Newtonian fluid, and p_y is the yield stress of the fluid. Let T_w, C_w be the temperature and concentration at the sheet respectively and temperature and concentration far away from the sheet is T_∞, C_∞. Also the reaction of species is the first order homogeneous chemical reaction with rate constant k.
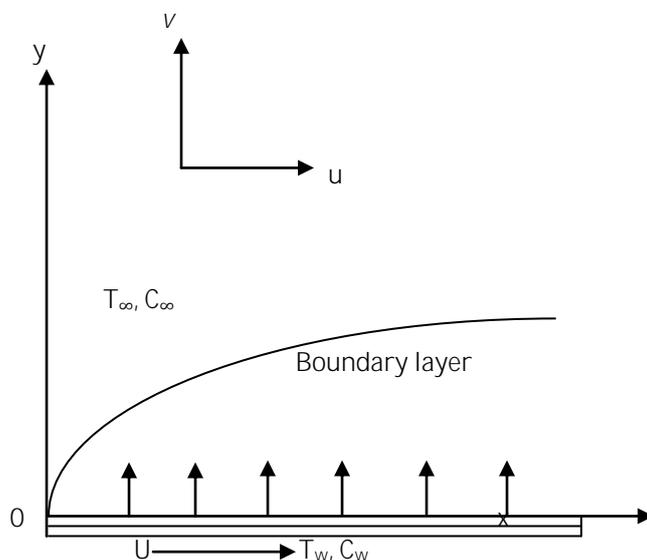

Fig A : Physical flow of the problem

Under these assumptions along with the Boussinesq and boundary layer approximations, the system of equations, which models the flow is given by
Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (2.1)$$

Linear momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} \qquad (2.2)$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (2.3)$$

Spices equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k\left(C-C_\infty\right) \qquad (2.4)$$

where u and v are the components of velocity respectively in the x and y directions, υ is the kinematic viscosity, ρ is the fluid density (assumed constant), $\beta=\mu_B\sqrt{2\pi_c}/p_y$ is parameter of the Casson fluid, ⍺ is the thermal conductivity of the fluid, D is the diffusion coefficient of the diffusing species in the fluid, $k=k_0e^{\frac{x}{L}}$ is the exponential reaction rate; k>0 stands for destructive reaction whereas k<0 stands for constructive reaction, k_0 is a constant.
The boundary conditions are

$$u=u_w+u_{slip}=U_0e^{\frac{x}{L}}+N\upsilon\frac{\partial u}{\partial y} \text{ at } y=0$$

$$v=v_w(x), T=T_w, C=C_w$$

$$u\to 0, T\to T_\infty, C\to C_\infty \quad \text{as } y\to\infty$$
$$(2.5)$$

Here U_0 is a reference velocity, L is the characteristic of length, and N is the slip factor having the dimension (seconds/metres), Moreover, $v_w(x)=V_0e^{\frac{x}{2L}}$ is the mass transfer due to suction $v_w(x) < 0$ or injection $v_w(x) > 0$.
We introduce also a stream functions ψ is defined by

$$u=\frac{\partial \psi}{\partial y}, v=-\frac{\partial \psi}{\partial x} \qquad (2.6)$$

Introducing the similarity variables as

$$\eta=\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}y, \psi=\sqrt{2\upsilon U_0 L}e^{\frac{x}{2L}}f(\eta)$$

$$u=U_0e^{\frac{x}{L}}f'(\eta), v=-\sqrt{\frac{\upsilon U_0}{2L}}\left(f(\eta)+\eta f'(\eta)\right)$$

$$T=T_\infty+T_0e^{\frac{x}{2L}}\theta(\eta), C=C_\infty+C_0e^{\frac{x}{2L}}\phi(\eta)$$
$$(2.7)$$

where $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature, $\phi(\eta)$ is the dimensionless concentration, η is the similarity variable, T_0, C_0 are the reference temperature and concentration respectively.

Substituting equations (2.6) and (2.7) in (2.2) – (2.5), we have

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-2f'^2=0 \qquad (2.8)$$

$$\theta''+\Pr\left(f\theta'-f'\theta\right)=0 \qquad (2.9)$$

$$\phi''+Sc\left(f\phi'-f'\phi-2kr\phi\right)=0 \qquad (2.10)$$

The corresponding boundary conditions are
$$f'(0)=1+\gamma f''(0), f(0)=fw, \theta(0)=1, \phi(0)=1$$

$$f'(\infty)=\theta(\infty)=\phi(\infty)=0 \qquad (2.11)$$

where the prime denotes differentiation with respect to η, $\Pr=\dfrac{\upsilon}{\alpha}$ is the Prandtl number, $Sc=\dfrac{\upsilon}{D}$ is the Schmidt number, $\gamma=N\sqrt{\dfrac{U_0\upsilon}{2L}}$ momentum slip parameter, $fw=-\dfrac{V_0}{\sqrt{\dfrac{U_0\upsilon}{2L}}}$ wall mass flux parameter, $kr=\dfrac{k_0L}{U_0}$ is the reaction rate parameter. Here kr>0 represents the destructive reaction, kr= 0 corresponds to no reaction, and kr<0 stands for the generative reaction.

The main physical quantities of interest are the skin friction coefficient $f''(0)$, the local Nusselt number $-\theta'(0)$ and the Sherwood number $-\phi'(0)$ which represent the wall shear stress, the heat transfer rate and mass transfer rate at the surface, respectively.

3 SOLUTION OF THE PROBLEM

The set of equations (2.8) to (2.11) were reduced to a system of first-order differential equations and solved using a MATLAB boundary value problem solver called bvp4c. This program solves boundary value problems for ordinary differential equations of the form $y' = f(x, y, p), a \le x \le b$, by implementing a collocation method subject to general nonlinear, two-point boundary conditions $g(y(a), y(b), p)$. Here p is a vector of unknown parameters. Boundary value problems (BVPs) arise in most diverse forms. Just about any BVP can be formulated for solution with bvp4c. The first step is to write the *ODEs* as a system of first order ordinary differential equations. The details of the solution method are presented in Shampine and Kierzenka[22].

4   RESULTS AND DISCUSSION

Due to the decoupled boundary layer (2.8) - (2.10), for $fw=\gamma = 0$ and $\beta=0$ (Newtonian fluid), it is found that there is a unique value of the skin friction coefficient, $f''(0) = 1.281816$, which is in very good comparison with the classical value $f''(0) = 1.281811$ by Sahoo and Poncet [23]. Table 1 presents the comparison between the present results with the previously reported results by Magyari and Keller [24], El-Aziz [25] and Sharma et al. [26] for various values of the Prandtl number Pr. Also, it is found that they are in good agreement.

Figures 1-13 document the graphical computations for velocity, temperature and concentration for the effects of the dictating thermophysical parameters.   Figs. 1-3 show that the effects of Casson parameter (β), momentum slip parameter (γ) and suction/injection parameter (fw) on velocity profiles. Velocity is found to decrease with increasing Casson parameter (Fig. 1), momentum slip parameter (Fig. 2) and suction/injection parameter (Fig. 3).   Increased momentum slip (Fig. 2) passes on a retarding effect to the flow at the wall and this influences the entire flow through the boundary layer, transverse to the wall. With increased mass flux into the sheet regime via the wall (fw<0, i.e., injection), the boundary layer flow is significantly accelerated, as observed in Fig. 3. The converse behaviour accompanies increasing mass flux of the boundary layer (fw>0, i.e., suction). Evidently the suction effect induces greater adherence to the wall of the fluid regime and this also increases momentum boundary layer thickness. With greater injection, the momentum boost results in a reduction in momentum boundary layer thickness.

Figs. 4-7 present the response of temperature profiles for a variation in Casson parameter (Fig. 4), momentum slip (Fig. 5), wall mass flux (Fig. 6) and Prandtl number(Fig. 7). Temperature of the fluid increases throughout the boundary layer with an increasing Casson parameter, momentum slip parameter, and consequently thermal boundary layer thickness enhanced and opposite results were found with an rising wall mass flux. From fig.7 it is found that as Pr increases, the temperature in the boundary layer decreases and the thermal boundary layer thickness also decreases. This is because for small values of the Prandtl number, the fluid is highly thermal conductive. Physically, if Pr increases, the thermal diffusivity decreases and these phenomena lead to the decreasing of energy ability that reduces the thermal boundary layer.

Figs. 8-11 present the response of temperature profiles for a variation in Casson parameter (Fig. 8), momentum slip (Fig. 9), wall mass flux (Fig. 10) and Schmidt number(Fig. 11). Concentration of the fluid increases throughout the boundary layer with an increasing Casson parameter, momentum slip parameter, and consequently boundary layer thickness enhanced and opposite results was found with a rising wall mass flux. From fig.11 it is found that as Sc increases, the concentration in the boundary layer decreases and boundary layer thickness also decreases. Physically, Sc=0.22 (hydrogen), Sc=0.62 (water vapour), Sc=0.78 (ammonia) and Sc=2.62 (Propyl Benzene), if Sc increases, the mass diffusivity decreases and these phenomena lead to the decreasing of spices ability that reduces the boundary layer.

Fig.12, 13 display the nature of concentration profiles for variable values of reaction rate parameter. In case of constructive chemical reaction (kr<0), concentration at a point on the sheet increases with increasing (absolute) values of reaction rate parameter [Fig.12] whereas for destructive chemical reaction (kr>0), opposite behaviour of concentration field is noted i.e. the reactive solute distribution decreases with increasing i.e. the reaction-rate parameter is a decelerating agent in this case [Fig.13]. As a result, the solute boundary layer, near the wall, becomes thinner. This is due to the fact that the conversion of the species takes place as a result of chemical reaction and thereby reduces the concentration in the boundary layer [Fig.13]. Concentration overshoot is noted for kr= -0.1, -0.15, -0.2 [Fig.12] but no overshoot is observed for destructive chemical reaction [Fig.13]. Note that the change in case of generative chemical reaction (kr< 0) is larger in comparison to the case of destructive chemical reaction (kr> 0). The concentration boundary layer decreases in case of destructive chemical reaction.

Figure 14 shows the effects of β and fw on skin friction coefficient. From Figure 14 it is seen that the skin friction decreases with an increase β or fw. The effect of β and γ on skin friction coefficient is shown in fig.15. It is

found that the skin friction coefficient enhanced with an increasing. The variations of β and Pr on local Nusselt number are shown in fig.16. It is observed that the local Nusselt number decrease with an increasing β, whereas local Nusselt number increases with a raising Pr. The variations of β and kr on local Sherwood number are shown in fig.17&18 in case of distractive chemical reaction and constrictive chemical reaction, respectively. It is observed that the local Sherwood number decrease **with an increasing β** or kr in distractive chemical reaction, whereas local Sherwood number increases with a raising kr in contractive chemical reaction.

5 CONCLUSIONS

In the present prater, Numerical solutions for a class of nonlinear differential equations arising in heat and mass transfer of a non-Newtonian fluid towards an exponentially stretching surface are obtained. Casson fluid model is used to characterize the non-Newtonian fluid behaviour. Numerical calculations are carried out

for various values of the dimensionless parameters of the problem. It has been found that

1. Momentum boundary layer thickness decreases with increasing Casson parameter and momentum slip parameter but the thermal and solute boundary layer thickness increases in this case.
2. Momentum and thermal as well as solute boundary layer thickness reduces with the influence of mass flux parameter.
3. Thermal boundary layer thickness decreases with a rising the Prandtl number.
4. With increasing Sc, concentration decreases for both constructive and destructive chemical reactions. Moreover, an increase in Schmidt number reduces the solute boundary layer thickness.
5. The concentration boundary layer and rate of mass transfer are decreases in case of destructive chemical reaction.

Table.1 Comparison for the values of $\theta'(0)$ for various values of Pr

Pr	$\theta'(0)$			
	Magyari and Keller [24]	El-Aziz [25]	Sharma *et al* [26]	Present results
1	-0.954782	-0.954785	-0.954789	-0.954811
2	---	---	-1.471461	-1.471454
3	-1.869075	-1.869074	-1.869073	-1.869069
5	-2.500135	-2.500132	-2.500125	-2.500128
10	-3.660379	-3.660372	-3.660350	-3.660369

Fig.1 Velocity for different values of $\boldsymbol{\beta}$

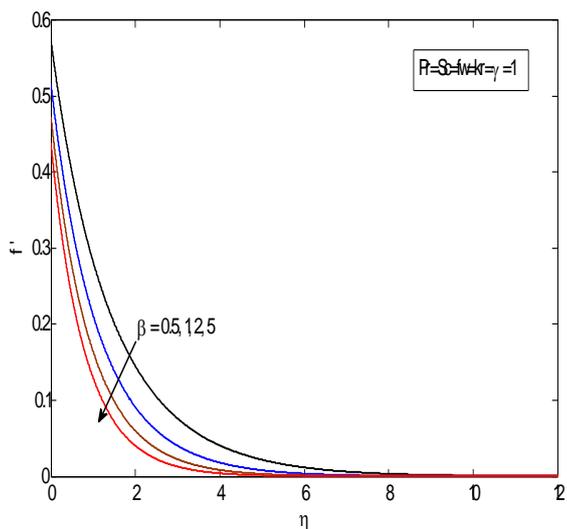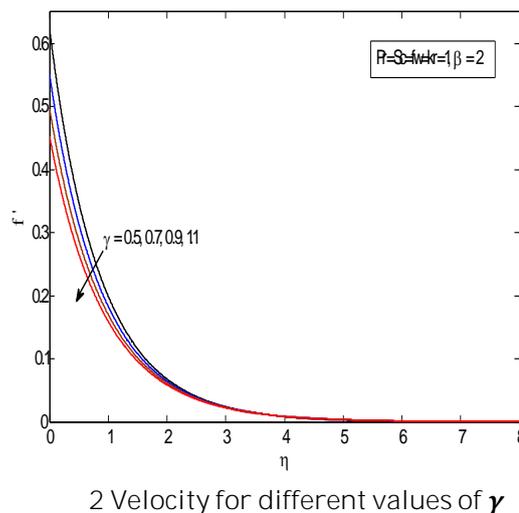Fig. 2 Velocity for different values of $\boldsymbol{\gamma}$
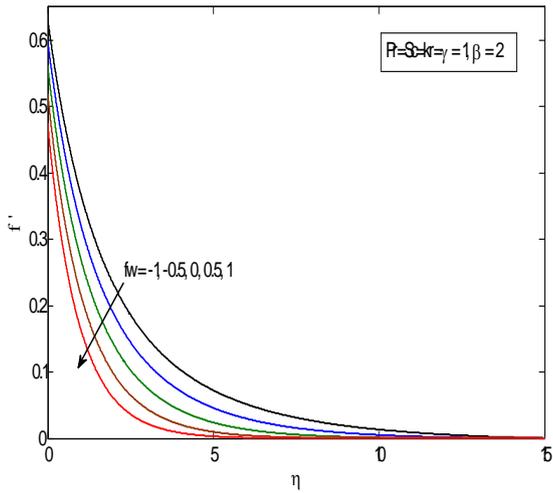
Fig.3 Velocity for different values of *fw*

Fig.4
Temperature for different values of **β**

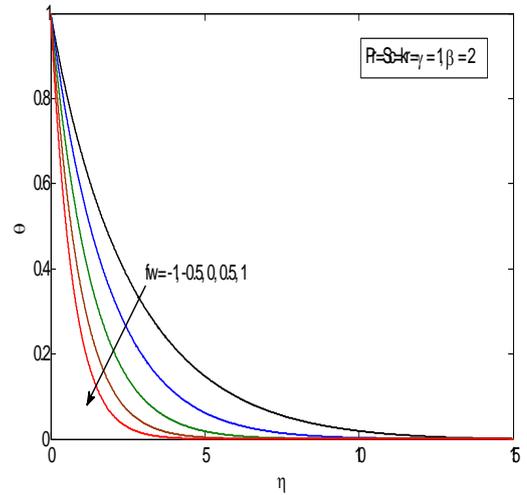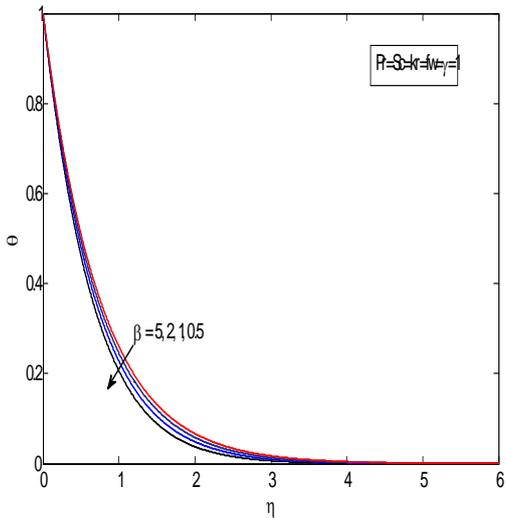Fig.5 Temperature for different values of **γ**

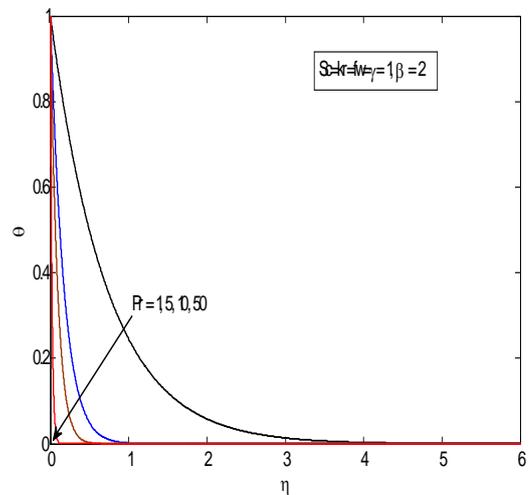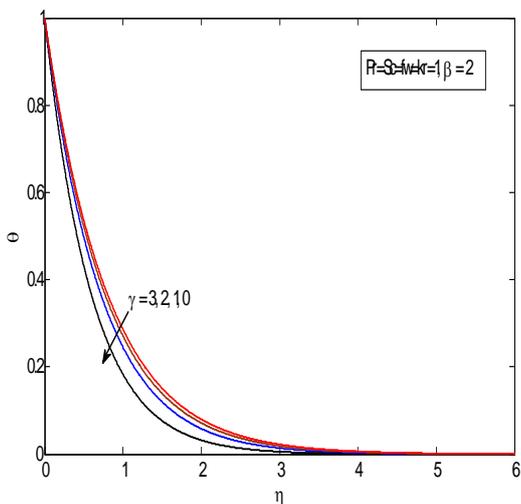Fig.6 Temperature for different values of *fw*

Fig.7 Temperature for different values of *Pr*
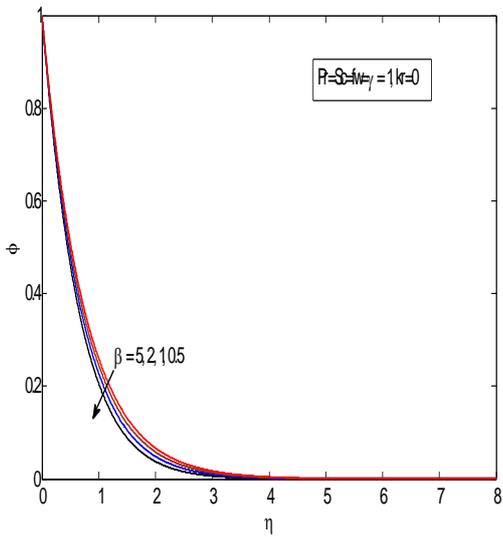
Fig.8 concentration for different values of $\boldsymbol{\beta}$

Fig.11 concentration for different values of Sc
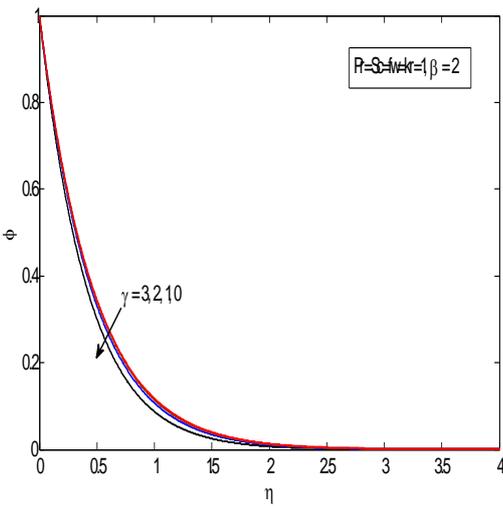
Fig.9 concentration for different values of $\boldsymbol{\gamma}$
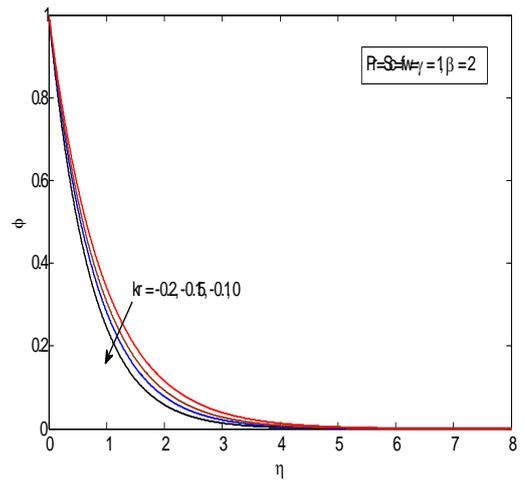

Fig.12 concentration for different values of kr in case of destructive chemical reaction

Fig.10 concentration for different values of fw

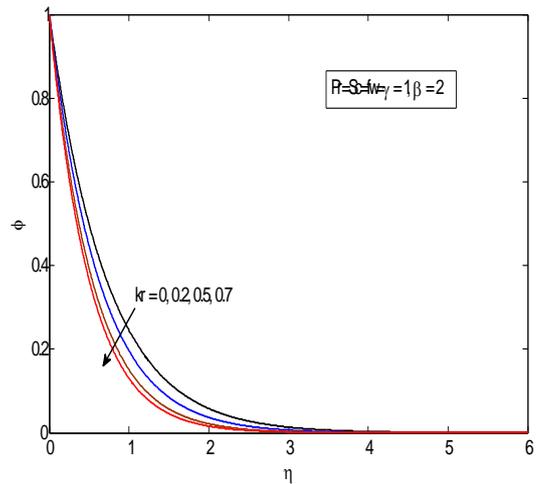Fig.13 concentration for different values of kr in case of constructive chemical reaction
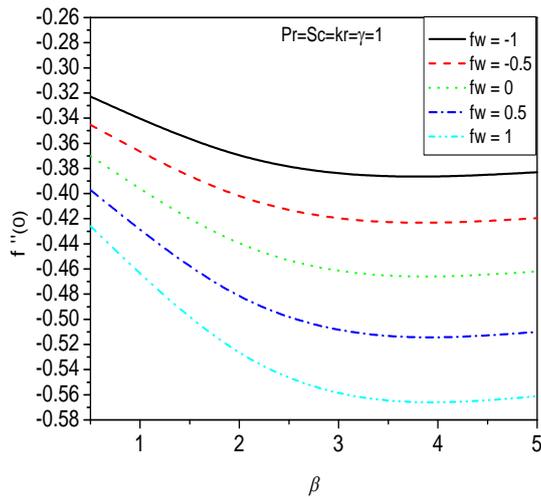
Fig.16 Local Nusselt number for different values of **β** and *Pr*

Fig. 14 Local Skin-friction for different values of **β** and *fw*

Fig.17 Local Sherwood number for different values of **β** and *kr* in case of destructive chemical reaction

Fig.15 Local Skin-friction for different values of **β** and **γ**
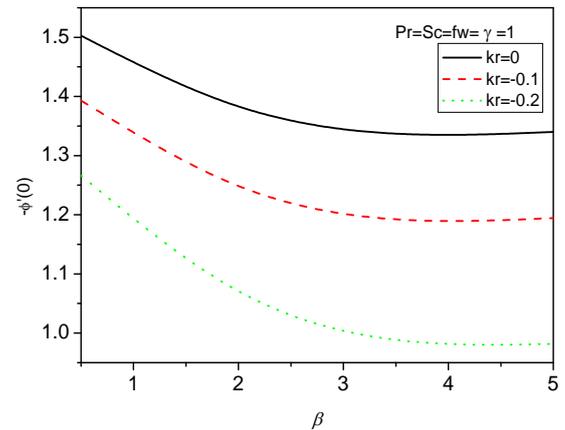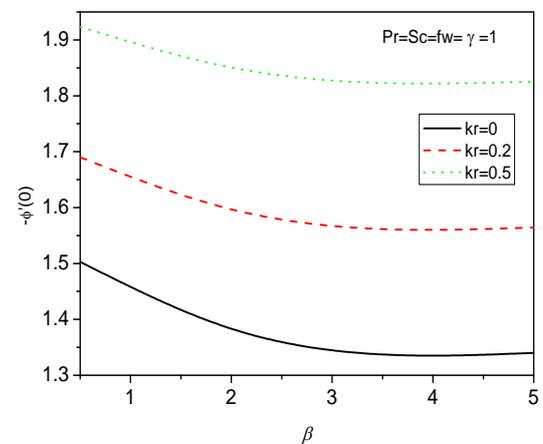
Fig.18 Local Sherwood number for different values of **β** and *kr* in case of constructive chemical reaction

REFERENCES

1. Prasad, K. V., Abel, S., & Datti, P. S., (2003), Diffusion of chemically reactive species of a non-Newtonian fluid immersed in a porous medium over a stretching sheet, Int. J. of Non-Linear Mech., Vol.38, pp.651 – 657.

2. Khan, S. K., (2006), Boundary layer viscoelastic fluid flow over an exponentially stretching sheet, Int. J. of Appl. Mech. and Engng., Vol.11, pp.321-335.

3. Sanjayanand, E., & Khan, S. K., (2006), On heat and mass transfer in a visco-elastic boundary layer flow over an exponentially stretching sheet, Int. J. of Ther. Sci., Vol.45, pp.819-828.

4. Sayed-Ahmed, M. E., Saif-Elyazal, A., and Iskander, L., (2009), Laminar Flow and Heat Transfer of Herschel-Bulkley Fluids in a Rectangular Duct; Finite-Element Analysis, Tamkang Journal of Science and Engineering, Vol. 12, No. 1, pp. 99-107.

5. Swati Mukhopadhyay, (2013), Casson fluid flow and heat transfer over a nonlinearly stretching surface, Chin. Phys. B Vol. 22, No. 7.

6. Mustafa, M., Hayat, T., Pop, I., and Aziz, A., (2011), Unsteady boundary layer flow of a Casson fluid due to an impulsively started moving flat plate, Heat Transfer-Asian Research, Vol.40 (6), pp.563-576.

7. Eldabe, N. T. M., & Salwa. M. G. E., (1995), Heat transfer of MHD non-Newtonian Casson fluid flow between two rotating cylinders, J Phys Soc Japan, Vol.64, pp.41-64.

8. Boyd, J., Buick, J. M., & Green, S., (2007), Analysis of the Casson and Carreau-Yasuda non-Newtonian blood models in steady and oscillatory flow using the lattice Boltzmann method, Phys Fluids, Vol.19, pp.93–103.

9. Pramanik, S., (2014), Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation, Ain Shams Engineering Journal, Vol.5, pp.205–212.

10. Bhattacharyya, K., (2011), Effects of radiation and heat source/sink on unsteady MHD boundary layer flow and heat transfer over a shrinking sheet with suction/injection, Front. Chem. Sci. Eng., Vol.5, pp.376–384.

11. Mukhopadhyay, S., (2012), Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation, Ain Shams Eng. J.

12. Iswar Chandra Mandal, Swati Mukhopadhyay, (2013), Heat transfer analysis for fluid flow over an exponentially stretching porous sheet with surface heat flux in porous medium, Ain Shams Engineering Journal, Vol.4, pp.103–110.

13. Swati Mukhopadhyay, (2013), MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium, Alexandria Engineering Journal, Vol.52, pp.259–265.

14. Chambre, P.L., and Young, J.D., (1958), On the diffusion of a chemically reactive species in a laminar boundary layer flow, Phys. Fluids flow, Vol.1, pp.48-54.

15. Akyildiz, F. T., Bellout. H., & Vajravelu. K., (2006), Diffusion of chemically reactive species in a porous medium over a stretching sheet, J. Math. Anal. Appl., Vol.320, pp.322–339.

16. Muthucumarswamy, R., and Ganesan, P., (2001), Effect of the chemical reaction and injection on the flow characteristics in an unsteady upward motion of an isothermal plate, J Appl. Mech.Tech.Phys.,Vol.42, pp.665-671.

17. Chamkha, A. J., (2003), Unsteady convective heat and mass transfer past a semi-infinite vertical moving porous plate with heat generation/absorption and chemical reaction, Int. Comm. Heat Mass Transfer, Vol.30, pp.413-422.

18. Seddeek, M.A., Darwish A. and Abdelmeguid M.S., (2007), Effects of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media with radiation, Communications in Non-linerar Science and Numerical Simulation, Vol.12, pp.195-213.

19. Gangadhar, K., (2013), Soret and Dufour Effects on Hydro Magnetic Heat and Mass Transfer over a Vertical Plate with a Convective Surface Boundary Condition and Chemical Reaction, Journal of Applied Fluid Mechanics, Vol. 6, No. 1, pp. 95-105.

20. Gangadhar, K., and Bhaskar Reddy, N., (2013), Chemically Reacting MHD Boundary Layer Flow of Heat and Mass Transfer over a Moving Vertical Plate in a Porous Medium with Suction, Journal of Applied Fluid Mechanics, Vol. 6, No. 1, pp. 107-114.

21. Gangadhar, K., Bhaskar reddy, N., Kameswaran, P. K., (2012), Similarity Solution of Hydro Magnetic Heat and Mass Transfer over a Vertical Plate with a Convective Surface Boundary Condition and Chemical Reaction, International Journal of Nonlinear Science, Vol.13, No.3, pp.298-307.

22. Shampine, L. F., and Kierzenka, J., (2000), Solving boundary value problems for ordinary differential equations in MATLAB with bvp4c, Tutorial Notes.

23. Sahoo, B., & Poncet, S., (2011), Flow and heat transfer of a third grade fluid past an exponentially stretching sheet with partial slip boundary condition, Int. J. of Heat and Mass Transfer, Vol. 54, pp.5010–5019.

24. Magyari, E., & Keller, B., (1999), Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface, J. of Phy. D: Appl. Phy., Vol.32, pp.577-585.

25. El-Aziz, M. A., (2009), Viscous dissipation effect on mixed convection flow of a micropolar fluid over an exponentially stretching sheet, Can. J. Phys., Vol.87, pp.359-368.

26. Sharma, R., Ishak, A., Nazar, R., Pop, I., (2014), Boundary Layer Flow and Heat Transfer over a Permeable Exponentially Shrinking Sheet in the Presence of Thermal Radiation and Partial Slip, Journal of Applied Fluid Mechanics, Vol. 7, No. 1, pp. 125-134.