Nonlinear Companding Transform Algorithm for Suppression of PAPR in OFDM Systems

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Abstract - *The main limitation in employing orthogonal frequency division multiplexing (OFDM) system is high peak-to-average power ratio (PAPR) of the transmitted signal. A new nonlinear companding algorithm that transforms the OFDM signals into the* ***desirable statistics form defined by a linear piecewise*** *function is proposed. The more adjustability in companding form and an effective trade-off between the PAPR and bit error rate (BER) performances can be* ***obtained by introducing an inflexion point and the*** *variable slopes in the target probability density function. Theoretical analyses for this algorithm is presented and expressions regarding the achievable signal attenuation factor and transform gain are produced. The selection criteria of transform parameters focusing on its robustness and performance aspects are also examined. The conferred* ***theoretical analyses are well verified via computer*** *simulations.*

Key Words: Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), nonlinear companding transform (NCT), high power amplifier (HPA).

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been extensively adapted in modern wireless communications due to its low vulnerability to the multipath propagation and high spectral efficiency [1]. However, its high peak-to-average power ratio (PAPR) is a major obstacle in OFDM based transmission systems, which gives rise to undesired in-band distortion and out-of-band radiation if the linear range of high power amplifier (HPA) is not adequate. The complex baseband representation of OFDM signal with *N* subcarriers is given by

$$x(t) = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_k . e^{j\frac{2\pi kt}{T}}, 0 \le t \le T, \qquad (1)$$

Where *T* is the symbol duration, $j = \sqrt{-1}$ and the vector $X = [X_0, X_1, X_2, \ldots, X_{N-1}]^T$ denotes the frequency-domain OFDM symbols. When N is large, $x(t)$ can be approximated as a complex Gaussian process based on the central limit theorem; thus, it is possible that the OFDM signals maximum amplitude may well exceeds its average amplitude. The different methods have been developed [2] for suppressing PAPR in OFDM signal among which nonlinear companding transform (NCT) is a dynamic solution. The first introduced concept of NCT uses µ-law companding which outperforms the traditional clipping. Up to date, several NCT methods have been recommended, e.g. µ-law companding, the uniform companding (UC), trapezoidal companding (TC), and exponential companding etc.

Probably, by enhancing small signals and compressing larger ones, both noise immunity of small signals and PAPR reduction can be achieved. However, NCT is an additional pre-distortion criteria applied to transmitted signals, which results in inflated sensitivity to the HPA and performance degradation. Due to drawbacks of nonlinear distortion, cautious design of such transform is mandatory so that extent of clipped signal is as small as possible. For this reason, the key challenge for a satisfactory designed NCT methodology is to reduce the effect of companding distortion. Furthermore, an effective and flexible trade-off among the implementation complexity, bit error rate (BER) and reduction in PAPR with respect to overall performance of OFDM systems should be considered.

In this paper, further persuaded by the perception above, a new NCT which undergo transformation of Gaussian distributed signal into a desirable form described by a linear piecewise function having an inflexion point is being proposed. Contrasted with the past methods, this NCT by choosing the proper transform parameters significantly diminishes the effect of companding distortion on BER achievement, also allows flexibility in companding form. The detailed expressions regarding selection basis of transform parameters, achievable PAPR reduction are derived and well verified via computer simulations.

The remainder of the paper is outlined as follows. Section 2 defines the characteristics of OFDM signal briefly, while Section 3 derives the generic formulas for the proposed algorithm. Section 4 represents the theoretical performance analysis while Section 5 shows the simulation results followed by conclusion in Section 6.

Notation: The maximal operator and expectations are denoted by max {.} and *E* {.}. Sgn (.) stands for sign function, N-point inverse fast Fourier transform (IFFT) is denoted by $IFFT_N\{.\}$. Probability of event A is denoted by Prob {A}. Bold letters denote vectors. We use $(.)^{-1}$, $[.]^T$ to denote inverse and transpose operations respectively.

2. CHARACTERIZATION OF OFDM SIGNAL

Usually, an OFDM signal is defined as sum of N independent data symbols with quadrature amplitude modulation (QAM) or phase shift keying (PSK) modulation. It is preferable to estimate the PAPR on an oversampled signal as in discrete time domain Nyquist rate samples might not represent the peaks of the continuous time signal. The oversampled time domain representation of OFDM symbols $\boldsymbol{X} = [x_0, x_1, x_2, \dots, x_{JN-1}]^T$ can be defined as

$$x_n = \frac{1}{\sqrt{JN}} \sum_{k=0}^{N-1} X_k . e^{j\frac{2\pi k t}{T}}, 0 \le n \le JN-1, \quad (2)$$

Where J is oversampling ratio, n = 0, 1... *JN-1* is time index. The PAPR of the continuous time signal is accurately defined, generally by using $J \ge 4$. The oversampling process can be attained by inserting *(J-1) N* zeros in the middle with extending X to a *JN*-length by performing *JN*-point IFFT, i.e.,

$$\boldsymbol{X_e} = [X_0, \dots, X_{N/2-1}, 0, \dots, 0, X_{N/2}, \dots, X_{N-1}]^T, \quad (3)$$

Its shows clearly that x $=IFFT_N\{\boldsymbol{X_e}\}$. For larger values of *N* (e.g. $N \ge 64$), the real, imaginary parts of x_n may be considered as Gaussian random variables with a variance σ^2 and zero mean. Based on this assumption, the signal amplitude $|x_n|$ follows a Rayleigh distribution with probability density function (PDF) is defined as

$$f_{|x_n|}(x) = \frac{2x}{\sigma^2} e^{\frac{-x^2}{\sigma^2}}, x \ge 0. \quad (4)$$

The cumulative density function (CDF) of $|x_n|$ is given as

$$F_{|x_n|}(x) = Prob\{|x_n| \le x\} = \int_0^x \frac{2y}{\sigma^2} e^{\frac{-y^2}{\sigma^2}} dy$$

$$= 1 - e^{\frac{-x^2}{\sigma^2}}, x \ge 0, \quad (5)$$

The PAPR of OFDM signal in a frame is therefore

$$PAPR_X = \frac{max_{n \in [0, JN-1]}\{|x_n|^2\}}{E\{|x_n|^2\}}, \quad (6)$$

It is more useful to assume PAPR as a random variable and exploit statistical description given by complementary cumulative density function (CCDF) defined as follows

$$CCDF_X(\gamma_0) = Prob\{PAPR_X > \gamma_0\}$$

$$= 1 - (1 - e^{-\gamma_0})^N, \quad (7)$$

The concept of NCT is defined as follows. The original signal x_n before converting into analog signal is companded, and then amplified by HPA. Such a companded signal is represented as $y_n = h(x_n)$, where companding function $h(.)$, only changes the amplitude of the original signal. In the presence of additive Gaussian white noise (AWGN) channel, the receiver signal can be regained by decompanding function $h^{-1}(.)$.

3. NEW ALGORIHTM DESCRIPTION

The fundamental idea of the intended algorithm is to alter the statistics of amplitude $|x_n|$ into the appropriate PDF which is described by a piecewise function that consists of two linear equations having an inflexion point *cA (0<c<1)* and cut-off point *A(A>0)* of the PDF of $|y_n|$. Hence, the target PDF can be defined as

$$f_{|y_n|} = \begin{cases} k_1 x, & 0 \le x \le cA \\ k_2 x + (k_1 - k_2)cA, & cA \le x \le A \end{cases}, \quad (8)$$

Here two slopes $k_1 > 0$ and $k_2 < 0$ are adjustable parameters that decide the ultimate PAPR, by regulating the average output power in this algorithm. PDF can be defined as $\int_{-\infty}^{+\infty} f_{|y_n|}(x)dx = 1$, we have

$$k_1 = \frac{2 - A^2 k_2 (c-1)^2}{A^2 c(2-c)} \quad (9)$$

From (8), the CDF of $|y_n|$ can be expressed as

$$F_{|y_n|}(x) = \begin{cases} \frac{k_1 x^2}{2}, & 0 \le x \le cA \\ \frac{k_2 x^2}{2} + (k_1 - k_2)cAx - \frac{(k_1 - k_2)}{2}(cA)^2, & cA < x \le A \\ 1, & x > A \end{cases} \quad (10)$$

As it is clear that, CDF is monotonic increasing function with equivalent inverse function expressed as

$$F^{-1}_{|y_n|}(x) = \begin{cases} \sqrt{\frac{2x}{k_1}}, & x \le \frac{k_1 (cA)^2}{2} \\ \frac{1}{k_2}\left(\begin{matrix} (k_2 - k_1)cA \\ + \sqrt{(k_1 - k_2)k_1 c^2 A^2 + 2k_2 x} \end{matrix} \right) & x > \frac{k_1 (cA)^2}{2} \end{cases} \quad (11)$$

As $h(x)$ is also monotonically increasing function we obtain the following expression.

$$F_{|x_n|}(x) = Prob\{|x_n| \leq x\} = F_{|y_n|}(h(x)). \quad (12)$$

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Hence, the intended companding transform function is given by (13) as shown below. Here $\chi_0 = \sigma(-ln(1-(k_1/2)c^2A^2))^{1/2}$. Also, in order to keep input signal and output signal at constant average power level we express as

$$A = \left(\frac{1}{2\zeta_2}((\zeta_1{}^2 - 4\zeta_0\zeta_1)^{\frac{1}{2}} - \zeta_1)\right)^{1/2} \quad (14)$$

Where $\zeta_0 = 12\sigma^2(c-2)$, $\zeta_1 = -2(c^3-4)$ and $\zeta_2 = -k_2(c^3-3c+2)$. At the receiver side the corresponding decompanded signal is used to recover companded signal as shown in (15) below. The expressions in (13) and (15) can be numeraically precomputed through look-up tables [3] which reduces implementation complexity significantly.

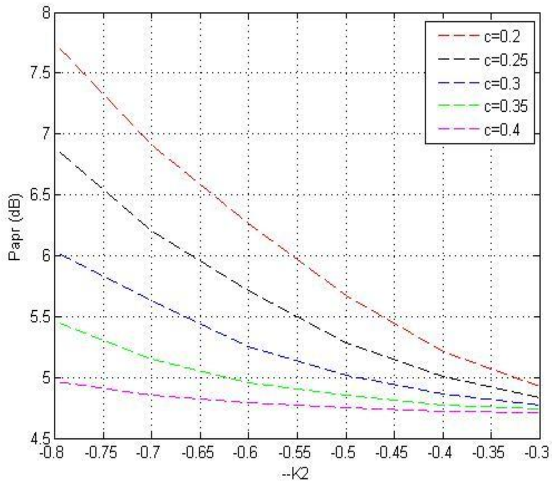
$$h(x) = sgn(x).F^{-1}_{|y_n|}\left(F_{|x_n|}(x)\right)\begin{cases} sgn(x)\sqrt{\frac{2}{k_1}\left(1-\frac{|x|^2}{\sigma^2}\right)}|x| \\ sgn(x)\frac{1}{k_2}\left((k_2-k_1)cA + \sqrt{(k_1-k_2)k_1c^2A^2}\right.\end{cases} \quad (13)$$

$$h^{-1}(x) = \begin{cases} sgn(x)\sigma\sqrt{-ln\left(1-k1\frac{|x|^2}{\sigma^2}\right)} & |x| \leq cA \\ sgn(x)\sigma\left(\sqrt{-ln\left(1-k1\frac{|x|^2}{\sigma^2}+(k_2-k_1)cA|x|+1-\frac{c^2A^2}{2}(k_2-k_1)\right)}\right), \end{cases} \quad (15)$$

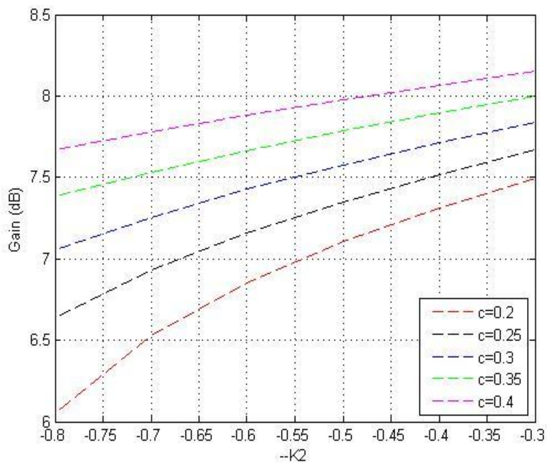
4. PERFORMANCE STUDY

The attainable reduction in PAPR and the impingement of companding distortion on the performance of BER at the receiver side are the two main evaluation criteria to characterize the theoretical analysis of the proposed algorithm.

A. Feasible Suppression in PAPR



(a)



(b)

Fig.1. Theoretical results of PAPR, G versus k_2 of the proposed algorithm. (a) The ultimate PAPR of companded signals. (b) Transform gain G.

By making proper substitution in (6), the new algorithm with the definitive PAPR of the companded signal is given as follows

$$PAPR_y = \frac{max_{n\epsilon[0,JN-1]}\{|y_n|^2\}}{E\{|y_n|^2\}} = \frac{A^2}{\sigma^2} \quad (16)$$

The theoretical results are illustrated in Fig. 1(a) and (b) respectively. As can be observed, this algorithm offers an acceptable flexibility in reduction of PAPR by altering the values of k_2 and *c*. Therefore, the ultimate PAPR is from 4.1 dB to 5.7 dB, else the attainable transform gain G can be restricted in the interval [6 dB, 7.7 dB].

Moreover, a transform gain G is expressed as the ratio of the primary signal to that of companded signal [4] defined as

$$G = 10log_{10}\frac{PAPR_X}{PAPR_y} \quad (17)$$

Furthermore, substituting (16) into (7), the CCDF of the PAPR can be expressed as

$$CCDF_y(\gamma_0) \ = Prob\{PAPR_X > \gamma_0\} \quad (18)$$

B. Impact of Companding Distortion

As known that, NCT applied to transmitter signal is extra nonlinear procedure. For this case, choosing the proper transform parameters and optimal companding form is the key challenge for minimizing the impact of companding distortion on performance of BER. The signal attenuation, companding noise b_n can be used to describe this impact [5], i.e.

$$y_n = \alpha x_n + b_n, \quad (19)$$

Where α attenuation factor is can be expressed

$$\alpha = \frac{1}{\sigma^2}\int_0^\infty xh(x)f_{|x_n|}(x)\,dx \quad (20)$$

The theoretical value of α is shown in Fig. 2, from that we can observe that α gradually tends to 1 as c and k_2 diminishes. As a result, Fig. 1, Fig. 2 illustrates that, to obtain typical PAPR reduction by choosing proper transform parameters to make signal undesired distortion as minimum as possible.

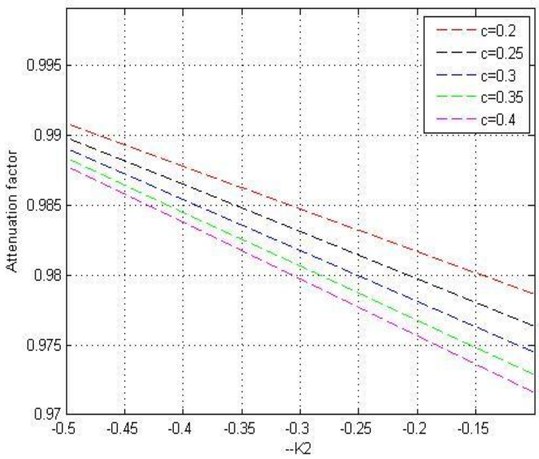


Fig-2: Theoretical result of attenuation factor of the proposed algorithm

5. SIMULATION RESULTS

For an OFDM system with N = 1024 subcarriers to assess general framework execution of the proposed transformation, computer simulations were executed. In the results it shows that to achieve better PAPR estimation, OFDM frames are modulated to acquire CCDFs, which have been calculated with an oversampling ratio *J = 4.*

A. Performance in PAPR suppression

Fig. 3 illustrates the simulation results of CCDF of the PAPR which are very close to theoretical results. This predicts that theoretical and experimental results almost similar. Moreover, the new transformation algorithm with *c = 0.25, k = -0.001* attains the maximum PAPR reduction and also observed with *c = 0.25, k = -0.45* tends to far less degradation of BER performance at the receiver.

B. BER Performance

Fig.4. depicts the BER versus E_b/N_0 under an AWGN channel using QAM being compared with exponential companding (EC) [6] transform.
It is clear that expected E_b/N_0s of the new algorithm are better to referred methods. As mentioned earlier, by selecting suitable transform parameters significantly reduces the impact of companding distortion on BER performance
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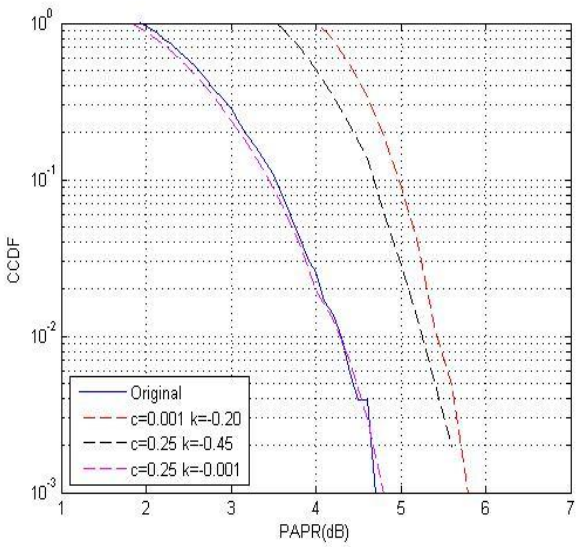

Fig-3: PAPR reduction performance for OFDM with N = 1024, QPSK modulation and oversampling ratio J = 4.

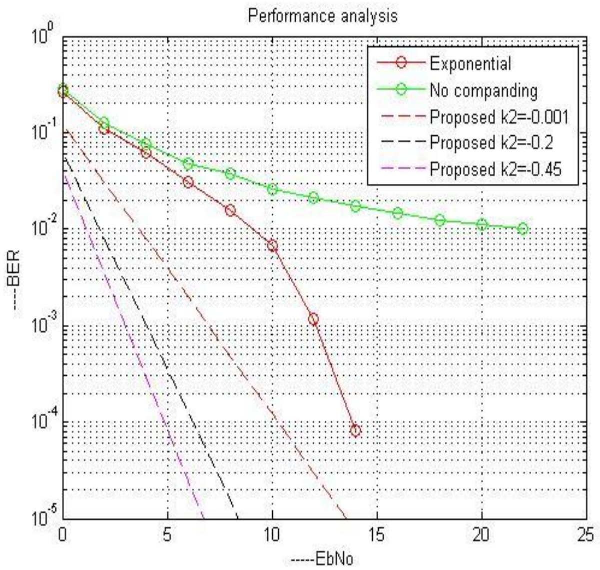

Fig-4: BER performance under an AWGN channel for the OFDM system with N = 1024 and 16QAM modulation

6. CONCLUSION

Due to its effectiveness and less complexity, to eliminate the PAPR in OFDM, NCT is a pleasing solution for the reduction of PAPR in OFDM signal. This new NCT algorithm alters the statistics of original signal into required PDF described by linear piecewise function. It is clearly seen that compared to original signal this algorithm provides 6.0 dB to 7.7dB transform gain of PAPR. Computer simulations illustrates that new algorithm substantially performs better than existing NCT methods in overall performance of OFDM systems. In addition, this algorithm allows adjustability of transform parameters, flexible trade-off between BER performance and PAPR.

REFERENCES

[1] T. Hwang, C. Yang, G. Wu, S. Li, and G. Y. Lee, "OFDM and its wireless application: A survey," IEEE Trans. Veh. Technol., vol. 58, no. 4, pp. 1673–1694, May 2009.

[2] T. Jiang and Y. Wu, "An overview: Peak-to-average power ratio reduction techniques for OFDM signals," IEEE Trans. Broadcast., vol. 54, no. 2, pp. 257–268, Jun. 2008.

[3] Y. Jiang, "New companding transform for PAPR reduction in OFDM," IEEE Commun. Lett, vol.14, no.4, pp.282–284, Apr.2010.

[4] X.Huang, J.Lu, J.Zheng, K.B.Letaief, and J.Gu, "Companding transform for reduction in peak-to-average power ratio of OFDM signals," IEEE Trans. Wireless Commun., vol. 3, no. 6, pp. 2030–2039, Nov. 2004.

[5]D.Dardari,V.Tralli and A.Vaccari, "A theoretical characterization of non-linear distortion effects in OFDM systems," IEEE Trans. Commun., vol. 48, no. 10, pp. 1755–1764, Oct. 2000.

[6] T. Jiang, Y. Yang, and Y. Song, "Exponential companding transform for PAPR reduction in OFDM systems," IEEE Trans. Broadcast., vol. 51, no. 2, pp. 244–248, June 2005.

BIOGRAPHIES

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