Comparative Study of Topological Optimization of Beam and Ring Type Structures under static Loading Condition

Vani Taklikar[1], Anadi Misra[2]

P.G. Student, Department of Mechanical Engineering, G.B.P.U.A.T, Pantnagar, Uttarakhand, India
Professor, Department of Mechanical Engineering, G.B.P.U.A.T, Pantnagar, Uttarakhand, India

Abstract- *We present ANSYS based Optimality Criterion approach for topology optimization of beam and ring type structures. Optimality Criterion is to be very efficient for solving the topology optimization problem. The optimization problem is formulated as to minimize the material compliance under volume constraints. Two linear elastic isotropic structures have been studied. These structures have been studied by using finite element solver software ANSYS. ANSYS employs topology optimization using the Solid Isotropic Material with Penalization (SIMP) scheme for the penalization of the intermediate design variables and the Optimality criterion for updating the design variables. All the structures have been optimized for minimum compliance and then other parameters like-stresses, displacement in x-y directions, von misses stress, deformed and un-deformed shapes are obtained. Further the structures have been studied by changing material from isotropic to orthotropic and then the results obtained by both the materials have compared.*

Keywords: ANSYS, Optimality criterion approach, Compliance

1. INTRODUCTION

In the present scenario, topology optimization is the most important structural optimization which gives the best distribution of material at the conceptual level. In topology optimization the optimal layouts are generated automatically to solve the design problems in the field of engineering. Optimization is the process of finding for feasible solution in a problem until no other best suitable solution can be found. In general, optimization is the process of minimization or maximization of an objective function subjected to given constraints (stress, deflection etc.) for the problem to be solved. In optimization, all the results obtained by all the candidates are compared and the best optimum result is obtained.

Topology optimization is concerned with seeking the optimum distribution of material in a given design domain that minimizes a given cost function while satisfies a series of constraints [1].

There are many topological optimization methods have been developed some of which are homogenization method, evolutionary structural optimization (ESO) method, solid isotropic method with penalization (SIMP) and other methods. In topology optimization there are mainly two types of regions, one is solid other is void. Solid region means the region with material and the void region means the region without material. Topology optimization gives the best suitable use of material over the structure or body such that an objective function (i.e. is to be maximized or minimized) subjected to given constraint should be satisfied.

The development of topological optimization can be attributed to Bendsoe and Kikuchi [2]. They assumed that the structure is formed by a set of non-homogenous elements which are composed of solid and void regions. They obtained optimal design under volume constraint through optimization process. This method requires a large amount of variables. To overcome this difficulty Bendsøe [9] introduces a solid isotropic material penalization. Suzuki *et al.* [3] studied the shape and topology optimization of linearly elastic material. Author done some modifications in the homogenization method and also clarified the strength of the present approach for plane structure. An evolutionary structural optimization (ESO) technique introduced by Xie *et al.* [7] in which material are gradually introduced or removed until the best condition is met. But once the material is removed it is not introduced again in the structure and this is the drawback of ESO technique. This difficulty was overcome by bi-directional evolutionary structural optimization (BESO) introduced by Querin *et al.*[8]. Many approaches have been studied to solve numerical instability. Further genetic algorithm, optimality criteria, adaptive refinement approach etc. have been developed by many researchers.

The present work is based on ANSYS based Optimality Criterion method. In this work, the minimization of compliance of the structures has been studied for both the isotropic and orthotropic material structures and then the results obtained by both the material have been compared.

2. THE OPTIMALITY CRITERION APPROACH

Optimality criteria are necessary conditions for minimality of the objective function and these can be derived by using either variational methods or extremum principles of mechanics. Optimality criteria (OC) method was analytically formulated by Prager and co-workers in 1960. It was later developed numerically and become a widely accepted structural optimization method (Venkaya et al. 1968).

The discrete topology optimization problem is characterized by a large number of design variables, N in this case. It is therefore common to use iterative optimization techniques to solve this problem, e.g. the method of moving asymptotes optimality criteria (OC) method, to name two. Here we choose the latter. At each iteration of the OC method, the design variables are updated using a heuristic scheme.

The Lagrangian for the optimization problem is defined as:

$$\mathcal{L}(x_j) = u^T Ku + \Lambda\left(\sum_{j=1}^{n} x_j v_j - V_o\right) + \lambda_1(Ku - F) + \sum_{j=1}^{n} \lambda_2^j (x_{min} - x_j) + \sum_{j=1}^{n} \lambda_3^j (x_j - 1) \quad (1)$$

Where, Λ, λ_1, λ_2, and λ_3 are Lagrange multipliers for the various constraints. The optimality condition is given by:

$$\frac{\partial \mathcal{L}}{\partial x_j} = 0 \qquad j = 1, 2, 3 \ldots . n \quad (2)$$

Now, Compliance,

$$C = u^T Ku \quad (3)$$

Differentiating eq. 3.8 w. r. t.x_j, the optimality condition can be written as:

$$B_j = \frac{-\frac{\partial c}{\partial x_j}}{\Lambda v_j} = 1 \quad (4)$$

The Compliance sensitivity can be evaluated as using eq. :

$$\frac{\partial C}{\partial x_j} = -p(x_j)^{p-1} u_j^T k_j u_j \quad (5)$$

Based on these expressions, the design variables are updated as follows:

$$x_j^{new} = \{\max(x_{min} - m), if\, x_j B_j^{\eta} \le (x_{min}, x_{min} - m\}$$
$$= \{x_j B_j^{\eta}, if \, \max(x_{min} - m) < x_j B_j^{\eta} < \min(1, x_j + m\}$$
$$\{\min(1, x_j + m, if \min(1, x_j + m) \le x_j B_j^{\eta}\} \quad (6)$$

Where, m is called the move limit and represents the maximum allowable change in x_j in a single OC iteration. Also, η is a numerical damping coefficient, and is usually taken to be $1/2$. The Lagrange multiplier for the volume constraintΛ, is determined at each OC iteration using a bisection algorithm. x_j is the value of the density variable at each iteration step. u_j is the displacement field at each iteration step determined from the equilibrium equations.

3. NUMERICAL EXAMPLES

In this section two example of optimization with pressure load and two example of optimization with point load are given. In these examples, all the four structures were solved by taking different isotropic properties for all the structures like, Young's modulus and Poisson's ratio and then all the four structures were solved for orthotropic material by taking Young's modulus 201GPa, 21.7GPa, 21.7GPa, Poisson's ratio 0.17 respectively and shear modulus 5.4GPa in all the direction respectively. For all the examples, volume usage fraction is 50% and a fixed mesh of 8-node quadrilateral element is used, the thickness of the structures is 1 mm and the convergence criteria are taken 0.0001.

3.1 Example 1

An overhanging beam of dimensions in the ratio of 6:1 is considered. The beam is subjected to pressure load of 200 N/mm and the geometry is shown in the Fig.1.1. Young's modulus is 200 GPa and Poisson's ratio is 0.29. The design domain is descretized into 7500 elements. The optimal solution obtained by proposed method is presented in Fig.1.2 (a) and (b). There is no checkerboard problem in the solution. The iteration histories for 50% volume fraction are presented in Fig. 1.3. The objective function decreases steadily and converges to 48.204 after 32 iterations for isotropic material and 48.798after 37 iterations for orthotropic material.

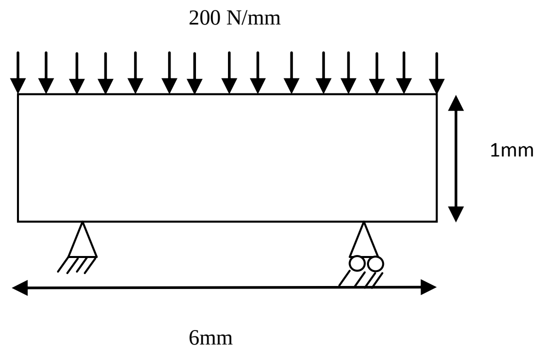
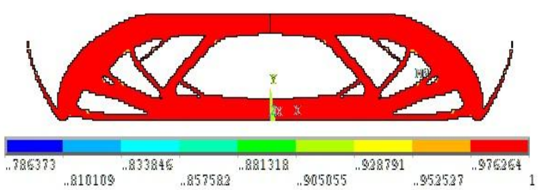
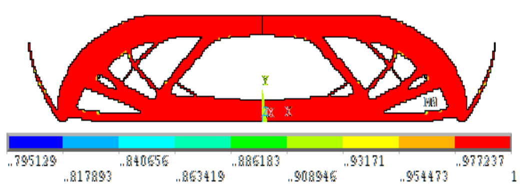


Fig.1.1 Geometry and boundary condition of overhanging beam

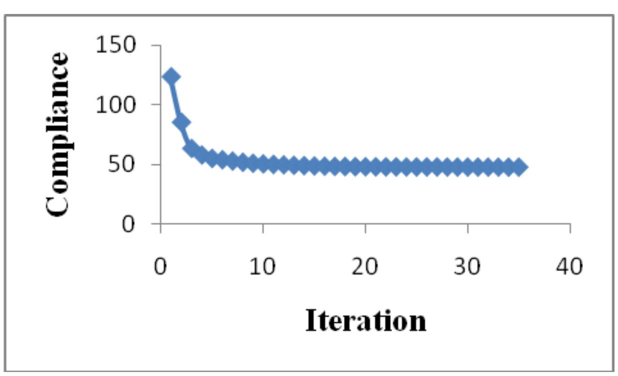


(a)



(b)

Fig. 1.2 Optimized shape for (a) Isotropic material (b) Orthotropic material



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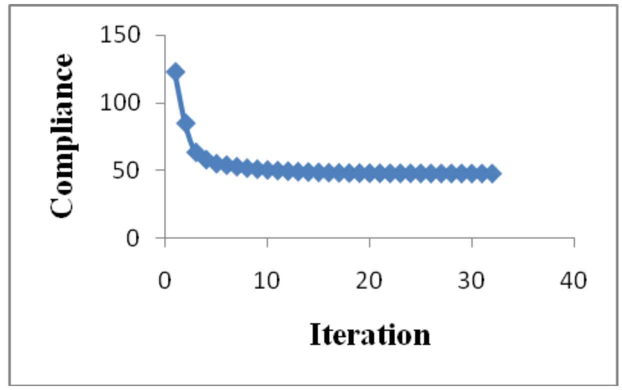
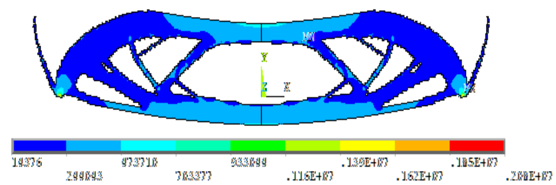


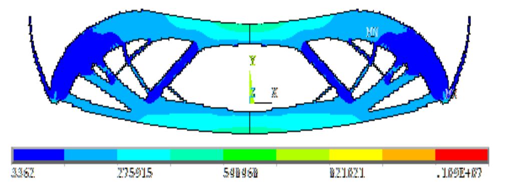
(b)

Fig. 1.3 Compliance v/s Iteration for overhanging beam (a) Isotropic (b) Orthotropic

After obtaining compliance and optimal shape we have obtained von misses stress. At the point of loading maximum stress occurred. Optimized structures for both the materials with von misses stress have shown in the Fig.1.4 (a) and (b) respectively. Maximum stress for isotropic material is 20814N/mm^2 and for orthotropic material is 12299N/mm^2. Now the deformed and un-deformed (initial structure) shape is shown in the Fig.1.5 (a) and (b) respectively below. The black portion shows the un-deformed structure and the blue portion shows the deformed structure of the beam.

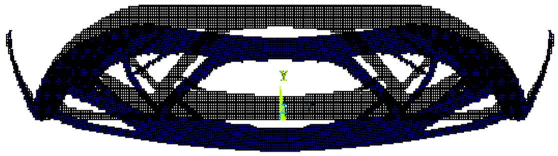


(a)

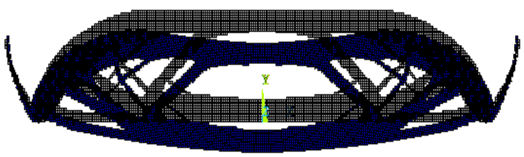


(b)

Fig. 1.4 Von misses stress induced in structure for (a) Isotropic (b) Orthotropic



(a)



(b)

Fig. 1.5 Deformed and un-deformed shape obtained for (a) Isotropic (b) Orthotropic

3.2 Example 2

A half ring of 10 mm outer and 3 mm inner radius is considered. The ring is subjected to pressure load of 1 N/mm and the geometry is shown in the Fig.1.6. Young's modulus is 1000 Pa and Poisson's ratio is 0.3. The design domain is descretized into 7381 elements. The optimal solution obtained by proposed method is presented in Fig.1.7 (a) and (b) respectively. There is no checkerboard problem in the solution. The iteration histories for 50% volume fraction are presented in Fig. 1.8 (a) and (b) respectively. The objective function decreases steadily and converges to 3.8219 after 31 iterations for isotropic material and 0.01899 after 34 iterations for orthotropic material.

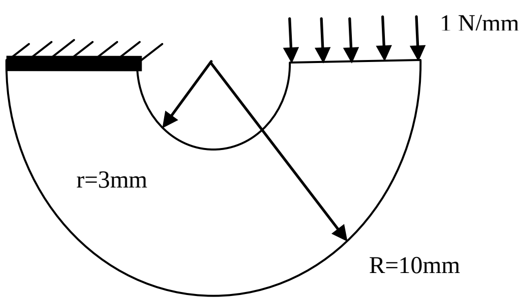
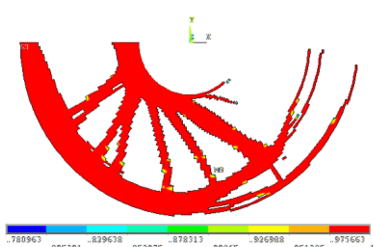
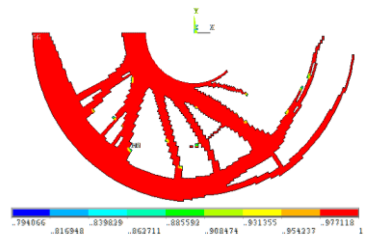


Fig.1.6 Geometry and boundary condition of overhanging beam

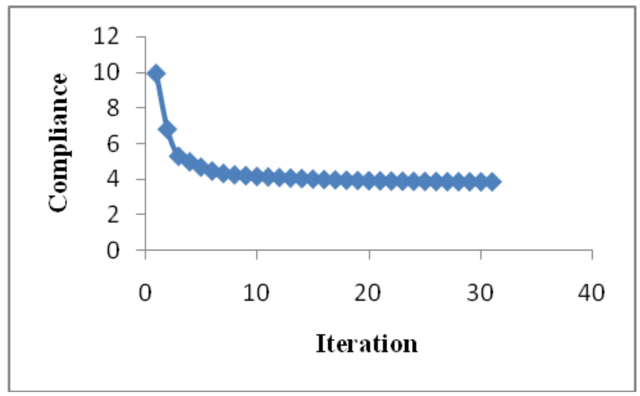


(a)



(b)

Fig. 1.7 Optimized shape for (a) Isotropic material (b) Orthotropic material



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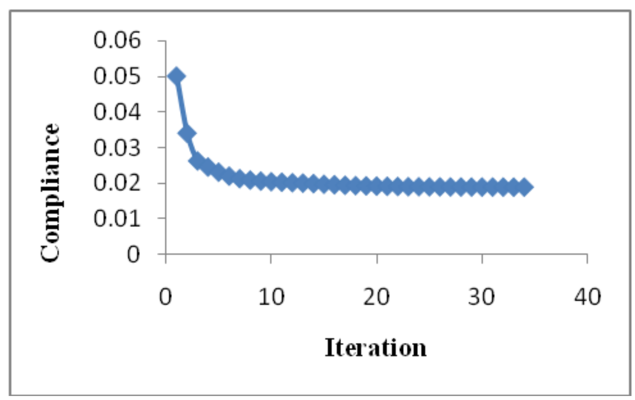
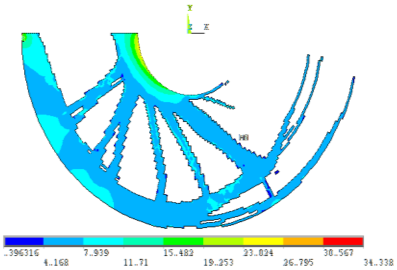


(b)

Fig. 1.8 Compliance v/s Iteration for Half ring (a) Isotropic (b) Orthotropic

After obtaining compliance and optimal shape we have obtained von misses stress. Near the boundary condition maximum stress occurred. Optimized structures for both the materials with von misses stress have shown in the Fig.1.9 (a) and (b) respectively. Maximum stress for isotropic material is 34.338 N/mm^2 and for orthotropic material is 13.273 N/mm^2. Now the deformed and un-deformed (initial structure) shape is shown in the Fig. 2 (a) and (b) respectively. The black portion shows the un-deformed structure and the blue portion shows the deformed structure of the beam.



(a)

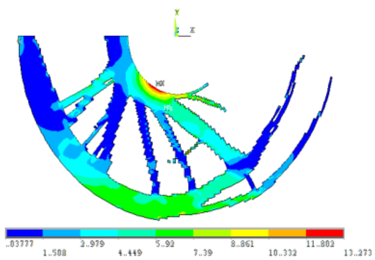


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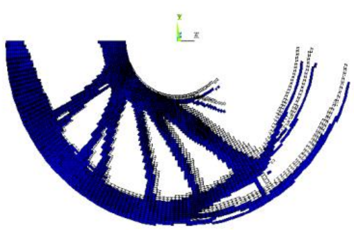
Fig.1.9 Von misses stress induced in structure for (a) Isotropic (b) Orthotropic



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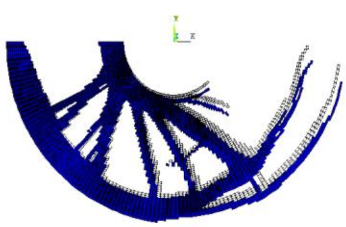


(b)

Fig. 2 Deformed and un-deformed shape obtained for (a) Isotropic (b) Orthotropic

3.3 Example 3

A Messerschmitt Bolkow Blohm (MBB) beam is considered. The beam is subjected to point load of 1 N and the geometry is shown in the Fig.2.1 Young's modulus is 100 Pa and Poisson's ratio is 0.3. The design domain is descretized into 1281 elements. The optimal solution obtained by proposed method is presented in Fig.2.2 (a) and (b) respectively. The iteration histories for 50% volume fraction are presented in Fig. 2.3 (a) and (b) respectively. The objective function decreases steadily and converges to 0.74065 after 39 iterations for isotropic material and 0.00036629 after 37 iterations for orthotropic material.

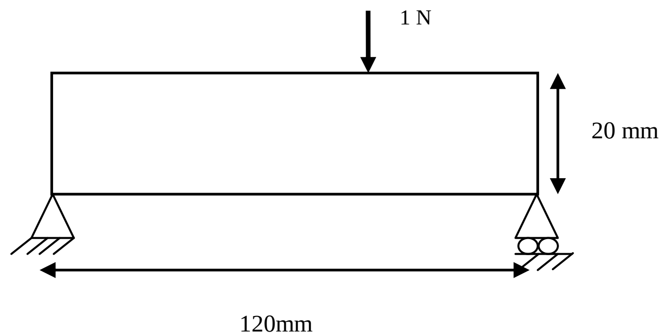
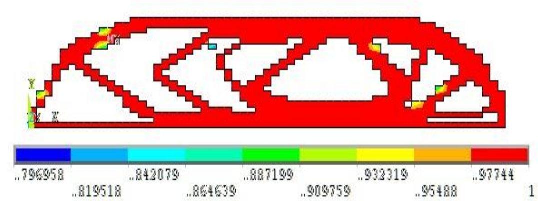
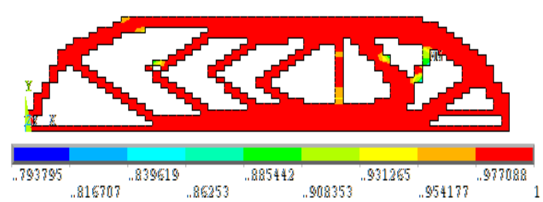


Fig.2.1 Geometry and boundary condition of overhanging beam



(a)



(b)

Fig. 2.2 Optimized shape for (a) Isotropic material (b) Orthotropic material

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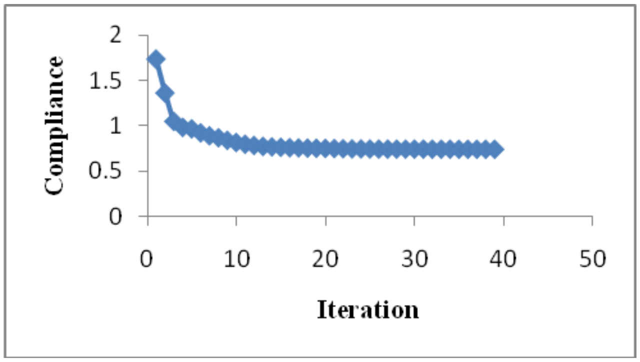

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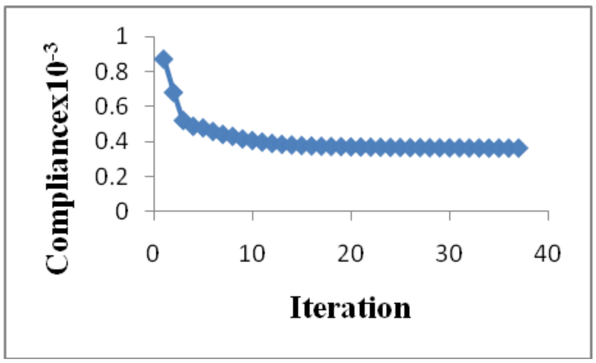
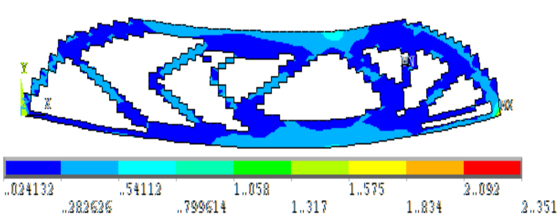


(b)

Fig. 2.3 Compliance v/s Iteration for MBB (a) Isotropic (b) Orthotropic

Now the von misses stress have calculated. At the point of loading maximum stress occurred. Optimized structures for both the materials with von misses stress have shown in the Fig.2.4 (a) and (b) respectively. Maximum stress for isotropic material is 2.351 N/mm² and for orthotropic material is 0.86758 N/mm². Now the deformed and un-deformed (initial structure) shape is shown in the Fig. 2.5 (a) and (b) respectively. The black portion shows the un-deformed structure and the blue portion shows the deformed structure of the beam.



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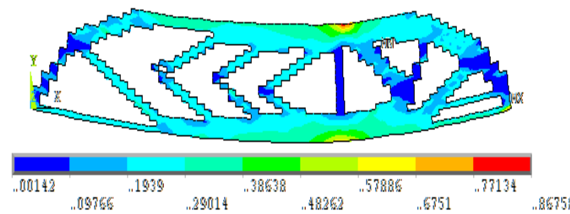

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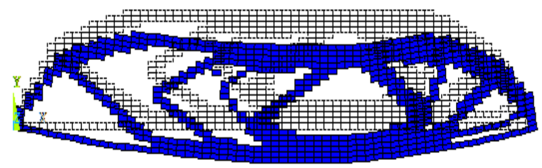
Fig. 2.4 Von misses stress induced in structure for (a) Isotropic (b) Orthotropic



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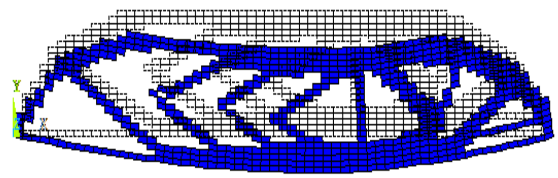


(b)

Fig. 2.5 Deformed and un-deformed shape obtained for (a) isotropic (b) orthotropic

3.4 Example 4

A Cantilever beam of dimension 45mm x 30mm is considered. The beam is subjected to point load of 1 N and the geometry is shown in the Fig.2.6. Young's modulus is 100 Pa and Poisson's ratio is 0.3. The optimal solution obtained by proposed method is presented in Fig.2.7 (a) and (b) respectively. The design domain is descretized into 3521 elements. The iteration histories for 50% volume fraction are presented in Fig. 2.8 (a) and (b) respectively.

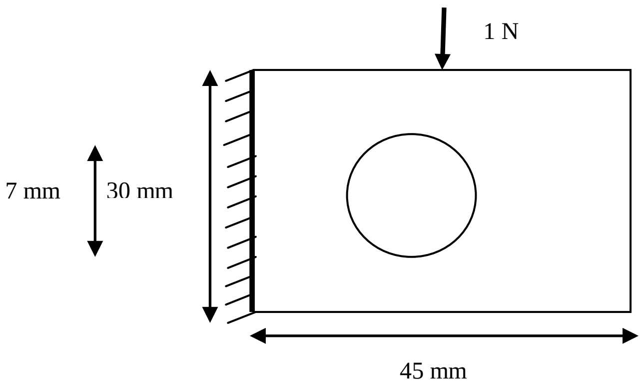
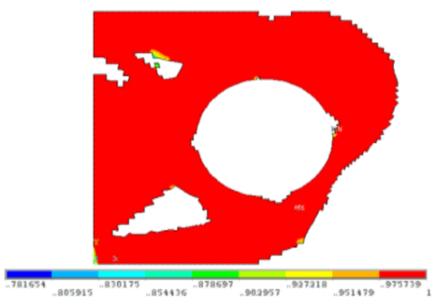


Fig.2.6 Geometry and boundary condition of overhanging beam



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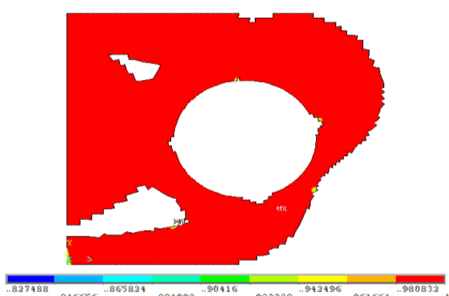

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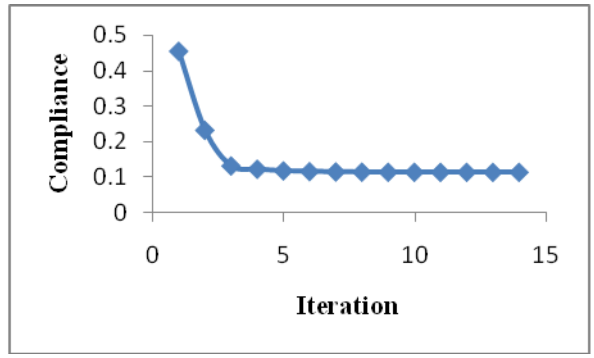
Fig. 2.7 Optimized shape for (a) Isotropic material (b) Orthotropic material



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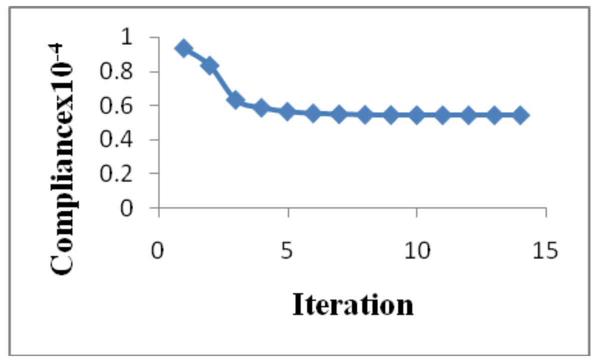
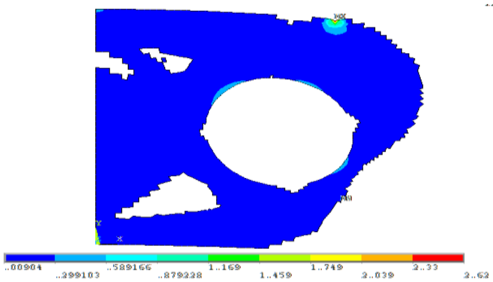


(b)

Fig. 2.8 Compliance v/s Iteration for MBB (a) Isotropic (b) Orthotropic

The objective function decreases steadily and converges to 0.11335 after 14 iterations for isotropic material and 0.54588x10^{-4} after 14 iterations for orthotropic material. Now the von misses stress have calculated. Near the point of loading maximum stress occurred. Optimized structures for both the materials with von misses stress have shown in the Fig.2.9 (a) and (b) respectively. Maximum stress for isotropic material is 2.351 N/mm^2 and for orthotropic material is 0.86758 N/mm^2. Now the deformed and un-deformed (initial structure) shape is shown in the Fig. 3 (a) and (b) respectively. The black portion shows the un-deformed structure and the blue portion shows the deformed structure of the beam.



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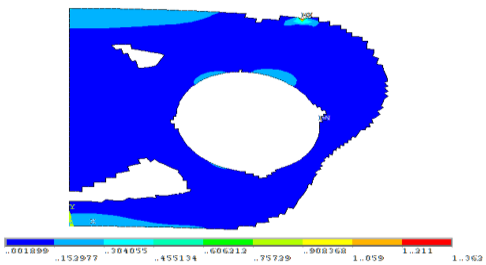

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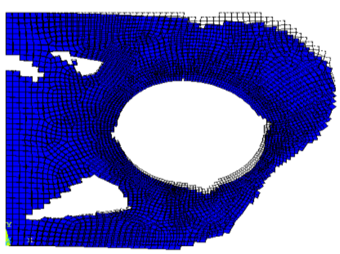
Fig. 2.9. Von misses stress induced in structure for (a) isotropic (b) orthotropic



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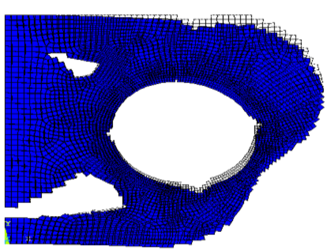


(b)

Fig. 3 Deformed and un-deformed shape obtained for (a) isotropic (b) orthotropic

4. EFFECT OF MESH SIZE/ NO. OF ELEMENTS ON COMPLIANCE AND TOPOLOGY OBTAINED

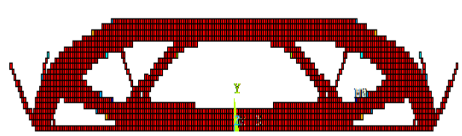
4.1 FOR ISOTROPIC MATERIAL OVERHANGING BEAM WITH UDL

In this section, effect of mesh size on the compliance and topology obtained has been studied for isotropic overhanging beam with UDL. The effect has been studied at the same volume fraction, load, Poisson's ratio, thickness, Young's modulus as taken in original model. To examine study the convergence characteristics of the OC in ANSYS, the compliance vs. Iteration plots for different number of elements have been plotted. The Table 1.1 below shows mesh size and compliance values.

Table 1.1: Variation of Compliance with number of elements

Mesh size (n_x, n_y, V_o)	Compliance	Iterations
120, 20, 0.5	49.777	32
130, 30, 0.5	48.866	31
140, 40, 0.5	48.659	37
150, 50, 0.5	48.204	32
160, 60, 0.5	47.209	30

For Overhanging with UDL values of compliance is decreasing as the number of elements are increasing. It is observed from Fig. 3.1 that the final optimal topology in each case is different and as the mesh size increases the optimal shape is much more finer then the previous one. It is observed that convergence rates are also random in nature for mesh size 120, 20 it takes 32 iterations to converge while it comes down to 31 iteration for mesh size 130, 30 and for mesh size 140, 40 it increases to 37 and is again decreases to 32 and 30 iterations for mesh size 150, 50 and 160, 60 respectively. The iteration histories for 50% volume as shown in Fig. 3.2



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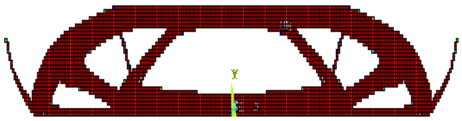
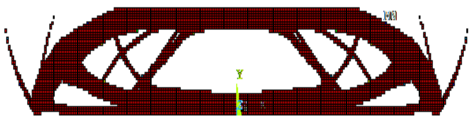
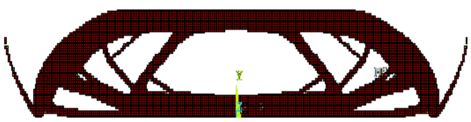
Mesh size 120, 20



Mesh size 130, 30



Mesh size 140, 40



Mesh size 150, 50

Mesh size 160, 60

Fig. 3.1 Optimal topologies for different mesh densities for IM Model 5 ($V_0 = 50\%$)

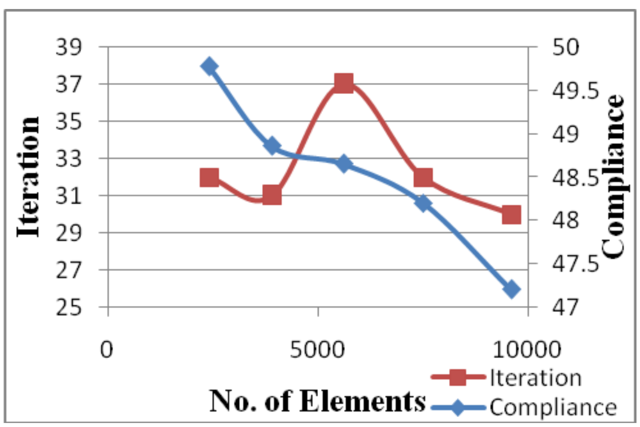


Fig. 3.2 Effect of number of elements on compliance and iterations for Isotropic material Model 5

4.2 FOR ORTHOTROPIC MATERIAL OVERHANGING BEAM WITH UDL

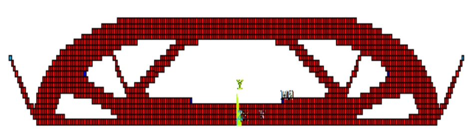
In this section, effect of mesh size on the compliance and topology obtained has been studied for Overhanging beam with UDL of orthotropic structures. The effect has been studied at the same volume fraction, Poisson's ratio, Thickness, Young's modulus. To examine study the convergence characteristics of the OC in ANSYS, the compliance vs. Iteration plots for different number of elements have been plotted.

The Table 1.2 below shows mesh size and compliance values.

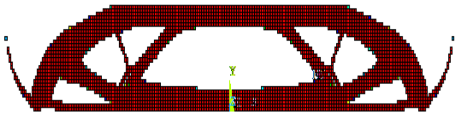
Table 1.2: Variation of Compliance with number of elements for OM Model 5

Mesh size (n_x, n_y, V_o)	Compliance	Iterations
120, 20, 0.5	49.596	25
130, 30, 0.5	48.509	29
140, 40, 0.5	48.651	28
150, 50, 0.5	47.898	35
160, 60, 0.5	48.513	30

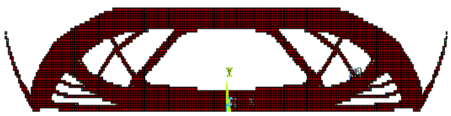
For Overhanging beam with UDL values of compliance is varying as the number of elements are increasing. Here the value of compliance increases from 49.596 N-mm again it decreases to 48.509 N-mm and then increases to 48.651 N-mm and further decreases to 47.898 N-mm and then increases to 48.513N-mm. It is observed from Fig.3.3 that the final optimal topology in each case is different and as the elements number is increased the optimal shape is much more finer then the previous one. It is observed that convergence rates are also random in nature for mesh size 120, 20 it takes 25 iterations to converge while it comes down to 29 iteration for mesh size 130, 30 and is again increased to 35 and 30 iterations for mesh size 150, 50 and 160, 60 respectively. The iteration histories for 50% volume as shown in Fig. 3.4



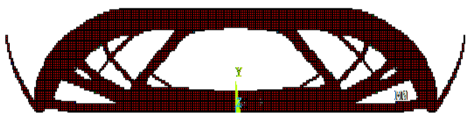
Mesh size 120, 20



Mesh size 130, 30



Mesh size 140, 40



Mesh size 150, 50

Mesh size 160, 60

Fig. 3.3 Optimal topologies for different mesh densities for Orthotropic material Model 5 ($V_0 = 50\%$)

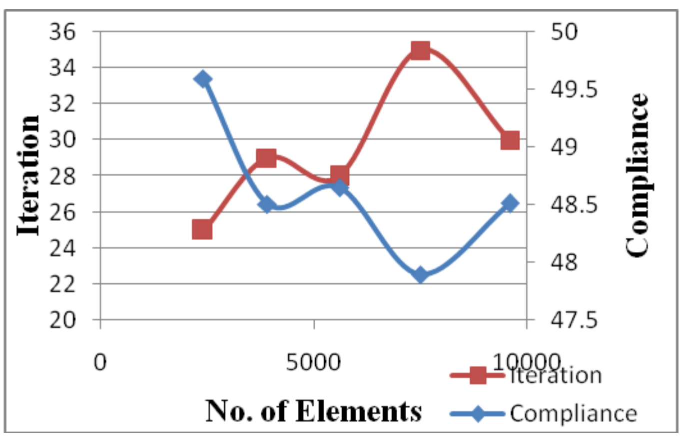


Fig. 3.4 Effect of number of elements on compliance and iterations for Orthotropic material Model 5

CONCLUSION

In the present paper, the topology optimization of structures which are subjected to pressure load and point load is solved by using an optimality criterion approach. Minimum compliance problem for optimization is considered. In this work, a commercially available finite element solver ANSYS 12.0 has been used to determine the optimal topology of the structures. The procedure is applied to a number of design problems. In all the cases, it is concluded that overall compliance decreases from initial to final value. The comparison of two materials shows that orthotropic materials structures have less compliance value than isotropic material structures in all the cases and that is why have stiffer structures. Von misses stresses are also less in orthotropic material structures than isotropic material structures. Compliance values decreases as the number of iterations increases for every model. As is evident from these plots, most of the compliance is dropped in the earlier iterations. Later on there is little variation in the compliance values.

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