

# Analysis of LFM and NLFM Radar Waveforms and their Performance Analysis

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**Abstract** - The linear frequency modulated (LFM) signal is extensively worn in practical radar systems. A weighting function is required for LFM in order to reduce the side lobes, which considerably decreases the SNR. In an attempt to attain low autocorrelation side lobes without applying weighting function, a different kind of radar waveform is needed, that exhibits high SNR and high range resolution. In this paper, a Non linear frequency modulated (NLFM) waveform is investigated. Different performance parameters for the NLFM and LFM are calculated with the help of computer simulations and comparative study is performed. Moreover Radar ambiguity function (RAF) is also analysed and used for studying various performance parameters of both NLFM and LFM waveforms.

**Key Words:** Linear frequency modulated waveform, non-linear frequency modulated waveform, radar ambiguity function.

## 1. INTRODUCTION

In the radar literature, LFM is known to be easily generated by a variety of technology and has a superior performance in pulse compression [1]. Pulse compression is used to increase the range resolution and signal to noise ratio. To transmit a long pulse that has a bandwidth corresponding to a short pulse, pulse compression technique is required. Despite the above advantages, LFM carries few limitations which cannot be ignored. A compressed LFM signal produces a first side lobe at a level of -13 dB to the peak of the main lobe at the receiver. But for this compression, the output SNR may get reduced typically by 1 to 2 dB. A single dB of SNR lost is equal to 25 % decrease in transmitter power [1]. Few more important corns of the LFM waveform are given below:

- Low signal to noise ratio (SNR).
- LFM always require a weighting function for pulse compression.
- Range resolution is not very good.

Due to this scientists concluded with use of a new waveform called non- linear frequency modulated (NLFM) waveform. NLFM waveform does not require any weighting function, they have inbuilt one. More over their range resolution in very good and their pulse shape is very much Doppler tolerant. In this paper, both the waveforms LFM and NLFM are thoroughly studied and all the experimental results are obtained. LFM waveform which is most commonly known as linear chirp is discussed and studied in next section.

## 2. THEORY

### 2.1 Linear frequency modulated waveform:

LFM waveform commonly known as linear chirps are the most commonly used waveform in radar systems as it can be easily generated, have good range resolution and more doppler tolerant than NLFM. A linear FM chirp has a linear time frequency description as its frequency varies linearly over the pulse duration of the signal. In case of LFM frequency increases (up chirp) or decreases (down chirp) linearly with time.

Following figure shows the time frequency characteristics of the signal. Fig 1 depicts the frequency versus time characteristics of a linear FM chirp.

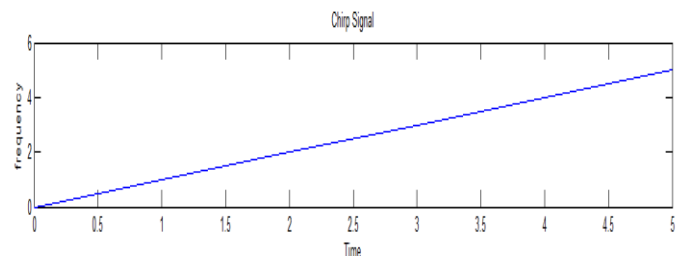


Fig.1 Time vs. frequency plot of linear FM chirp

The complex exponential version of the LFM chirp waveform is given as [2]

$$s(t) = \exp(j\phi(t)) \tag{1}$$

where  $\phi(t)$  is the instantaneous phase given by the equation as below

$$\phi(t) = 2\pi(f_0 + kt^2) \tag{2}$$

The instantaneous frequency as a linear function of time is expressed as [2]

$$f_i = 1/2\pi(d\phi/dt) \tag{3}$$

$$f_i = f_0 + kt \tag{4}$$

where  $k$  is the slope.

$f_0$  is the fundamental frequency.

LFM waveforms can reduce side lobe level by -13dB by applying various methods but at the cost of reduced SNR. Therefore to overcome this we opt for a new waveform NLFM which can do the same with good SNR and low cost. In the next section NLFM waveform is thoroughly described.

### 2.2 Non linear frequency modulation (NLFM) waveform:

NLFM is considered to be capable of achieving fine resolution, good SNR, low cost and high-quality interference mitigation [3]. NLFM is having superior detection rate characteristics and is more precise in range determination than LFM [3]. In case of NLFM, it is observed that the time frequency characteristic is non-linear in nature.

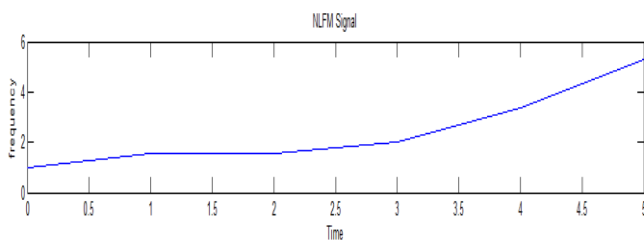


Fig. 2 Time vs. frequency plot of linear FM chirp

The complex exponential of NLFM waveform is given by [4]

$$s(t) = \exp(j\phi(t)) \tag{5}$$

Here in this case, the instantaneous frequency  $f_i$  is given as [4]

$$f_i = f_0 + kt + \cos(t) \tag{6}$$

where  $f_0$  is the fundamental frequency

$t$  is the instantaneous time

$k$  is the slope

Non linear frequency modulation waveform does not require weighting function as they have an in built weighting function. NLFM is a waveform which cannot be easily generated, to understand the principle of generation of NLFM one must understand the principle of stationary phase which is described in the next section.

### 2.3 Principle of stationary phase :

The method for evaluation of integrals of the type is given by stationary phase principle [5]

$$I = \int_{-\infty}^{\infty} F(x) e^{-j\phi(x)} dx \tag{7}$$

where  $\phi(x)$  is function of  $x$  which is rapidly varying over most of the range of integration,  $F(x)$  is gradually varying

The cancellation of sinusoids with rapidly varying phases is the basic idea of the stationary phase method. Rapidly varying phases means that the factor  $I$  is roughly zero above those regions of the integrand and the only significant non zero contribution to the integral occurs in region of the integration where  $(d\phi/dx)=0$

Points of stationary phase are labeled  $x_s$  and is defined by  $\phi'(x_s) = 0$

It should be noted that  $F(x) = F(x_s)$

Now expanding  $\phi(x)$  in a Taylor series near the point  $x_s$  and keeping only the first two non zero terms given by

$$\phi(x) = \phi(x_s) + 1/2\phi''(x_s)(x - x_s) \tag{8}$$

Substituting this in the integral

$$I = \sqrt{2\pi/j\phi''(x_s)} F(x_s) e^{-j\phi(x_s)} \tag{9}$$

### 2.4 Radar ambiguity function:

The foremost tool for analyzing radar signals is ambiguity function. The ambiguity function is primarily used to gain an understanding of how a signal processor responds to a given returned signal. It defines the range and doppler resolution. The ambiguity function of the waveform  $s(t)$  can be defined in terms of the cross-correlation of a Doppler-shifted version of the waveform, that is  $s(t)\exp(j2\pi f_d t)$  with the un shifted waveform.[6]

$$x(\tau, f_d) = \int_{-\infty}^{\infty} [s(t) e^{j2\pi f_d t}] [s^*(t - \tau)] dt \tag{10}$$

where  $\tau$  is the delay time

$f_d$  is the Doppler frequency shift

$$s(t) = 1 \quad ; \text{ if } 0 \leq t \leq t_s$$

$$= 0 \quad ; \text{ elsewhere}$$

As the function describes more about the waveform than just its ambiguity properties, hence the name ambiguity function is quite misleading.

Properties of radar ambiguity function:

1. Maximum at (0,0)  
 $|x(\tau, f_d)| \leq |x(0,0)| = 1$   
 Ambiguity function is highest at the origin
2. Constant volume  
 $\iint_{-\infty}^{\infty} |x(\tau, \nu)|^2 d\tau d\nu = 1$
3. Symmetry with respect to origin  
 $|x(-\tau, -\nu)| = |x(\tau, \nu)|$
4. Linear FM effect [7]  
 $u(t) \exp(j\pi k t^2) = |x(\tau, \nu - k\tau)|$

### 2.5 Radar ambiguity functions for LFM signal:

The LFM complex envelope signal given by [8]

$$s(t) = \left(\frac{1}{\sqrt{T}}\right) \text{rect}\left(\frac{t}{T}\right) e^{j\pi u t^2} \tag{11}$$

For computing LFM complex envelope we take the following case:

When  $0 \leq \tau \leq T'$

Therefore the integration limits becomes  $-\left(\frac{T'}{2}\right)$  to  $\left(\frac{T'}{2}\right) - \tau$

Now the ambiguity function is given by [8]

$$x(\tau; f_d) = \int_{-\infty}^{\infty} s(t) s^*(t + \tau) e^{j2\pi f_d t} dt \tag{11}$$

Now substituting equation 1 in equation 2

$$x(\tau, f_d) = \left(\frac{1}{T}\right) \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{T}\right) \text{rect}\left(\frac{t+\tau}{T}\right) e^{j\pi u t^2} e^{-j\pi u (t+\tau)^2} e^{j2\pi f_d t} dt \tag{12}$$

And the above equation follows that

$$x(\tau, f_d) = \left(\frac{e^{-j\pi u \tau^2}}{T}\right) \int_{-\tau/2}^{\tau/2} e^{-j2\pi (u\tau - f_d)t} dt \tag{13}$$

after applying integration steps we get the following equation

$$x(\tau; f_d) = e^{-j\pi u \tau^2} \left(1 - \frac{\tau}{T}\right) \frac{\sin(\pi \tau' (u\tau + f_d) (1 - \frac{\tau}{T}))}{\pi \tau' (u\tau + f_d) (1 - \tau/\tau')} \tag{14}$$

Similarly, examination for the case when  $-\tau \leq \tau \leq 0$  be able to be carried out where the integration limits will become  $(-\frac{\tau}{2}) - \tau$  to  $\tau'/2$ . Similar result can be obtained by means of the symmetry property of the ambiguity function ( $|x(-\tau, -f_d)| = |x(\tau, f_d)|$ ). It follows an expression for  $x(\tau; f_d)$  that is suitable for any  $\tau$  is given by

$$x(\tau; f_d) = e^{j\pi u \tau^2} \left(1 - \frac{|\tau|}{T}\right) \frac{\sin(\pi \tau' (u\tau + f_d) (1 - \frac{|\tau|}{T}))}{\pi \tau' (u\tau + f_d) (1 - \frac{|\tau|}{T})} \tag{15}$$

and the LFM ambiguity function is

$$x(\tau; f_d) = \left| \left(1 - \frac{|\tau|}{T}\right) \frac{\sin(\pi \tau' (u\tau + f_d) (1 - \frac{|\tau|}{T}))}{\pi \tau' (u\tau + f_d) (1 - \frac{|\tau|}{T})} \right| \tag{16}$$

### 3. RESULTS AND DISCUSSIONS

For linear frequency modulated waveform:

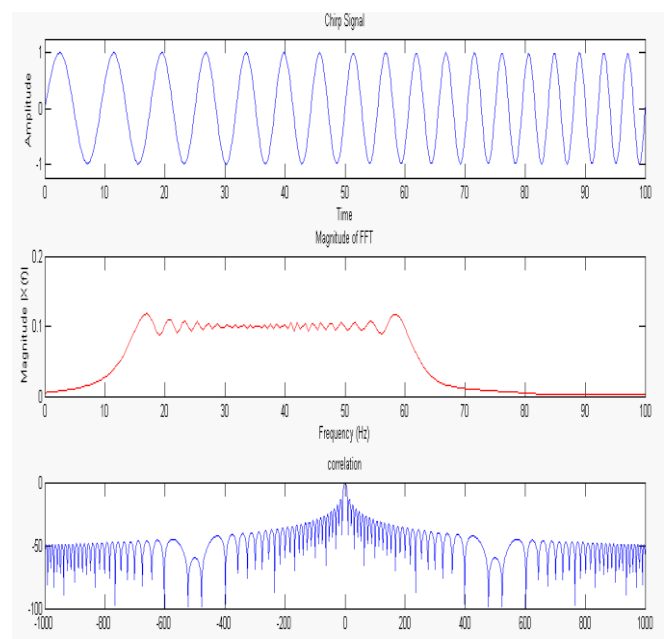


Fig. 3 shows chirp signal, its magnitude spectrum and its matched filter response respectively.

Here in fig. 3 chirp signal is generated and its corresponding magnitude spectrum is plotted. The matched filter response is the sinc function which is the autocorrelation function of the original signal. The side lobes of this matched filter response comes about -13dB.

For non linear frequency modulated waveforms:

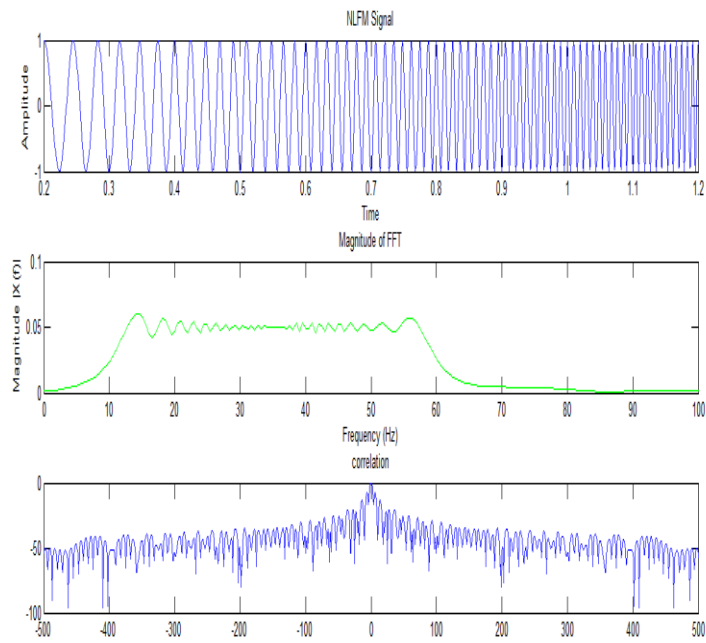


Fig. 4 shows NLFM signal, its magnitude spectrum and its matched filter response respectively.

Here in fig. 4 NLFM signal is generated and its corresponding magnitude spectrum is plotted. The matched filter response is the sinc function which is the autocorrelation function of the original signal. The side lobes of this matched filter response comes about -40dB.

Ambiguity plots :

For linear frequency modulated waveforms:

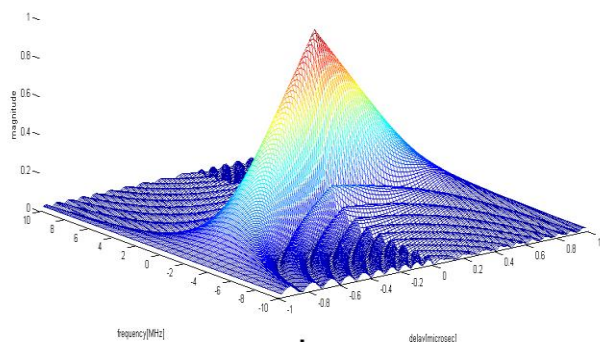


Fig. 5(a) ambiguity plot of LFM

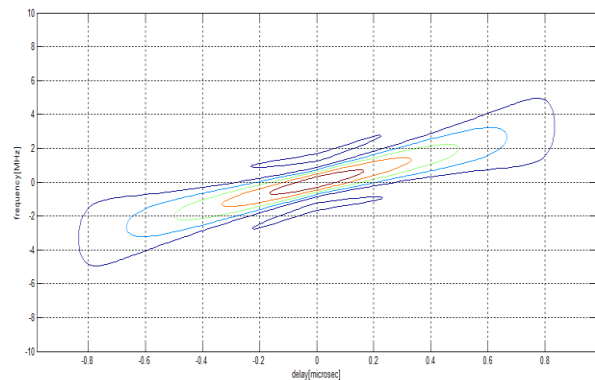


Fig. 5(b) contour plot of LFM

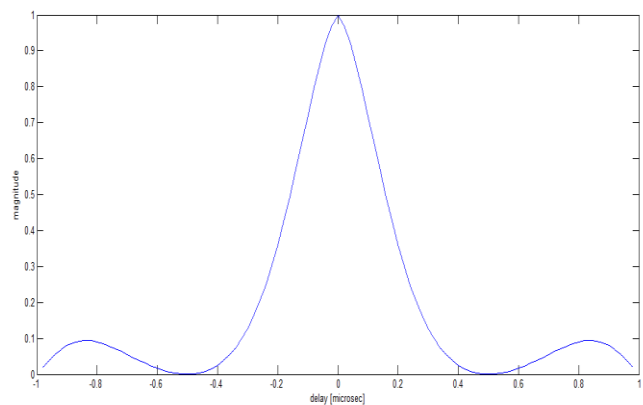


Fig. 5(c) Doppler cut plot of LFM

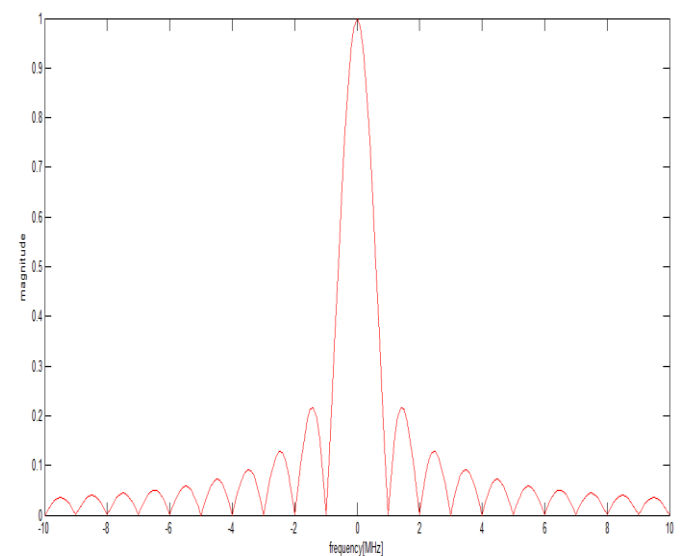


Fig. 5(d) delay cut plot of LFM

For non- linear frequency modulated waveforms :

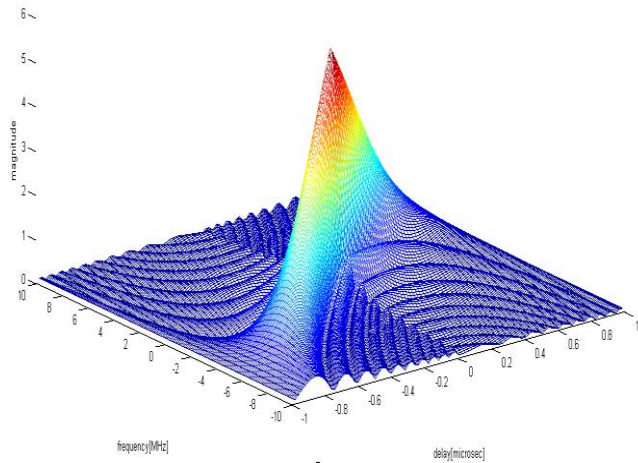


Fig. 6(a) ambiguity plot of NLFM

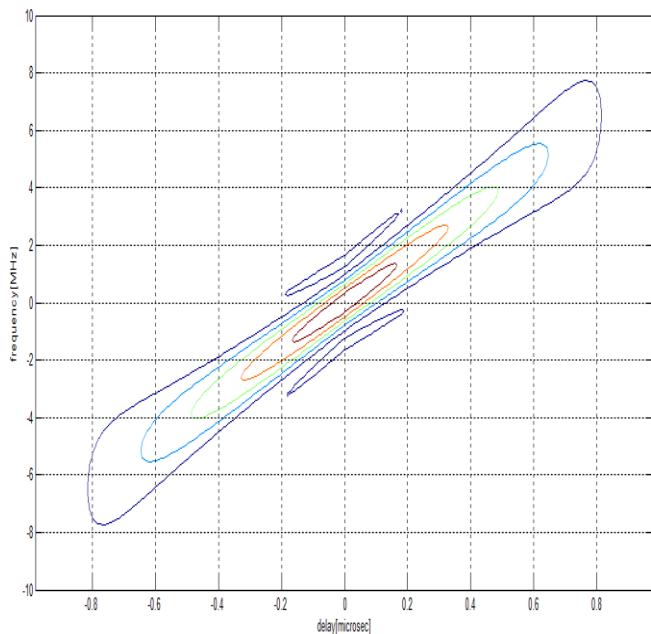


Fig. 6(b) contour plot of NLFM

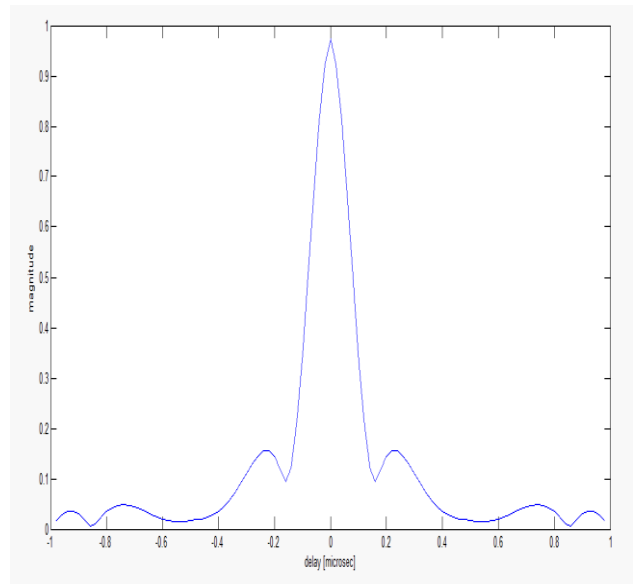


Fig. 6(c) doppler cut plot of NLFM

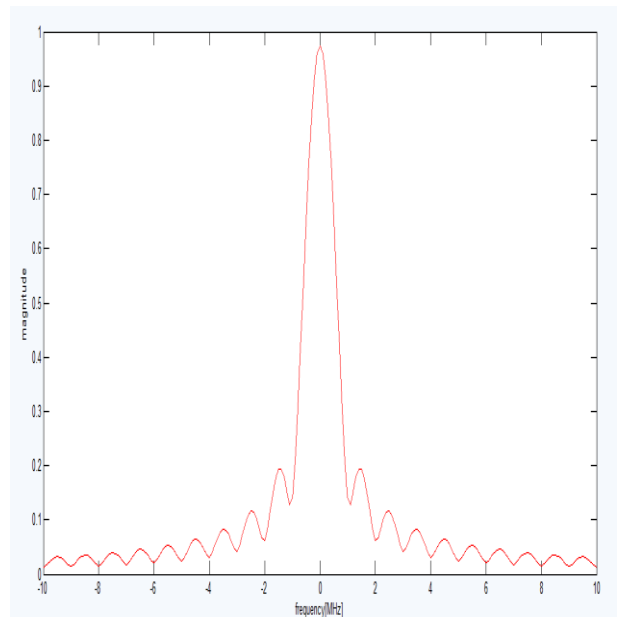


Fig . 6(d) delay cut plot of NLFM

The above shown plots are the ambiguity function plots for LFM and NLFM waveforms. Fig 5(a) shows the ambiguity plot. Contour plot of the ambiguity function for LFM is shown in Fig 5(b), contour plot is the projection of the ambiguity function. By default, contour traces are assigned a colour from the rainbow colour. Red is having the minimum magnitude and violet is having maximum magnitude. Fig 5(c) shows the zero doppler cut of the ambiguity function plot which is obtained by putting  $f_d = 0$  in the ambiguity function . Doppler cut plot is of great importance as it tells about the range resolution of the waveform. Here in case of the LFM range resolution

comes out to be  $1/B$  which is equal to  $(1/0.5) = 2$ . Fig 5(d) shows the delay cut plot of the ambiguity function for LFM.

Correspondingly, Fig 6(a) shows the ambiguity function plot for the NLFM waveform, Fig 6(b) shows the projection of the ambiguity function which is known as the contour plot. Fig6(c) shows the zero doppler cut of the ambiguity function plot, it tells about the range resolution of the waveform and here in case of NLFM range resolution comes out to be  $(1/B)=(1/0.1)=10$ . Fig 6(d) shows the delay cut plot of the ambiguity function for NLFM.

### 3.1 Comparative analysis of LFM and NLFM waveforms:

There exist certain performance parameters that distinguish LFM and NLFM waveforms. They are mentioned as PSL, Range resolution, Doppler shift. Table 1 below shows different calculated values of these performance parameters via computer simulations.

Table 1 Main performance parameters of the LFM and NLFM waveforms:

Parameter	LFM	NLFM
PSL(peak side lobe)	-13dB	-40dB
Range resolution	2	10
Doppler shift	1 MHz	1.3 MHz

### 4. CONCLUSIONS

This paper presents the detailed description of LFM and NLFM and their performance parameters. The main parameters embrace PSL, range resolution and doppler shift. Moreover the matched filter response of both the waveforms is approximated by a sinc function; only difference lies in the side lobe suppression. In case of LFM, we have sinc function with -13dB side lobes and in case of NLFM, we have sinc function with -40 dB side lobes. Doppler shift means that if two targets are at the same range, they need to have a difference of 1 MHz(LFM) and 1.3MHz(NLFM) in the doppler domain to be separated. In summary, NLFM came out to be a more practical,

constructive and functional radar signal waveform. It has been an effective technique for side lobe suppression. If the doppler intolerance of the NLFM is enhanced in a controlled manner, NLFM can be worn as the most used practical radar waveform in future.

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