DECAY OF MAXIMALLY ENTANGLED BELL STATES

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Abstract

Strangeness of quantum physical laws is very fascinating aspect for all of us. In various composite quantum systems it was found that the finite time disentanglement called Entanglement sudden death takes place.Entangalment sudden death is a very well known phenomena in which qubits die after in a particular time. In our case we have studied two qubit quantum system as maximally entangled systems. We can expand it for many qubit systems also. We examine the behaviour of these systems under amplitude damping noise. The other environmental noises such as phase damping and depolarisation were also studied. All these noises were applied on single qubit as well as on both the qubits. Class of X-states has been used for the study.

Keywords: Decoherence, Qubits, Quantum entanglement, Amplitude damping, ESD

1.Introduction

There are two fundamental features of quantum world, the superposition and entanglement which can distinguish it from classical world. Quantum coherence acts as a major resource in quantum information technology, but it is fragile in some senses, as when a quantum system is in contact with an environment with many degrees of freedom , it can be subjected to decoherence[1]. Decoherence is a crucial issue in quantum information processing and has been a subject of active research since the 1980s. The presence of noise in a quantum channel [3] or the decoherence effects of qubits interacting with an environment [4] will transform an idealized pure state into a mixed one.Quantum entanglement has been properly studied from many years, due to its fundamental **significance in quantum theory**[5,6] and its utility as a resource for quantum communication and quantum information science [7].Entanglement is a major   resource for quantum information processing , In a variety of processes such as quantum computation, quantum cryptography [8] or quantum teleportation [9] entangled qubit states are used. In recent papers of T. Yu and J.H.Eberly [10-14] it was found that for theoretically studying the dynamics of entanglement between atoms One of the simplest procedure is that two atoms which are isolated from one another and interact **with electromagnetic fields individually, also they** found that reservoir-induced decay of entangledqubitcan be faster than the corresponding decay of single qubit coherence. Sudden and asymptotically gradual decay of entanglement were predicted in amplitude [12] and phase damping channels [13]. The sudden disentanglement of two qubit system in finite time was called 'entanglement sudden death' (ESD). experimentalvarifiation of ESD was reported for an optical setup [15] and atomic ensembles [16]. It was observed that ESD was found at first for entangled two qubit states., later it was also explored in a multipartite systems and higher dimensional Hilbert spaces [16-21]. A significant contribution is also given by atomicphysics which offers a domain with sufficient control of the system and isolation from noise.ESD is a undesirable process and is counterintuitive in many senses so its study is more intresting topic in current research in information science.

Recently ,Entanglement sudden death is found for class of X-states,and subset of X-states[22].In our latter we re-examine this X-state under amplitude damping, phase damping and depolarisation quantum channels. The latter is arranged as follows: in section 2 we examine the X-state under amplitude damping quantum chennal when one qubit is subjected alone as well as both the qubits are under the influence of this quantum chennal .The phase damping effect on one qubit as well as on both the qubit is explained in section 3. Section 4,gives the details of depolarisation effect on quantum entanglement of one qubit alone as well as of both the qubits. The conclusion is drawn on the section 5.

2.Entanglement decay in a class of X-states

Here we consider the subset of so called X-class of states

$$\rho(0) = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & z & 0 \\ 0 & z* & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix} \tag{1}$$

Where a+b+c+d = 1, z=x+iy [15].It is a combination of pure states as all bell states and mixed states sometimes called werner state and isotropic states. The measure of entanglement known as concurrence for the above states [23] is given by C=2Max [0,$(|z|$- $\sqrt{ad})$]. The noiseychennals which are considered here are amplitude damping , phase damping, and depolarising. Starting with the Lindblad master equation[7].For the effect of each of these noises on a quantum system we use operator sum representation throught this paper.

2.1. AmplitudeDamping : Applied on single qubit

When only one qubit is experiences amplitude damping:

The dynamics of an atom which spontaneously emits a photon is well characterised by a quantum operation known as amplitude damping. It provides the knowledge about energy dissipation for quantum systems.We have the Lindblad master equation of the form

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \sum_j \left[2 L_j \rho L_j^\dagger - \{L_j^\dagger L_j, \rho\} \right] \tag{2}$$

Where H is the system Hamiltonian, a hermitian operator representing the coherent part of the dynamics, $\{L_j^\dagger L_j, \rho\}$ is denotes an anticommutator and L_j are the Lindblad operators ,representing the coupling of the system to the environment. Solving this equantion we get the time dependent matrix and the results are expressed by operators in operator sum representation.[24]

The operators are taken as follows

$$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D} \end{pmatrix}, \qquad A_1 = \begin{pmatrix} 0 & \sqrt{D} \\ 0 & 0 \end{pmatrix} \tag{3}$$

Where $D = (1 - e^{-2t\gamma})$ and γ is decay rate.

When the first qubit is subjected to amplitude damping decoherence leaving the second qubit noise free the operators are taken as

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$E_2 = \begin{pmatrix} 0 & \sqrt{D} \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Then the time evolution of density matrix is given by

$$\rho(t) = E_{01}(t) \rho(0) E_{01}^\dagger + E_{12}(t)\rho(0) E_{12}^\dagger \tag{3}$$

we get

$$\rho(t) = \begin{pmatrix} a+cD & 0 & 0 & 0 \\ 0 & b+dD & z\sqrt{1-D} & 0 \\ 0 & z^*\sqrt{1-D} & c(1-D) & 0 \\ 0 & 0 & 0 & d(1-D) \end{pmatrix} \tag{4}$$

It is evident from (4) that this equation preserves its X-form as in (1). Hence the concurrence is given by

$$C_{A1} = 2\,\mathrm{Max}[0,(\sqrt{1-D}\;|z| - \sqrt{(a+cD)(d-dD)})] \tag{5}$$

From (5) it is clear that we get a completely separable state if

$$\sqrt{1-D}\;|z| - \sqrt{(a+cD)(d-dD)} \leq 0$$

Or $\quad t \geq \dfrac{1}{2\gamma}\ln\dfrac{cd}{d(a+c)-|z|^2} \tag{6}$

But for pure states of the form

$$|\varphi\rangle^{\pm} = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$$

$a = d = 0$ and $c=b=|z| = \frac{1}{2}$ we get the concurrence

$$C_{A1} = \mathrm{Max}[0, e^{-\gamma t}] \tag{7}$$

From (7) we can say that there is exponential decay for bell states $|\varphi\rangle^{\pm}$. For probability parameter a = 0.1, b = 0.4, c = 0.4, d = 0.1, $|z| = 0.2$ we found entanglement sudden death at γt = 0.693.

2.2 When both the qubits are experiences the amplitude damping noise.

The operators are given as

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D_A} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D_B} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-D_B} & 0 & 0 \\ 0 & 0 & \sqrt{1-D_A} & 0 \\ 0 & 0 & 0 & \sqrt{1-D_A}\sqrt{1-D_B} \end{pmatrix}$$

$$E_2 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D_A} \end{pmatrix} \otimes \begin{pmatrix} 0 & \sqrt{D_B} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \sqrt{D_B} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{D_B}\sqrt{1-D_A} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$E_3 = \begin{pmatrix} 0 & \sqrt{D_A} \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D_A} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \sqrt{D_A} & 0 \\ 0 & 0 & 0 & \sqrt{D_A}\sqrt{1-D_B} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$E_4 = \begin{pmatrix} 0 & \sqrt{D_A} \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & \sqrt{D_B} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \sqrt{D_A}\sqrt{D_B} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (8)$$

Then the time evolution of density matrix as given by (3)

$$\rho(t) = E_1(t)\rho(0)E_1^\dagger + E_2(t)\rho(0)E_2^\dagger + E_3(t)\rho(0)E_3^\dagger + E_4(t)\rho(0)E_4^\dagger$$

$$\rho(t) = \begin{pmatrix} a + bD_B + cD_A + dD_A D_B & 0 & 0 & 0 \\ 0 & b(1-D_B) + (1-D_B)dD_A & z\sqrt{1-D_B}\sqrt{1-D_A} & 0 \\ 0 & z^*\sqrt{1-D_B}\sqrt{1-D_A} & c(1-D_A) + (1-D_A)dD_B & 0 \\ 0 & 0 & 0 & d(1-D_A)(1-D_B) \end{pmatrix}$$

(9)

the above equation preserves the X-form hence we have the concurrence,

$$C_{A2} = 2\max\left[0, |z|\sqrt{1-D_B}\sqrt{1-D_A} - \sqrt{d(1-D_A)(1-D_B)(a+bD_B+cD_A+dD_A D_B)}\right] \quad (10)$$

the concurrence vanishes when,

$$|z|\sqrt{1-D_B}\sqrt{1-D_A} - \sqrt{d(1-D_A)(1-D_B)(a+bD_B+cD_A+dD_A D_B)} \le 0$$

Or $\quad t \ge \dfrac{1}{2\gamma}\ln\dfrac{bd+cd}{ad+bd+cd-|z|^2}$ $\qquad$ (11)

Here for simplicity we choose $D_A = D_B = D$. For maximally entangled bell state $|\Psi\rangle^\pm = \dfrac{|0\rangle|1\rangle \pm |1\rangle|0\rangle}{\sqrt{2}}$ there is always exponential decay of entanglement. $C_{A2} = \max[0, e^{-2\gamma t}]$. Entanglement sudden death occurs for a = 0.1, b = 0.2, c = 0.3, d = 0.4 with $|z| = 0.2$.

3.1 Phase damping channel: Applied on single qubit

The Kraus operators are:

$$P_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D} \end{pmatrix}, P_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{D} \end{pmatrix} \quad (12)$$

Where $D = 1 - e^{-2\lambda t}$ and λ is decay rate for phase damping noise

When only first qubit is subjected to phase damping noise then we have the operators of the form, $E_1 = P_0 \otimes I$, $E_2 = P_1 \otimes I$

Where I is identity operator I=$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{1-D} & 0 \\ 0 & 0 & 0 & \sqrt{1-D} \end{pmatrix} \quad (13)$$

$$E_2 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{D} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{D} & 0 \\ 0 & 0 & 0 & \sqrt{D} \end{pmatrix} \quad (14)$$

The density operator after interaction is

$$\rho(t) = \sum_i^2 E_i \rho(0) E_i^\dagger = E_1(t)\rho(0)E_1^\dagger + E_2(t)\rho(0)E_2^\dagger \quad (15)$$

$$\rho(t) = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & z\sqrt{1-D} & 0 \\ 0 & z^*\sqrt{1-D} & cD + c(1-D) & 0 \\ 0 & 0 & 0 & dD + d(1-D) \end{pmatrix} \quad (16)$$

X-form is preserved we get the concurrence,

$$C_{P1} = 2\,Max[0, |z|\sqrt{(1-D)} - \sqrt{a\,d}\,] \quad (17)$$

concurrence vanishes when, $\sqrt{1-D}|z| - \sqrt{(ad)} \leq 0$

Or $t \geq \frac{1}{2\lambda} \ln \frac{|z|^2}{ad}$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (18)$

For maximally entangled bell state $|\Psi\rangle^\pm = \frac{|0\rangle|1\rangle \pm |1\rangle|0\rangle}{\sqrt{2}}$ there is always an exponential decay of entanglement.

$$C_{P1} = max[0, e^{-\gamma t}]$$

For a = 0.1, b = 0.4, c = 0.4, d = 0.1 with $|z|$ varies from 0.2 to 0.9 we get entanglement sudden death.

3.2 Phase damping effect on both the qubits

The operators are,

$$E_1 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D_A} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D_B} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{1-D_B} & 0 & 0 \\ 0 & 0 & \sqrt{1-D_A} & 0 \\ 0 & 0 & 0 & \sqrt{1-D_B}\sqrt{1-D_B} \end{pmatrix}$$

$$E_2 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D_A} \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{D_B} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \sqrt{D_B} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{D_B}\sqrt{1-D_B} \end{pmatrix}$$

$$E_3 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{D_A} \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-D_B} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{D_A} & 0 \\ 0 & 0 & 0 & \sqrt{D_B}\sqrt{1-D_B} \end{pmatrix}$$

$$E_4 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{D_A} \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{D_B} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{D_B}\sqrt{D_B} \end{pmatrix}$$

The density matrix after interaction is given by:

$$\rho(t) = E_1(t)\rho(0)E_1^\dagger + E_2(t)\rho(0)E_2^\dagger + E_3(t)\rho(0)E_3^\dagger + E_4(t)\rho(0)E_4^\dagger$$

Adding (3.107a), (3.107b), (3.107c),and (3.107d) we get ,

$$\rho(t) = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & z\sqrt{1-D_A}\sqrt{1-D_B} & 0 \\ 0 & z^*\sqrt{1-D_A}\sqrt{1-D_B} & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix} \qquad (19)$$

It is evident from above equation that density matrix retains its X-form, then the concurrence is given by,

$$C_{P2} = 2Max[0, |z|\sqrt{1-D_A}\sqrt{1-D_B} - \sqrt{ad}\,]$$

concurrence should be vanishes when

$$|z|\sqrt{1-D_A}\sqrt{1-D_B} - \sqrt{ad}\,] \le 0$$

Or $t \ge \frac{1}{4\lambda}\ln\frac{|z|^2}{ad}$ \qquad (20)

choosing $D_A = D_B = D$ .For maximally entangled bell state $|\Psi\rangle^{\pm} = \frac{|0\rangle|1\rangle \pm |1\rangle|0\rangle}{\sqrt{2}}$ there is always exponential decay of entanglement.

$$C_{P2} = max[0, e^{-2\lambda t}] \qquad (21)$$

For a = 0.1, b = 0.2, c = 0.3, d = 0.4 with $|z|$ varies from 0.2 to 1.0 we get entanglement sudden death.

4.1 Depolarising noise: Applied on singlequbit

For it we have the Kraus operators:

$$E_1 = D_1 \otimes I, E_2 = D_2 \otimes I, E_3 = D_3 \otimes I, E_4 = D_4 \otimes I$$

$$E_1 = \sqrt{1-\frac{3P}{4}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \sqrt{1-\frac{3P}{4}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_2 = \sqrt{\frac{P}{4}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \sqrt{\frac{P}{4}}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$E_3 = \sqrt{\frac{P}{4}}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \sqrt{\frac{P}{4}}\begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$E_4 = \sqrt{\frac{P}{4}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \sqrt{\frac{P}{4}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The state after decoherence

$$\rho(t) = \sum_i^4 E_i \rho(0) E_i^\dagger = E_1(t)\rho(0)E_1^\dagger + E_2(t)\rho(0)E_2^\dagger + E_3(t)\rho(0)E_3^\dagger + E_4(t)\rho(0)E_4^\dagger$$

$$\rho(t) = \begin{pmatrix} a + \frac{p(c-a)}{2} & 0 & 0 & 0 \\ 0 & b + \frac{p(d-b)}{2} & \frac{(2-p)z}{2} & 0 \\ 0 & \frac{(2-p)z^*}{2} & c + \frac{p(a-c)}{2} & 0 \\ 0 & 0 & 0 & d + \frac{p(b-d)}{2} \end{pmatrix}$$

concurrence vanishes when,

$$(2-p)|z| - \sqrt{(2a + p(c-a))\ (2d + p(b-d)\ )} \le 0$$

or

$$t \ge \frac{1}{2\tau} \ln \frac{\sqrt{bc}}{\sqrt{bc} + \sqrt{|z|^2 - ad + ab + cd}} \tag{22}$$

For maximally entangled bell state $|\Psi\rangle^\pm = \frac{|0\rangle|1\rangle \pm |1\rangle|0\rangle}{\sqrt{2}}$ there is always exponential decay of entanglement.

$$C_{D1} = \max[0, e^{-\frac{\tau t}{2}}] \tag{23}$$

Parameters for which entanglement sudden death is found when depolarizing noise is applies to one of the two qubits, are , a=0.5 , b=0, c=0, d=0.5 and $|z| = 0.5$. This is one of the pure entangled bell state given by $|\varphi\rangle^\pm = \frac{|0\rangle|0\rangle \pm |1\rangle|1\rangle}{\sqrt{2}}$

4.2 Depolarising noise: Applied on both qubits

The Kraus operators are,

$$E_1 = D_{1A} \otimes D_{1B} = \sqrt{1 - \frac{3P_A}{4}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \sqrt{1 - \frac{3P_B}{4}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \sqrt{1 - \frac{3P_A}{4}} \sqrt{1 - \frac{3P_B}{4}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E_2 = D_{1A} \otimes D_{2B} = \sqrt{1 - \frac{3P_A}{4}} \sqrt{\frac{P_B}{4}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \sqrt{1 - \frac{3P_A}{4}} \sqrt{\frac{P_B}{4}} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$E_3 = D_{1A} \otimes D_{3B} = \sqrt{1 - \frac{3P_A}{4}} \sqrt{\frac{P_B}{4}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$= \sqrt{1 - \frac{3P_A}{4}} \sqrt{\frac{P_B}{4}} \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}$$

$$E_4 = \sqrt{1 - \frac{3P_A}{4}} \sqrt{\frac{P_B}{4}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \sqrt{1 - \frac{3P_A}{4}} \sqrt{\frac{P_B}{4}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$E_5 = D_{2A} \otimes D_{1B} = \sqrt{1 - \frac{3P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \sqrt{1 - \frac{3P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$E_6 = D_{2A} \otimes D_{2B} = \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \frac{P_A}{4} \frac{P_B}{4} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$E_7 = D_{2A} \otimes D_{3B} = \sqrt{\frac{P_A}{4}} \sqrt{\frac{P_B}{4}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} =$$

$$= \sqrt{\frac{P_A}{4}} \sqrt{\frac{P_B}{4}} \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$E_8 = D_{2A} \otimes D_{4B} = \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$E_9 = D_{3A} \otimes D_{1B} = \sqrt{1 - \frac{3P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \sqrt{1 - \frac{3P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$E_{10} = D_{3A} \otimes D_{2B} = \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$E_{11} = D_{3A} \otimes D_{3B} = \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$= \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$E_{12} = D_{3A} \otimes D_{4B} = \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}$$

$$E_{13} = D_{4A} \otimes D_{1B} = \sqrt{1 - \frac{3P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \sqrt{1 - \frac{3P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$E_{14} = D_{4A} \otimes D_{2B} = \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$E_{15} = D_{4A} \otimes D_{3B} = \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$= \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}$$

$$E_{16} = D_{4A} \otimes D_{4B} = \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \sqrt{\frac{P_B}{4}} \sqrt{\frac{P_A}{4}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \left(\frac{16+4P^2-16p}{16}\right)a + (b+c)\left(\frac{8P-4P^2}{16}\right) + \frac{4P^2 d}{16} & 0 & & 0 \\ 0 & \left(\frac{16+4P^2-16p}{16}\right)b + (a+d)\left(\frac{8P-4P^2}{16}\right) + \frac{4P^2 c}{16} & & \left[(1-\frac{3P}{4})^2 + (1-\frac{3P}{4})\frac{P}{4}\right]z \\ 0 & \left[(1-\frac{3P}{4})^2 + (1-\frac{3P}{4})\frac{P}{4}\right]z^* & \left(\frac{16+4P^2-16p}{16}\right)c + (a+d)\left(\frac{8P-4P^2}{16}\right) + \frac{4P^2 b}{16} & \\ 0 & 0 & & 0 \end{pmatrix}$$

(24)

This equation also preserves the X- form so we have concurrence for this,

$$C_{D2} = 2\max$$

$$\left[0, |z|(1-\frac{3P}{4})^2 + (1-\frac{3P}{4})\frac{P}{4} - \sqrt{\left[\left(\frac{16+4P^2-16p}{16}\right)a + (b+c)\left(\frac{8P-4P^2}{16}\right) + \frac{4P^2 d}{16}\right]\left[\left(\frac{16+4P^2-16p}{16}\right)d + (b+c)\left(\frac{8P-4P^2}{16}\right) + \frac{4P^2 a}{16}\right]}\right]$$ (25)

the concurrence for maximally entangled bell state $|\Psi\rangle^{\pm} = \frac{|0\rangle|1\rangle \pm |1\rangle|0\rangle}{\sqrt{2}}$, for which $b = c = 0.5$, $|z| = 0.5$, $a = d = 0$, comes out to be zero at $\tau t = 1.695$ or $P \geq \frac{4}{7}$, for simplicity we should consider $P_A = P_B = P$. For maximally entangled bell state $|\varphi\rangle^{\pm} = \frac{|0\rangle|0\rangle \pm |1\rangle|1\rangle}{\sqrt{2}}$ we have, $b = c = 0, |z| = 0.5$, $a = d = 0.5$, we observe entanglement sudden death at $\tau t = 3.847$ or P = 0.175.

5.Conclusion

From the above study we can say that when a two qubit system is subjected to various kind of noises, there is exponential as well as sudden decay of entanglement occurs. In case of amplitude damping and phase damping the bell state of the singlet form shows entanglement sudden death at different values of probability parameters, but for maximally entangled states we found exponential decay of entanglement. In case depolarising noise applied on single qubit, the entanglement sudden occurs for maximally entangled bell states and for singlet states the qubit entanglement exponentially deacays. When depolarising noise applied on both the qubits we found entanglement sudden death for maximally entangled bell states. From here we conclude that entanglement sudden death can not be avoided in maximally entangled bell states.z

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