International Research Journal of Engineering and Technology (IRJET) e-ISSN: 2395 -0056
Volume: 02 Issue: 04 | July-2015 www.irjet.net p-ISSN: 2395-0072

Moon shadow eclipse prediction of a lunar orbiting spacecraft

Shivali Kulshrestha[1], M.K. Bhaskar[2]*

[1]M.E. Student, Department of Electrical Engineering, M.B.M. Engineering College, J.N.V. University, Jodhpur-342001, India; Email: shivali.kulshrestha@gmail.com
[2]Associate Professor, Department of Electrical Engineering, M.B.M. Engineering College, J.N.V. University, Jodhpur-342001, India; Email: mkb31@rediffmail.com

---***---

Abstract - *In this paper, we describe a line of intersection conical shadow model for predicting the Moon shadow eclipse of a lunar orbiting spacecraft. Spherical and non-spherical shapes of the Moon are considered in the described model in order to show the effect of oblateness of the Moon. To observe the variation in the eclipse duration, two different altitudes of a fictitious spacecraft are considered. Obtained results from the study provide significant insights beneficial to a mission planner and designer.*

Key Words: Lunar orbiting spacecraft, Sun vector, lunar shadow.

1. INTRODUCTION

When a lunar orbiting spacecraft (S/C) passes through the dark side of the Moon, it experiences partial or full eclipse of the Moon. Apart from the occultation of the Sun by the Moon, the Earth may also cast shadow on the satellite; however these events occur less frequently due to the Earth is constantly in motion with respect to the satellite orbit, unlike the Moon-eclipse case where the occulting body is fixed at the focus of the orbit. Computation of eclipse conditions is generally applied for the Moon as the occulting body. These partial or total Moon shadow eclipse conditions correspond to the regions known as penumbra and umbra, respectively. Umbra is the region that is in total eclipse by the Moon and penumbra encompasses the region only partially eclipsed by the Moon or other primary body. The shape and size of the Moon's umbra and penumbra regions mainly depend on the Moon and the Sun sizes, and the distance between them.

Two particular categories of shadow prediction method for the Moon orbiting S/C exist: a simplified cylindrical shadow model and a conical shadow model. Cylindrical shadow model totally ignores the partial region which is not true in reality. Similar to the Earth's shadow eclipses of any Earth and Mars orbiting spacecraft [1-10], the Moon's shadow eclipses on a lunar orbiting spacecraft can be classified as either a Moon umbra or penumbra. In this paper, we describe the well known line of intersection method model to predict the Moon shadow eclipse of any lunar orbiting spacecraft. In Section 2, the mathematical model is described while results and discussion is given in Section 3. A concluding discussion is presented in Section 4.

Table1: Spacecraft properties of a lunar orbiting spacecraft.

Characteristics	Lunar Orbiter
Eccentricity	0
Inclination (degree)	90
RAAN (degree)	0
Argument of Perigee (degree)	0
True Anomaly (degree)	0

2. MATHEMATICAL MODEL

In this section, we describe a line of intersection conical shadow model [5, 6, 7, 9, 10] briefly for predicting the Moon shadow eclipse of any lunar orbiting spacecraft. Let $\vec{a}^m, \vec{r}_{\odot}^m$ and $\vec{r}_{\oplus}^m$ represent the spacecraft (S/C), Sun and Earth position vectors from the seleno-center, respectively. The magnitude $\left\|\vec{r}_{\oplus}^m\right\|$ gives the Earth to Moon distance.

Fig. 1 depicts the line of intersection method to predict the Moon's shadow eclipses occurring on a lunar orbiting spacecraft. Note that Fig. 1 is exaggerated for the sake of clarity. Approximating the Moon's surface as a spheroid:

$$\frac{\left(x^2+y^2\right)}{R_{\otimes e}^2}+\frac{z^2}{R_{\otimes p}^2}=1$$

(1)

where (x,y,z) are the coordinates of a point on the Moon's surface from the seleno-centered (Moon centered) frame, $R_{\otimes e}$ and $R_{\otimes p}$ are the equatorial and polar radius of the Moon, respectively. Note that when $R_{\otimes e} = R_{\otimes p}$, Eq. (1) reduces to the spherical lunar surface.

Assume Line 1 (Line 2) represents the line passing through the S/C to the Sun-edge 1 (Sun-edge 2). Sun-edge 1 to S/C vector, and Sun-edge 2 to S/C vector are denoted by $\vec{b}$ and $\vec{c}$, respectively. Any point (x,y,z) on Line 1 and Line 2 satisfies

$$\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3} \qquad (2)$$

and

$$\frac{x-a_1}{c_1} = \frac{y-a_2}{c_2} = \frac{z-a_3}{c_3}, \qquad (3)$$

respectively.

Assume that the vectors $\vec{r}_{s1}^{\,m}$ and $\vec{r}_{s2}^{\,m}$ define the lines from the seleno-center to the Sun-edge 1 and 2, respectively. To evaluate the vectors $\vec{b}$ and $\vec{c}$, the coordinates of the Sun-edges are required. Let the vector $\vec{r}_{\odot}^{\,m}$ represents the Sun position vector from the seleno-center. The vector $\hat{S}_p^m$, which is the unit vector orthogonal to $\vec{r}_{\odot}^{\,m}$ lying in the plane defined by the seleno-center, Sun, and S/C, is used to find these Sun-edges. The vector $\hat{S}_p^m$ is expressed as:

$$\hat{S}_p^m = \frac{\hat{S}_p^m}{\left\|\hat{S}_p^m\right\|} = \frac{\vec{r}_{\odot}^{\,m} \times \vec{r}_i^{\,m}}{\left\|\vec{r}_{\odot}^{\,m} \times \vec{r}_i^{\,m}\right\|}, \ where\ \vec{r}_i^{\,m} = \vec{r}_{\odot}^{\,m} \times \vec{a}^{\,m}. \qquad (4)$$

Thus, the Sun-edge points can be expressed as:

$$\left.\begin{aligned} \vec{r}_{s1}^{\,m} &= \vec{r}_{\odot}^{\,m} + R_{\odot}\hat{S}_p^m \\ \vec{r}_{s2}^{\,m} &= \vec{r}_{\odot}^{\,m} - R_{\odot}\hat{S}_p^m \end{aligned}\right\} \qquad (5)$$

Hence, the vectors $\vec{b}$ and $\vec{c}$ in the seleno-centered frame are given by:

$$\left.\begin{aligned} \vec{b} &= \vec{a}^{\,m} - \vec{r}_{s1}^{\,m} \\ \vec{c} &= \vec{a}^{\,m} - \vec{r}_{s2}^{\,m} \end{aligned}\right\} \qquad (6)$$

Combining Eqs. (1) and (2), the following relation can be found for the intersection between the lunar surface and Line 1:

$$x^2 R_{\otimes p}^2 + \left[\frac{b_2}{b_1}(x-a_1)+a_2\right]^2 R_{\otimes p}^2 + \left[\frac{b_3}{b_1}(x-a_1)+a_3\right]^2 R_{\otimes e}^2 = R_{\otimes}^2 \qquad (7)$$

Eq. (7) can be re-written as:

$$A_{Line1}x^2 + B_{Line1}x + C_{Line1} = 0 \qquad (8)$$

Similarly, the intersection of lunar surface and Line 2 gives:

$$A_{Line2}x^2 + B_{Line2}x + C_{Line2} = 0 \qquad (9)$$

When the conditions

$$\left.\begin{aligned} (B_{Line1})^2 - 4A_{Line1}C_{Line1} &\ge 0,\ \text{Line 1} \\ (B_{Line2})^2 - 4A_{Line2}C_{Line2} &\ge 0,\ \text{Line 2} \end{aligned}\right\} \qquad (10)$$

are satisfied, Eqs. (8) and (9) give the real solutions. If the condition (10) is satisfied, an intersection will occur. When the distance between the point of intersection and the Sun is less than the distance between the Sun and the Moon, S/C is not in the Moon's shadow. Otherwise, if both lines interest the lunar surface, S/C is in the Moon's umbra; if only one line intersects the lunar surface, S/C is in the Moon's penumbra else it is not in the Moon's shadow.

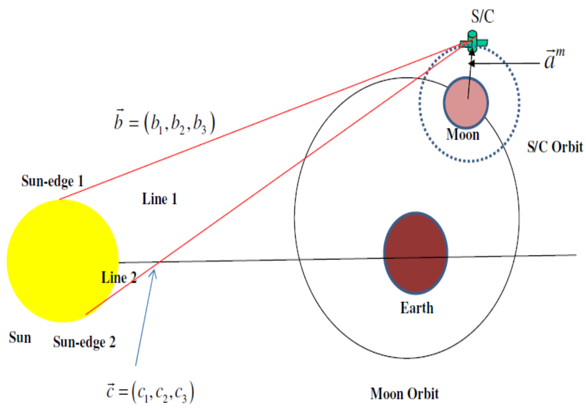

Fig. 1: Moon eclipse prediction of a lunar orbiter mission.

3. RESULTS AND DISCUSSION

The mathematical model discussed in Section 2 predicts when a lunar orbiting spacecraft is in the shadow (umbra/penumbra) of the Moon using the Sun vector $\vec{r}_{\odot}^{\,m}$, the Earth vector $\vec{r}_{\oplus}^{m}$, and the spacecraft vector $\vec{a}^{m}$ in the seleno-centered frame. The model is simulated for a typical trajectory of a fictitious lunar orbiting spacecraft of altitudes 100, and 200 km whose orbital characteristics are given in Table 1. State vectors (position and velocity) of the spacecraft, in the seleno-centric J2000 reference frame, are generated using two body equations of motion of the spacecraft at 1 sec interval with initial parameters as given in Table 1. The Sun and Earth state vectors are taken from the Jet Propulsion Laboratory planetary (JPL) ephemerides DE421.

Table 2 shows the Moon's shadow umbra and penumbra entry and exit timings on a lunar orbiting spacecraft on the mentioned date. From Table 2, it can be seen that lunar penumbra occurs for a few seconds while lunar umbra exists for some minutes (~ 46 minutes) for lunar orbiting spacecraft. From Table 2, it can be noticed that there is almost a constant difference of 4 seconds between umbra/penumbra entry and exit using both spherical and non-spherical models at different altitudes of the spacecraft.

We note that the difference between the equatorial and polar radius of the Moon is almost 2 km. As the altitude of the spacecraft increases Moon's oblateness effect remains almost the same. Further, as the altitude of the spacecraft increases, the Moon's shadow eclipse duration also remains almost the same (46 minutes).

Table 2: Shadow eclipse duration of the Moon for a lunar orbiter mission on 28/Sep/2015 at different altitudes.

Altitude (km)	Model	Penumbra Start Time (UTCG)	Umbra Start Time (UTCG)	Umbra Stop Time (UTCG)	Penumbra Stop Time (UTCG)
100	Non-spherical	00:42:05	00:42:15	01:28:26	01:28:36
		02:39:54	02:40:03	03:26:14	03:26:24
	Spherical	00:42:01	00:42:11	01:28:30	01:28:40
		02:39:50	02:39:59	03:26:18	03:26:28
200	Non-spherical	00:31:21	00:31:32	01:16:23	01:16:34
		02:38:54	02:39:05	03:23:56	03:24:07
	Spherical	00:31:18	00:31:28	01:16:26	01:16:37
		02:38:51	02:39:01	03:23:59	03:24:10

4. CONCLUSIONS

In this work, the well known line of intersection conical shadow model is considered for predicting the Moon shadow eclipse of any lunar orbiting spacecraft. From the study, we notice that the Moon's penumbra lasts for a few seconds while Moon's penumbra lasts for some minutes. As the altitude of the spacecraft increases, the Moon's oblateness remains almost the same. It can also be noticed that as the altitude of the spacecraft increases, the Moon's shadow eclipse duration remains almost the same. The output of the carried out study highlights the importance of modeling the oblate shape of the Moon when predicting the lunar shadow events of a lunar orbiting spacecraft while choosing its altitude.

REFERENCES

[1] J. R. Wertz, Spacecraft Attitude Determination and Control System, Kluwer, 2002 (Reprint).
[2] O. Montenbruck, E. Gill, Satellite orbits: models, methods, and applications, Springer, (2005).
[3] D. A. Vallado, Fundamental of Astrodynamics and Applications, Fourth Edition, Microcosm, Hawthorne, CA, 2013.
[4] D. Vokrouhlicky, P. Farinella, F. Mignard, Solar radiation pressure perturbations for Earth Satellites: IV. Effects of the Earth's polar flattening on the Shadow structure of the Penumbra transitions, Astron. Astrophys., 1996 (307) 635-644.
[5] S. Adhya, A. Sibthorpe, M. Ziebart, P. Cross, Oblate Earth Eclipse State Algorithm for Low Earth-Orbiting Satellites, J. Spacecr. Rockets, 41(2004) 157-159.
[6] V.K. Srivastava, Ashutosh, M. Pitchaimani, B.S. Chandrasekhar, Eclipse Prediction Methods for LEO satellites with Cylindrical and Cone geometries: A Comparative study of ECSM and ESCM to IRS satellites, Astron. Comput., 2 (2013) 11-17.
[7] V. K. Srivastava, Ashutosh, M. V. Roopa, B. N. Ramakrishna, M. Pitchaimani, B. S. Chandrasekhar, Spherical and Oblate Earth conical Shadow models for LEO satellites: Applications and comparisons with real time data and STK to IRS satellites, Aerosp. Sci. Technol. 33 (2014) 135-144.
[8] V. K. Srivastava, S. M. Yadav, Ashutosh, J. Kumar, B. S. Kushvah, B. N. Ramakrishna, P. Ekambaram, Earth conical shadow modeling for LEO satellite using reference frame transformation technique: A comparative study with existing earth conical shadow models, Astron. Comput., 9 (2015) 34-39.
[9] V.K. Srivastava, J. Kumar, S. Kulshrestha, A. Srivastava, M.K. Bhaskar, B.S. Kushvah, P. Shiggavi, D.A. Vallado, Lunar shadow eclipse prediction models for the Earth orbiting spacecraft: Comparison and application to LEO and GEO spacecrafts, Acta Astronautica, 110 (2015) 206-213.

[10] V.K. Srivastava, J. Kumar, S. Kulshrestha, B.S. Kushvah, M.K. Bhaskar, Somesh S., M.V. Roopa, B.N. Ramakrishna, Eclipse Modeling for the Mars Orbiter Mission, Advances in Space Research, 56 (4) (2015) 671-679.

BIOGRAPHIES

Mrs. Shivali Kulshrestha has done B.E. in Electrical Engineering from JECRC, Jodhpur. Currently, she is pusuing her M.E. in Electrical Engineering with specialization in power system from M.B.M. Engineering College, J.N.V. University, Jodhpur, India.

Dr. M.K. Bhaskar is currently working as an associate professor in Electrical Engineering, M.B.M. Engineering College, J.N.V. University, Jodhpur, India. He is having teaching and R&D experience more than two decades.