

Heat Transfer in MHD Nano Fluid along a Stretching flat Plate With Chemical Reaction and Injection/Sink

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Abstract - Heat, mass, and momentum transfer in the laminar boundary layer flow on a stretching sheet are important due to its applications to polymer technology and metallurgy. The chemical reaction of metal particle with the solvent is natural and decomposition of the metal particle is a natural phenomenon. But, this decomposition affects the viscosity and the fluid flow. The inclusion of chemical reaction rate parameter in the expression of viscosity given by Graham A.L (Applied Science Research, 37(3), pp.275-286(1981) address the behavior of the viscosity of the Nano fluid in its flow and heat transfer. The heat transfer is more for large volume fraction of the solid due to the decomposition of metal particle. The destructive chemical reaction ($K > 0$) gives the lesser temperature than the generative chemical reaction ($K < 0$).

Key Words: Cu-H₂O Nano - fluid, viscosity, vertical Stretching sheet, Heat transfer.

1. INTRODUCTION

The boundary layer flow over a stretching sheet plays an important role in aerodynamic, extrusion of plastic sheet, metal-spinning, manufacture of plastic and rubber sheets, paper production etc and thus, remains at the leading edge of technology development. In the industrial operation, metal or more commonly an alloy, is heated until it is molten, whereupon it is poured into a mould or dies which contains a cavity, of required shape. The study of convective Nano fluid heat transfer dominated the heat transfer through fluids, due to strong physical properties of the mater (solid + base fluid). Literature survey proved that the study of Nano fluid heat transfer was worth enough to implement in industries. Sarit Kumar Das (7) and Xiang- Qi Wang (9) stressed the lack of heat transfer mechanisms through Nano fluids and the interaction of metal particle with solvent. However, at later stages many authors attempted to study the thermal conductivity, viscosity and physical parameters. Tiwari Arun Kumar (8) studied the thermal conductivity and viscosity of Nano

fluids with various models and concluded that existing literature was still unclear to match the experimental and theoretical results.

Recently Gbadeyan, J.A (1) reports that the increase in Brownie motion is reduced heat transfer rate. M.A.A.Hamad (6) concluded that the Nano particle inclusion in the base fluid changes the flow patron significantly. To match the experimental and theoretical results Hassan A.M. (4) and GVPN Srikanth (3) attempted by including the chemical reaction coefficients in the diffusion equation.

So we made an attempt here to analyze the convective heat and mass transfer of a nano fluid, past a permeable oscillating stretching sheet by considering the size of the particle using viscosity due to Graham [2] and the thermal conductivity due to Jong & Choi.

2. MATHEMATICAL FORMULATION

Consider the unsteady free convection flow of a nano-fluid past a vertical permeable semi-infinite stretching sheet in the presence of an applied magnetic field with constant heat source, radiation and suction. We consider a Cartesian coordinate system $(\bar{x}, \bar{y}, \bar{z})$, the flow is assumed to be in the \bar{x} direction, which is taken along the sheet, and \bar{z} - axis is normal to the sheet. We assume that the sheet has an oscillatory movement on time \bar{t} and frequency \bar{n} with the velocity $u(0,t)$, which is given $u(0,t) = U_0 (1 + x + \epsilon \cos(nt))$, where ϵ is a small constant parameter ($\epsilon \ll 1$), x is the rate of stretching and U_0 is the characteristic velocity. We consider that initially ($t < 0$) the fluid as well as the sheet is at rest. A uniform external magnetic field B_0 is taken to be acting along the \bar{z} -axis. Also assume that the induced magnetic field is small compared to the external magnetic field B_0 . The surface temperature is assumed to have the constant value T_w

while the ambient temperature has the constant value T_∞ , where $T_w > T_\infty$. The conservation equation of current density $\nabla \cdot \mathbf{J} = 0$ gives $J_z = \text{constant}$. Since the sheet is electrically non-conducting, this constant is zero. It is assumed that the sheet is infinite in extent and hence all physical quantities do not depend on \bar{x} and \bar{y} but depend only on \bar{z} and \bar{t} , the Schematic Diagram of this is shown in Fig.1.

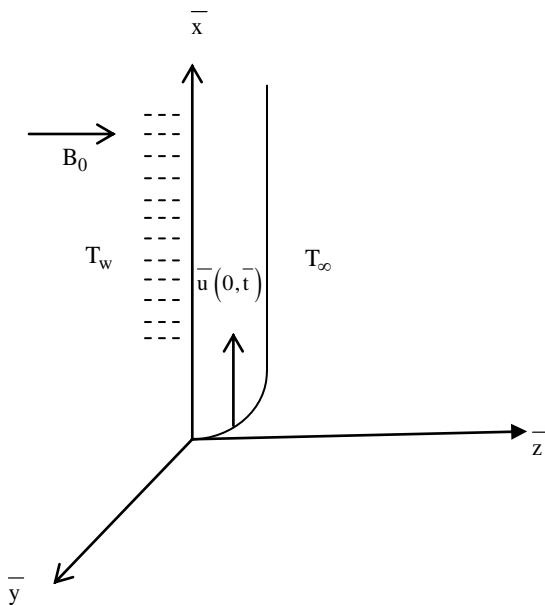


Fig-1: Schematic Diagram

$$\text{i.e. } \frac{\partial \mathbf{u}}{\partial \bar{x}} + \frac{\partial \mathbf{v}}{\partial \bar{y}} = 0$$

It is further assumed that the regular fluid and the suspended nano-particles are in thermal equilibrium and no slip occurs between them. Under Boussinesq and boundary layer approximations, the boundary layer equations governing the flow and temperature are,

$$\frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} = \frac{1}{\rho_{nf}} \left[\mu_{nf} \frac{\partial^2 u}{\partial z^2} + (\rho \beta_T)_{nf} g (T - T_\infty) \cos \gamma - \sigma B_0^2 u \right] \tag{2}$$

$$\frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} = \alpha_{nf} \frac{\partial^2 T}{\partial z^2} - \frac{Q}{(\rho c_p)_{nf}} (T - T_\infty) - \frac{1}{(\rho c_p)_{nf}} \frac{\partial q_f}{\partial z} \tag{3}$$

The appropriate initial and boundary conditions for the problem are given by

$$u(z, t) = 0, T = T_\infty \text{ for } t < 0 \forall z$$

$$\left. \begin{aligned} u(0, t) &= U_0 \left[1 + x + \frac{\varepsilon}{2} (e^{int} + e^{-int}) \right], T(0, t) = T_w \\ u(\infty, t) &\rightarrow 0, T(\infty, t) \rightarrow T_\infty, \varepsilon \ll 1 \end{aligned} \right\} \tag{4}$$

Thermo-Physical properties are related as follows:

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$(\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_s$$

$$(\rho \beta)_{nf} = (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_s$$

$$\frac{\mu_{nf}}{\mu_f} = 1 + 2.5 \phi + 4.5 \left[\frac{1}{\left(\frac{h}{d_p} \left(2 + \frac{h}{d_p} \right) \right) \left(1 + \frac{h}{d_p} \right)^2} \right]$$

$$k_{nf} = k_f (1 - \phi) + \beta_1 k_p \phi + c_1 \frac{d_f}{d_p} k_p \text{Re}^2 d_p \text{pr} \phi \tag{5}$$

Due to chemical Reaction by the solvent and the metal particle the rate of change of the size of the particle can be considered as:

$$\frac{dd_p}{dt} = -K d_p$$

$$\Rightarrow d_p = d_0 e^{-Kt}$$

Where d_0 is the initial size of the particle (Assumed as $d_0 = 50 \text{ nm}$) and K is the reaction rate:

$$\frac{\mu_{nf}}{\mu_f} = 1 + 2.5 \phi + 4.5 \left[\frac{1}{\left(\frac{h e^{Kt}}{50} \right) \left(2 + \frac{h e^{Kt}}{50} \right) \left(1 + \frac{h e^{Kt}}{50} \right)^2} \right]$$

$$k_{nf} = k_f (1 - \phi) + \beta_1 k_p \phi + \frac{c_1}{50} d_f e^{Kt} k_p \text{Re}^2 d_p \text{pr} \phi$$

Where $\beta_1 = 0.01$ is a constant for considering the Kapitza resistance per unit area

$c_1 = 18 \times 10^6$ is proportionality constant

$$\text{Re} d_p = \frac{d_p}{\nu_f} \frac{\kappa T}{3 \pi \mu_f d_p l_f} = \frac{1.381 \times 10^{23} T}{\nu_f 3 \pi \mu_f (0.738)}$$

$$d_f = 0.384 \text{ nm for water}$$

$$\text{Pr} = \text{Prandtl number} = \frac{\nu_f}{\alpha_f}$$

$$l_f = \text{Mean free path} = 0.738$$

$$k = \text{Boltzmann constant, } T = 300 \text{ K}$$

We consider the solution of Esq. (1) as $w = -w_0$ (6)

Where the constant w_0 represents the normal velocity at the plate which is negative for injection ($w_0 < 0$)

Thus, we introduce the following dimensionless variables:

$$z = \left(\frac{v_f}{U_0} \right) Z, \quad t = \left(\frac{v_f}{U_0^2} \right) t^*, \quad n = \left(\frac{U_0^2}{v_f} \right) \eta,$$

$$u = x U U_0, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad q_r = - \frac{4\sigma_1}{3\delta} \frac{\partial T^4}{\partial y}$$

We assume that the temperature differences within the flow are sufficiently small. So that, the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4$$

By using above, we obtain $\frac{\partial q_r}{\partial z} = - \frac{16\sigma_1}{3\delta} \frac{\partial^2 T^4}{\partial z^2} T_\infty^3$ (7)

Using equations 4,5,6,7 the Equations 2 and 3 can be written in the following dimensionless form:

$$\left[1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right) \right] \left(\frac{\partial U}{\partial \tau} - S \frac{\partial U}{\partial Z} \right) = 1 + 2.5\phi + 4.5 \left[\frac{1}{\left(\frac{heKt}{50} \right) \left(2 + \frac{heKt}{50} \right) \left(1 + \frac{heKt}{50} \right)^2} \right] \frac{\partial^2 U}{\partial Z^2}$$

$$+ \left[1 - \phi + \phi \left(\frac{\rho \beta_T}{\rho \beta_T} \right) \right] Gr \theta \cos \gamma \frac{1}{x} - MU$$

$$\left[1 - \phi + \phi \left(\frac{\rho c_p}{\rho c_p} \right) \right] \left(\frac{\partial \theta}{\partial \tau} - S \frac{\partial \theta}{\partial Z} \right) = \frac{1}{Pr} \left[1 - \phi + 0.01 \phi \left(\frac{k_p}{k_f} + \frac{k_p}{k_f^2} \phi \frac{\rho_f^2 c_{pf} e^{Kt}}{50 \mu_f^3} - 28632.9991 \times 10^{-52} \right) \right] \frac{\partial^2 \theta}{\partial Z^2}$$

$$- \frac{1}{Pr} Q_H \theta + \frac{1}{Pr} \frac{1}{3 Ra} \frac{\partial^2 \theta}{\partial z^2}$$

Where the corresponding boundary conditions (4) can be written in the dimensionless form as:

$$U(z, t) = 0, \quad \theta(z, t) = 0 \quad \text{for } t < 0 \quad \forall z$$

$$U(0, t) = U_0 \left[1 + x + \frac{\epsilon}{2} \left(e^{int} + e^{-int} \right) \right], \quad \theta(0, t) = 1 \quad \left. \vphantom{U(0, t)} \right\} \forall t \geq 0$$

$$U(\infty, t) \rightarrow 0, \quad \theta(\infty, t) \rightarrow 0$$

Here Pr is the Prandtl number, S is the injection ($S < 0$) parameter, M is the magnetic parameter, Ra is the

Radiation parameter and Q_H is the heat source parameter, Gr is the Grashof number, which are defined as:

$$Pr = \frac{v_f}{\alpha_f}, \quad S = \frac{w_0}{U_0}, \quad M = \frac{\sigma B_0^2 v_f}{\rho_f U_0^2}, \quad Ra = \frac{4 \alpha \sigma_1 T_\infty^3}{\delta k_{nf}}$$

$$Q_H = \frac{Q v_f^2}{k_f U_0^2}, \quad Gr = g \beta_{Tf} (T_w - T_\infty) v_f$$

Where the velocity characteristic U_0 is defined as

$$U_0 = [g \beta_f (T_w - T_\infty) v_f]^{1/3}$$

The local Nusselt number Nu in dimension less form:

$$Nu = -x \frac{k_{nf}}{k_f} \theta'(0)$$

3. RESULTS

The momentum decreases with increase in volume fraction (ϕ) from Fig.1. The 5% of solid volume fraction significantly decreases the momentum when compared with usual fluid. From Fig. 2 As the injection increases, the momentum increases more rapidly compared with no suction. The increase in h drastically drops the momentum due to the adhesion of the layer around the Nano-particle are more packing fraction of the particles which can be seen in Fig.3. From Fig. 4 the increase in thermal buoyancy (Gr) will increase the momentum unlike the other profiles of the momentum. From Fig. 5 the magnetic field (M) affects the momentum very much. The momentum is almost linear in the absence of the magnetic field and decreases with increase in M. From Fig. 6 the momentum decreases slightly with increase in inclination angle (γ). From Fig. 7 the heat source (Q_H) does not show much impact on momentum but, the momentum decreases with increase in heat source. From Fig. 8 it shows the increase in radiation decreases the momentum. It is also found that there is no impact on momentum for $Ra > 0.4$. From Fig. 9 the momentum variation is observed for generative ($K < 0$) and destructive ($K > 0$) chemical reactions. The generative chemical reaction produces almost the uniform momentum whereas; the destructive chemical reaction tends to exponential decay in the momentum along the Stretching sheet. Fig. 10 shows the variation of velocity with stretching parameter (x). The increase in x retains the fluid leading to enhance the momentum.

The temperature rises with increase in the solid volume fraction (ϕ) from Fig. 11. From Fig. 12 interestingly the injection (S) enhance the temperature along the plate almost exponentially. From Fig. 13 the layer (h) acts as a semimetal to enhance the temperature along the plate. From Fig. 14 the inclination angle (γ) of the plate reduces the temperature. The thermal boundary layer has thinned by layer around the nano- particle and the inclination of the plate. From Fig. 15 the heat source (Q_H) decreases the temperature. From Fig. 16 the radiation drastically drops the temperature even for slightly higher values ($Ra > 0.1$). From Fig. 17 the generative chemical reaction (K) has no impact on temperature whereas, the destructive chemical reaction reduces the temperature slightly. Fig. 18 shows the variation of temperature with stretching parameter (x). The increase in x retains the fluid leading to reduction in temperature.

Fig. 19 depicts the variation of heat transfer by M. A. A. Hamad and I.Pop (6). Fig. 20 depicts the variation of heat transfer in the present study for $K = 0.5$ at the base of the plate. As the suction and heat source increases the heat transfer rate increases for $\phi = 0.15, 1$. However, the heat transfer rate has no much variation for different S . The heat transfer is more for large volume fraction of the solid due to the decomposition of metal particle.

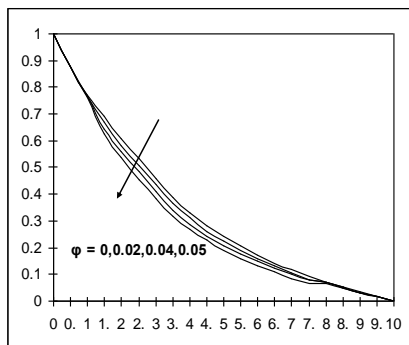


Fig. 1 Variation of U with ϕ

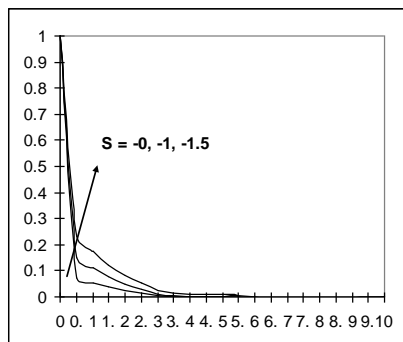


Fig. 2 Variation of U with S

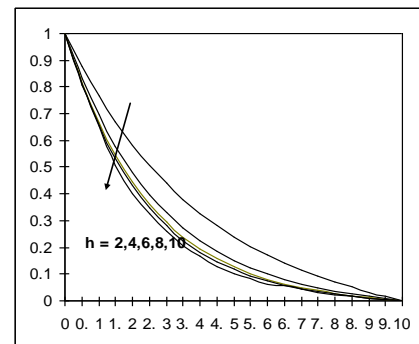


Fig. 3 Variation of U with h

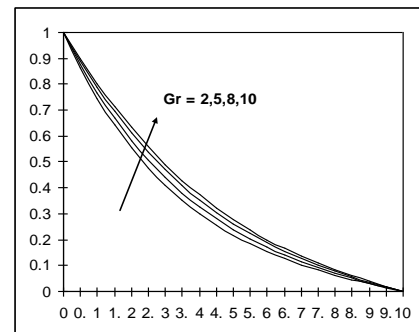


Fig. 4 Variation of U with Gr

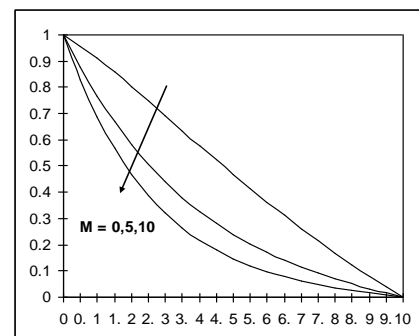


Fig. 5 Variation of U with M

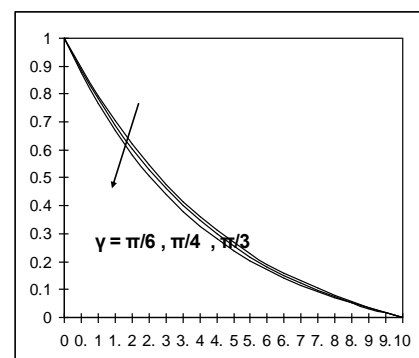


Fig. 6 Variation of U with γ

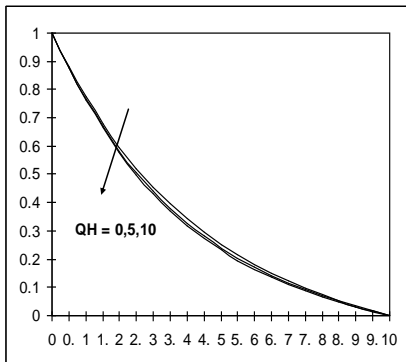


Fig. 7 Variation of U with Q_H

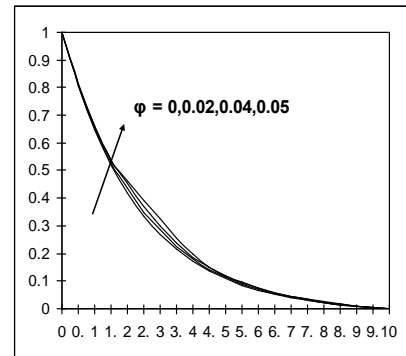


Fig. 11 Variation of θ with ϕ

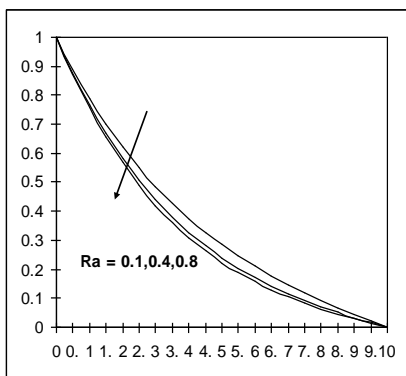


Fig. 8 Variation of U with Ra

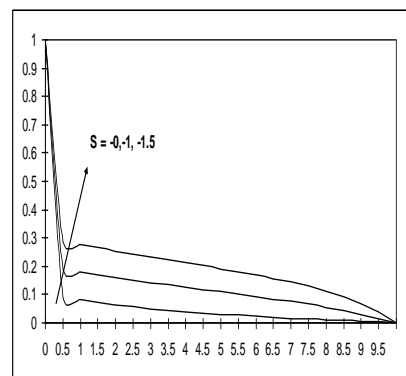


Fig. 12 Variation of θ with S

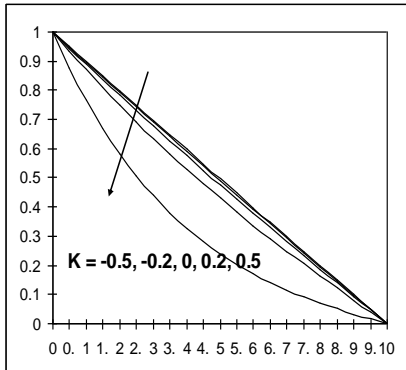


Fig. 9 Variation of U with K

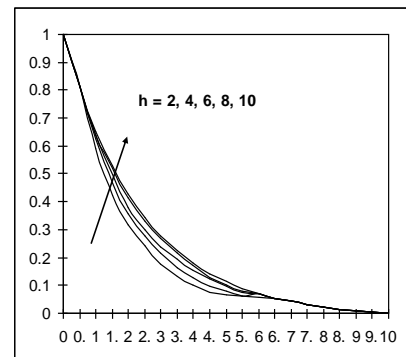


Fig. 13 Variation of θ with h

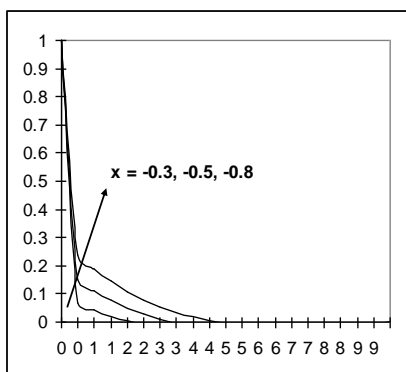


Fig. 10 Variation of U with x

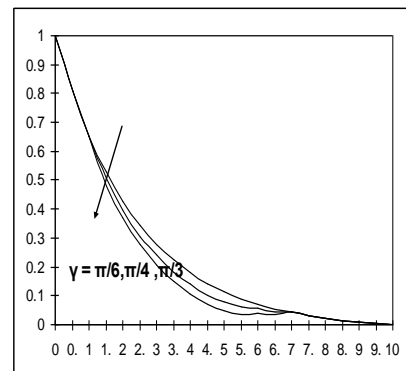


Fig. 14 Variation of θ with γ

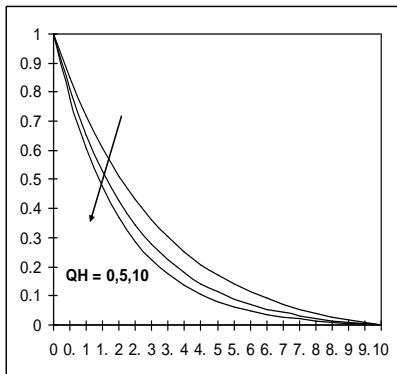


Fig. 15 Variation of θ with Q_H

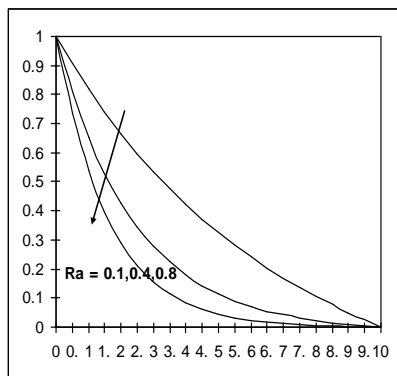


Fig. 16 Variation of θ with Ra

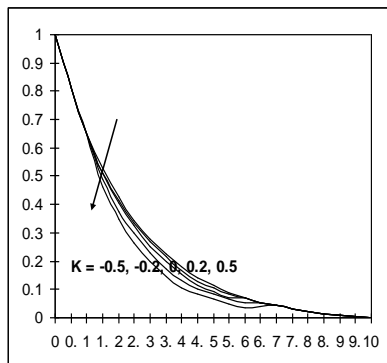


Fig. 17 Variation of θ with K

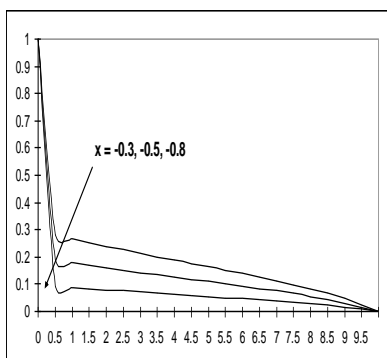


Fig. 18 Variation of θ with x

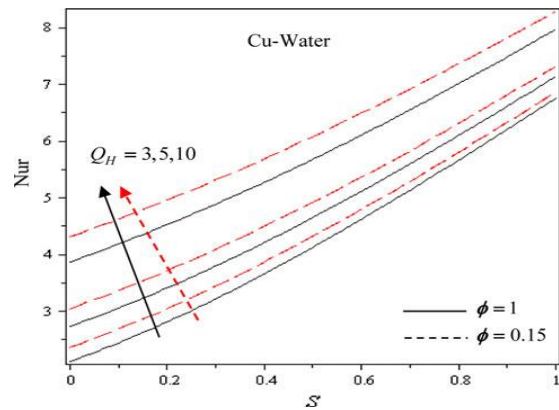


Fig. 19 Variation of Nusselt Number (M. A. A. Hamad)

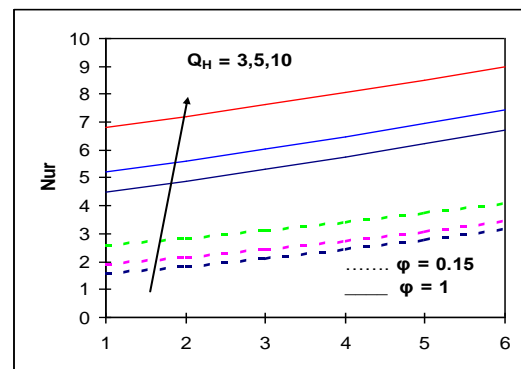


Fig. 20 Variation of Nusselt Number

4. CONCLUSION

The destructive chemical reaction exponentially decreases the velocity. Both types of chemical reaction (destructive and generative) reduce the temperature along the plate and the variation is very narrow.

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BIOGRAPHIES



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