# STRUCTURAL ANALYSIS AND TOPOLOGY OPTIMIZATION OF CONTINUOUS LINEAR ELASTIC ORTHOTROPIC STRUCTURES USING OPTIMALITY CRITERION APPROACH IN ANSYS

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**Abstract –** *The structural topology optimization of linear elastic continuous structures with orthotropic material properties is performed in this paper, for given design domain and boundary conditions of mentioned structures. The various parameters are obtained such as optimal topology, compliance, displacement and stresses using optimality criterion approach using ANSYS. The orthotropic material properties for all four structures are taken same as of Kelvar material. The present paper also consists of the comparison between the results of optimal topologies obtained for Isotropic Material and Orthotropic Material in ANSYS using O C approach. The study is done on four illustrative examples.*

*Key Words: Topology Optimization, Simple Column, Beam, Short Pressurized Beam, Flat Plate with Central Circular Hole, ANSYS, Compliance, OC Approach, OM, IM, etc…*

## 1. INTRODUCTION

This paper presents the topology optimization of linear elastic continuous orthotropic material (OM) structures. The optimal design of a simple column, a beam, a three point supported pressurized short beam and a flat plate with a central circular hole are obtained in ANSYS. The plane state of stress condition is considered for the mentioned problems. The optimal design is performed by ANSYS software which gives the optimum topology of the structures mainly it is achieved by reducing material in the design domain. For the optimization, the finite element method is used to discretize the structures and topology is performed by removing parts of elements to get a continuum design with holes. The models are considered to be linearly elastic isotropic material (IM) structures whose analysis has not been done so far by using optimality criterion approach in ANSYS. The work presented in the paper is obtaining compliance value, optimal topology, deformed shape, displacements of optimized shapes with deformed and undeformed edges, stress distribution in the optimized topology and von-Mises stresses variation of the structures.

The research in the area of topology optimization is extremely active recent years. Several topology optimization methods have been proposed, and used for the design of practical problem. However, there still exist a number of problems such as checkerboard, mesh-dependence, and local minima being investigated currently [1].

The topology optimization of continuum structures corresponds to finding the connectedness, shape and number of holes such that the objective-function is extremized [1].

Bendsøe and Kikuchi [2] introduced a periodic microstructure to the material through the use of so-called homogenization approach to topology optimization that allows the volume density of material to cover the complete range of values from 0 to 1 by changing the size of microstructure. To use this method, it is necessary to determine the effective material characteristic by homogenization, and results are obtained with large regions of perforated microstructure or composite materials ($0< p <1$, $p$ is the density function, 0 is void and 1 is material). Another approach that is called density function method [3] disregards the details of the microstructure and defines the elasticity tensor as a function of density of material directly. The SIMP (Simple Isotropic Material with Penalization) approach [4] is kind of density function method, in which the stiffness tensor of the intermediate density material is penalized with an exponential function of density to somehow approach a 0-1 design.

To control the value of $p$ can control the speed of convergence and the rate of intermediate density material in the result design. It is a popular method and has also been widely used because of its simplicity. In this study, the SIMP approach is improved in order to be more efficient in optimization process. Using the concept of Michell truss, we assume material to be a pseudo orthotropic continuum and introduce new penalties to the Young's modulus[5 & 6]. Four examples are attached for topology optimization using optimality criterion approach in ANSYS.

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Gunwant et al. (2013), obtained topologically optimal configuration of sheet metal brackets using Optimality Criterion approach through commercially available finite element solver ANSYS and obtained compliance versus iterations plots for various aspect ratio structures (brackets) under different boundary conditions [8].
Chaudhuri, worked on stress concentration around a part through hole weakening a laminated plate by finite element method. Peterson has developed good theory and charts on the basis of mathematical analysis. Patle et al. determined stress concentration factors in plate with oblique hole using FEM. Various angle of holes have been considered to evaluate stress concentration factors at such holes. The stress concentration factors are based on gross area of the plate[9].
The goal of topological optimization is to find the best use of material for a body such that an objective criterion (i.e. global stiffness, natural frequency, etc.) attains a maximum or minimum value subject to given constraints (i.e. volume reduction).
In this work, maximization of static stiffness has been considered. This can also be stated as the problem of minimization of compliance of the structure. Compliance is a form of work done on the structure by the applied load. Lesser compliance means lesser work is done by the load on the structure, which results in lesser energy is stored in the structure which in turn, means that the structure is stiffer.
ANSYS employs gradient based methods of topology optimization, in which the design variables are continuous in nature and not discrete. These types of methods require a penalization scheme for evolving true, material and void topologies. SIMP (Solid Isotropic Material with Penalization) is a most commonly penalization scheme, and is explained in the next section.

## 2. MATERIALS AND METHODS

Topology optimization aims to answer the question, what is the best domain in which to distribute material in order to optimize a given objective function subject to some constraints?
Topology optimization is an incredibly powerful tool in many areas of design such as optics, electronics and structural mechanics. The field emerged from structural design and so topology optimization applied in this context is also known as structural optimization.

Table-1: Nine Independent Elastic Constants for Orthotropic Material (Kelvar) [16]

| Orthotropic Properties of Kelvar | | | | | |
|---|---|---|---|---|---|
| Young's Modulus (GPa) | | Poisson's Ratio | | Shear Modulous (GPa) | |
| $E_x$ | 150 | $\upsilon_{xy}$ | 0.35 | $G_{xy}$ | 1.5 |
| $E_y$ | 4.2 | $\upsilon_{yz}$ | 0.35 | $G_{yz}$ | 2.9 |
| $E_z$ | 4.2 | $\upsilon_{xz}$ | 0.35 | $G_{xz}$ | 1.5 |

Applying topology optimization to structural design typically involves considering quantities such as weight, stresses, stiffness, displacements, buckling loads and resonant frequencies, with some measure of these defining the objective function and others constraining the system.
In structural design, topology optimization can be regarded as an extension of methods for size optimization and shape optimization. Size optimization considers a structure which can be decomposed into a finite number of members. Size optimization then seeks to find the optimal values of the parameters defining the members.
Shape optimization is an extension of size optimization in that it allows extra freedoms in the configuration of the structure such as the location of connections between members. The designs allowed are restricted to a fixed topology and thus can be written using a limited number of optimization variables. The topology optimization is performed using optimality criteria method through ANSYS software.
Analytical method provides accurate solutions with applications limited to simple geometries. Experimental methods are used to test prototypes or full scale models. However they are costly and may not be feasible in certain cases.
In this paper, we treat the problem of maximum stiffness of structures with the given amount of material. Design for maximum stiffness of statically loaded linearly elastic structures is equivalent to design for minimum compliance defined as the work done by the set of given loads against the displacements at equilibrium[1].
This process leads to a set of linear algebraic simultaneous equations for the entire system that can be solved to yield the required field variable (e.g., strains and stresses). As the actual model is replaced by a set of finite elements, this method gives an approximate solution rather than exact solution. However the solution can be improved by using more elements to represent the model.

### 2.1 The Optimality Criterion approach

The discrete topology optimization problem is characterized by a large number of design variables, N in this case. It is therefore common to use iterative optimization techniques to solve this problem, e.g. the method of moving asymptotes, optimality criteria (OC) method, to name two. Here we choose the latter. At each iteration of the OC method, the design variables are updated using a heuristic scheme.
Optimality criteria (OC) method was analytically formulated by Prager and co-workers in 1960. It was later developed numerically and become a widely accepted structural optimization method (Venkaya et al. 1968).OC methods can be divided into two types. One type is rigorous mathematical statements such as the Kuhn-Tucker conditions. The other is algorithms used to resize the structure for satisfying the optimality criterion.

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Different optimization problems require different forms of optimality criterion.

This paper considers the maximization of static stiffness through the inbuilt topological optimisation capabilities of the commercially available FEA software to search for the optimum material distribution in two plane stress structures.

The optimum material distribution depends upon the configuration of the initial design space and the boundary conditions (loads and constraints).

The goal of the paper is to minimize the compliance of the structure while satisfying the constraint on the volume of the material reduction.

Minimizing the compliance means a proportional increase in the stiffness of the material. A volume constraint is applied to the optimisation problem, which acts as an opposing constraint [10].

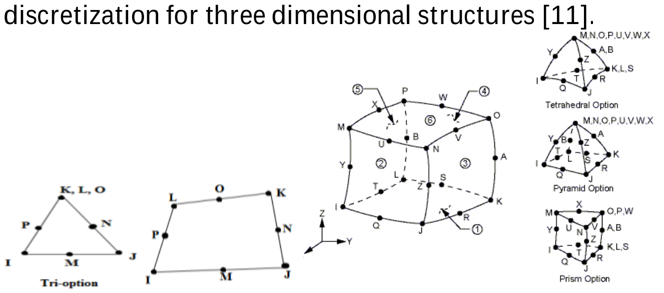
### 2.1.1 Element Type

Selection of element type is one of the most important features in topology optimization through ANSYS. Topological optimization in ANSYS supports 2-D and 3-D solid elements. By this technique the model can be discretized into following element type:

(a). 2-D Solids: Plane 82

(b). 3-D Solids: Plane 95

Plane 82: This is an 8-node element and is defined by eight nodes having two degree of freedom at each node. Translations in the nodal x and y directions (Figure-1a). The element may be used as a plane element or as an axi-symmetric element. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. In the present paper structures are considered to be 2 D, so here plane 82 8-node type element is taken for discretization. Figure-1b shows the discretization for three dimensional structures [11].




(a) Plane 82 (b) Plane 95

Fig-1: Element type (a) for 2 D and (b) for 3 D

To visualize, more the volume of material, lower will be the compliance of the structure and higher will be the structural stiffness of the structure. For implementation of this, APDL codes for various beam modelling and topological optimisation were written and run in ANSYS [11].

## 2.2 Specimen Geometry and Boundary Conditions

In the present investigation, the orthotropic material (OM) properties are used for topology optimization and structural analysis on all four specimen geometries and boundary conditions applied have been used as shown in the figures below. The specimen 1 is taken from the research work of Philip Anthony Browne [12], specimen 2 is taken from Huang *et al.* [13], specimen 3 is taken from the research work of H. Zhang *et al.* [14] And specimen 4 is taken from Kishan Anand *et al.* [15]. All the four models are under plane state of stress. The used OM properties of Kelvar are given in table-1 [16].

2.2.1 Centrally loaded column (Model 1): Example 1 is a stiffness topology optimization problem for a simple column structure. Here is presented a somewhat trivial optimization problem which is included for comparison with results of OM and IM properties. The design domain is square and a unit load is applied vertically downwards at the centre of the top of the design domain and the base is fixed, as shown in Figure-2.

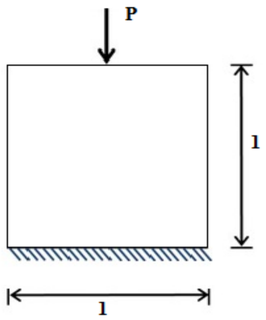



Fig-2: Design domain of model column problem. This is a square domain with a unit load acting vertically at the midpoint of the upper boundary of the space.

2.2.2 Model 2: Example 2 is a stiffness topology optimization problem for a beam structure which is supported by both ends and vertically loaded (P = 100 N) in the middle of its upper edge as depicted in Fig.3. The computations are performed in the domain with 200 × 100 four-node plane stress elements.

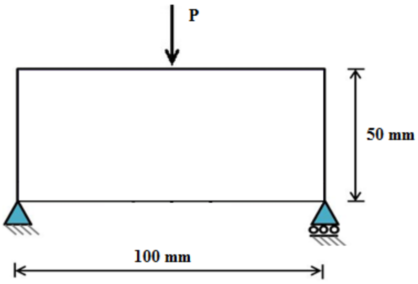



Fig-3: Geometry and boundary conditions of a beam (Model-2)

2.2.3 Model 3: A general case of more than two support points is considered. As shown in Fig. 4, another support point is added in the middle bottom of the beam and unit pressure is applied on top surface of short beam. The elastic properties and volume fraction of solid material are the unchanged. Eight hundred square elements are used to discretize the design domain. The optimal topology of structure is shown in Fig. 8(a & b).

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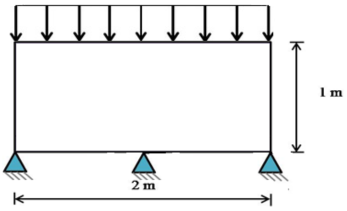



Fig-4: A three-point supported short beam design, a Pressurized short beam sketch.

2.2.4 Model 4: In structure 4, the topology optimization and nodal analysis of a rectangular plate with a central circular hole of dimensions 400mm x 100mm with central hole of diameter 10mm under transversely downwards static load at right end of top edge of magnitude 1000 N has been analyzed using optimality criterion approach in ANSYS. In the fig-5, centre of plate depicts a fixed circular hole, taking hole as constraint in all DoF (considering it as fixed for some purpose).

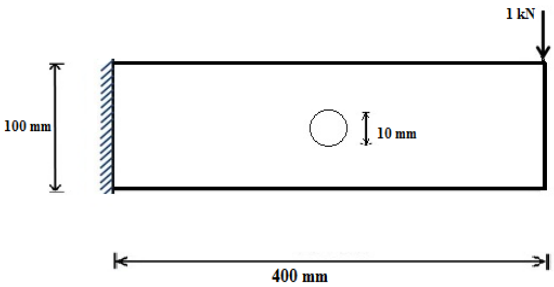



Fig-5: The optimized topology with a new kind of element based search scheme of load surfaces method

## 3. RESULTS

In this section the optimal topology of structures are shown obtained from the Optimality Criteria Approach through ANSYS. Further the iteration versus values of compliances for all the structures are shown in the charts [1, 2 & 3]. Chart shows the graph between Compliance and iterations.

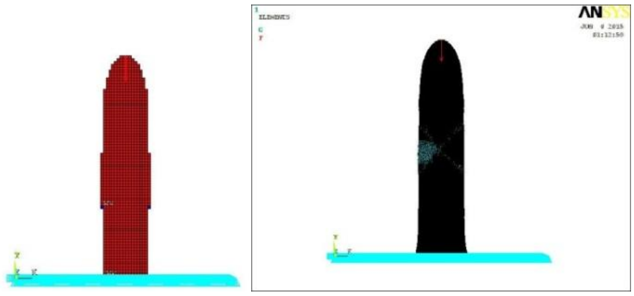
### 3.1 Structure Compared:

In this section, final compliance and optimal shape of the models obtained with the help of ANSYS based Optimality Criterion for orthotropic material (OM) properties are compared with isotropic material (IM) properties.

### 3.2 Optimized Shape:

In this section, the optimal topologies are shown for orthotropic material properties and isotropic material properties (E & υ are shown in table-3) of the four structures which are mentioned above.

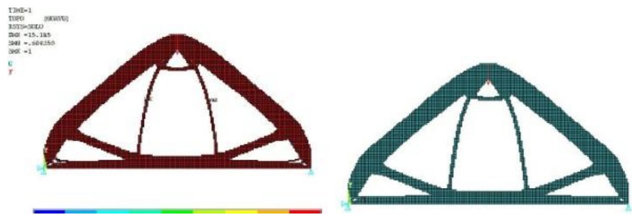
Figure 6 (a), shows the topology optimization through OC method in ANSYS for OM and figure 6 (b) represents the optimized topology of same structure using isotropic material (IM) properties, which are nearly same, for the simple column structure under the given boundary and loading conditions.



(a)OC in ANSYS for OM (b) OC in ANSYS for IM

Fig-6: Optimal design for Model 1 using optimality criteria approach

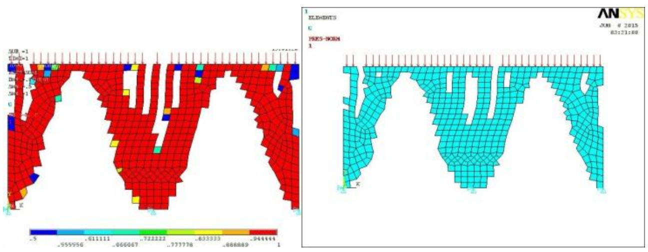
The topologically optimized shape as obtained for the beam (model 2) under the given boundary conditions is obtained by using optimality criteria using ANSYS. Figure 7 (a), shows the topologically optimized shape for OM and fig-7 (b), shows optimal shape for IM in ANSYS.



(a)OC in ANSYS for OM (b) OC in ANSYS for IM

Fig-7: Optimal design for Model 2 using optimality criteria approach

The topologically optimized shape as obtained for the three point supported pressurized short beam (model 3) under the given boundary conditions is obtained by using optimality criteria using ANSYS. Figure 8 (a), shows the topologically optimized shape for OM and fig-8 (b), shows optimal topology for IM in ANSYS.



(a)OC in ANSYS for OM (b) OC in ANSYS for IM

Fig-8: Optimal design for Model 3 using optimality criteria approach

The topologically optimized shape as obtained for the flat plate with central circular hole (model 4) under the given boundary conditions is obtained by using optimality criterion using ANSYS. Figure 9 (a & b), shows the topologically optimized shape for OM and IM respectively through ANSYS.

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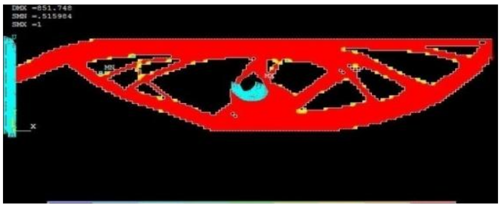
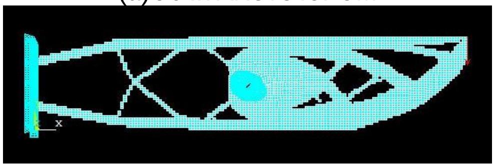


(a)OC in ANSYS for OM



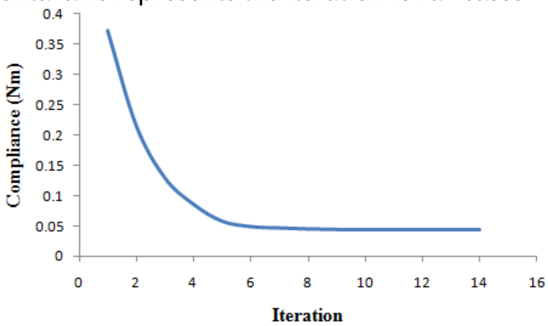
(b) OC in ANSYS for IM

Fig-9: Optimal design for Model 4 using optimality criteria approach

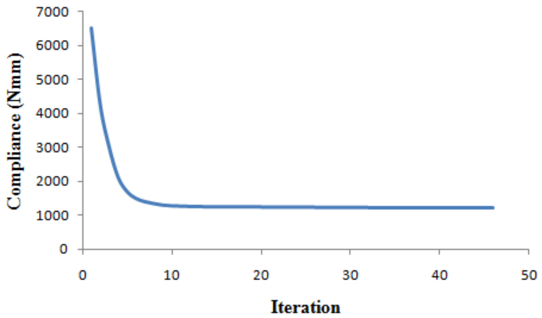
The optimal topology through ANSYS for all the four structures are nearly same as obtained for structures with orthotropic material as well as isotropic material properties.

### 3.3 Compliance:

For structure 1, the initial value of compliance was 0.372197 and the final value as obtained after 14 iterations is 0.0427 for mesh size of 200. Variation of compliance with iteration is shown in the graph 1(a) below. Vertical axis represents the compliance and the horizontal axis represents the iteration for all cases.




(a) Simple Column

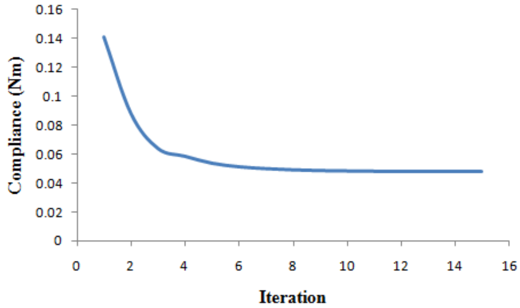



(b) Beam

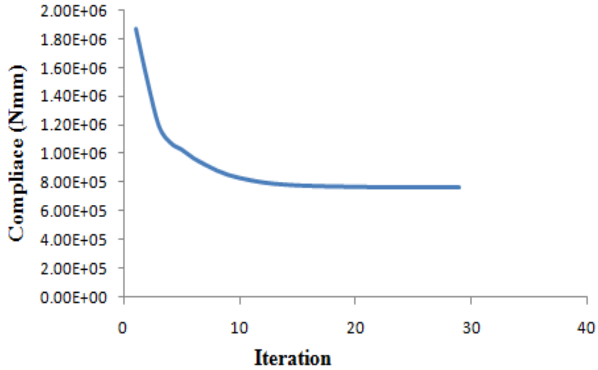
Chart -1: Compliance and iteration plot for (a) Model 1 and (b) Model 2

For structure 2, the initial value of compliance was 6518.5 and the final value as obtained after 46 iterations is 1222.3. Variation of compliance with iteration is shown in the graph 1(b) above.

For structure 3, the initial value of compliance was 0.141215 and the final value as obtained after 15 iterations is 0.04805. Variation of compliance with iteration is shown in the graph 2(a) below.




(a) Three Point Support Short Beam




(b)Plate with Central Circular Hole

Chart -2: Compliance and iteration plot for (a) Model 3 and (b) Model 4

For structure 4, the initial value of compliance was 1866840 and the final value as obtained after 29 iterations is 763574. Variation of compliance with iteration is shown in the graph 2(b) above. Vertical axis represents the compliance and the horizontal axis represents the iteration.

The compliance obtained by ANSYS is nearly same as that obtained for OM and IM properties.

Table-2: Comparison of Compliances between Optimal Topologies for Structures of Orthotropic Materials and Isotropic Materials for Given Volume Fraction

| S. N. | Structure | Final Compliances by OC in ANSYS | | Volume Fraction |
|---|---|---|---|---|
| | | OM | IM | |
| 1. | Simple Column | 0.04272 | 7.50157 | 0.2 |
| 2. | Beam | 1222.3 | 183.89 | 0.3 |
| 3. | Three Point Supported pressurized Short beam | 0.04805 | 0.07159 | 0.5 |
| 4. | Plate with Central Hole | 763574 | 489.87 | 0.5 |

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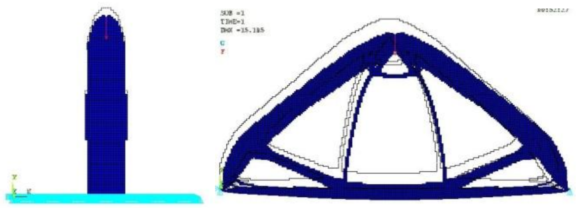
As we have seen from the above problems that the optimized shape obtained for the linearly elastic isotropic structures with ANSYS are nearly same and comparable with the optimized shape obtained for the linearly elastic orthotropic structures. Thus, we can say that optimal topology is independent of material properties.

### 3.4 Structural Analysis (Nodal Solution using ANSYS)

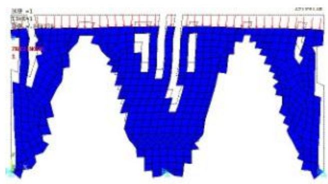
The structural analysis has been also done for the above mentioned structures with orthotropic material properties. The table-3 given below shows the numerical values obtained by ANSYS software of vector-sum displacement (maximum deformation occurred) and von-Mises stress for all the four structures.



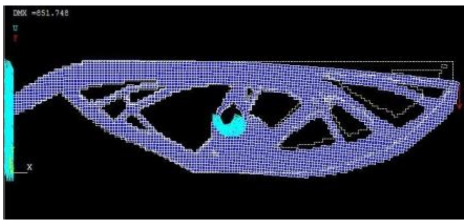
(a).Deformed Shape for Column (b).Deformed Shape for Beam

Fig-10: Deformed shape with un-deformed edges for OM (a) Model 1 & (b) Model 2

From the figure-10 (a and b), we can see the deformed shapes with undeformed edge for the simple column and beam respectively.



(a).Deformed Shape for Column



(b).Deformed Shape for Beam

Fig-11: Deformed shape with un-deformed edges for OM (a) Model 3 & (b) Model 4

Figure- 11(a), shows the deformed shape with un-deformed edge for three point support short pressurized beam and figure-11(b) represents the deformed shape for Model-4 (flat plate with central circular hole).

The numerical values of maximum deformation and von-Mises stresses are presented in the table-3 along with the Isotropic Material (IM) properties which are used to compare the optimal topology with that of Orthotropic material (OM) properties of all four structures mentioned in this paper.

Table -3: Material Properties of Structures and Nodal Solutions (displacements and von-Mises stress)

| S. N. | Structure | E | υ | Displacement Vector Sum | Stress von-Mises |
|---|---|---|---|---|---|
| 1. | Simple Column | 1 Pa | 0.3 | 0.048435 | 59.032 |
| 2. | Beam | 1 GPa | 0.3 | 15.185 | 168 |
| 3. | Three Point Supported pressurized Short beam | 100 Pa | 0.3 | 0.038178 | 16.145 |
| 4. | Plate with Central Hole | 210 Gpa | 0.3 | 851.748 | 641.636 |

The table-3 shown above also shows Young's modulus and Poisson's ratio of the structures for Isotropic Material properties. The optimal topologies of all four structures with these IM properties are compared with the OM properties of same structures using OC approach in ANSYS.

## 4. CONCLUSIONS

The following conclusions can be drawn from the present investigation:

- The optimized shape of all four models for the orthotropic material properties are nearly same as that of structures for isotropic material properties for same boundary and loading conditions.
- Further the value of compliance is obtained and found enormous variations. Also the compliance obtained for isotropic as well as orthotropic always convergent for all the mentioned structures. Thus ANSYS is an effective tool for topological optimization and the results obtained by ANSYS for isotropic and orthotropic material properties are same for optimized topology but different in compliance values.
- For further work structural analysis has been done for the above mentioned structures for orthotropic material properties.

In this paper, a simple method for topology optimization of linearly elastic continuum structures with Orthotropic Material properties is presented.

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