

Application of Classical Lamination Theory and Analytical Modeling of Laminates

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Abstract - Extensive research has been carried out on FRP (Fiber Reinforcement Polymer) composites and on variety of laminates. In this paper, work has been done on unsymmetric $0^\circ/30^\circ/-45^\circ$ graphite/epoxy laminate and symmetric $0^\circ/90^\circ/0^\circ$ graphite/epoxy laminate with different mechanical and thermal loading conditions. Theoretical model is developed to show the mechanical response of a laminate. Determination of response of a laminate involves developing stress-strain relations for a composite plate. The macromechanical analysis has been carried out for a laminate. Based on applied in-plane loads of extension, shear, bending and torsion, stresses and strains are found in the global axes of each ply. Stresses and strains in each ply are calculated for the laminates subjected to thermal loads. Analytical models developed helps to calculate fairly accurately mechanical properties of each lamina. Finally results are computed through CLT (Classical Lamination Theory) and ANSYS. Conclusion is drawn based on computed results.

Key Words: CLT, graphite/epoxy, laminate and stress

1. Introduction

A lamina is a thin layer of a composite material that is generally of a thickness of the order of 0.125 mm. A laminate is constructed by stacking a number of such lamina in the direction of the lamina thickness (Figure 1). The design and analysis of laminated structures demands knowledge of the stresses and strains in the laminate. Also, design tools, such as failure theories, stiffness models, and optimization algorithms, need the values of these laminate stresses and strains. However, the building blocks (Figure 2) of a laminate are single lamina, so understanding the mechanical analysis of a lamina precedes understanding that of a laminate. A lamina is unlike an isotropic homogeneous material. For example, if the lamina is made of isotropic homogeneous fibers and an isotropic homogeneous matrix, the stiffness of the lamina varies from point to point depending on whether the point is in the fiber, the matrix, or the fiber-matrix interface. Accounting for these variations will make any kind of mechanical modeling of the lamina very complicated. For this reason, the macromechanical analysis of a lamina is

based on average properties and considering the lamina to be homogeneous [1].

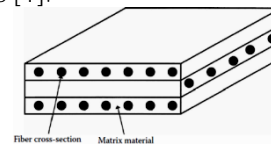
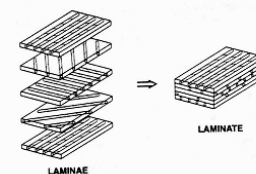


Fig -1: Laminate with 3 lamina showing fiber and matrix

The thermo mechanical properties required for the analysis were obtained from tests on unidirectional laminates [2]. The fundamental solutions of concentrated forces and moments on an infinitely extended Unsymmetric laminate are developed and discussed [3]. It should be recognised that similar behaviour can also be achieved using less sophisticated designs, such as applying off-axis material



1. Fig -2: Lamina stacked to make laminate

alignment to otherwise balanced and symmetric laminates or by using un-balanced and symmetric design [4], Unsymmetric CFRP laminates are studied. These hybrid laminates are studied using analytical, finite element and experimental techniques [5]. Usually, uncoupling is obtained using symmetric stacking sequences, but in this case bending orthotropy is very difficult to be obtained [6]. The sensitivity of the FEM to model inputs and manufacturing accuracy was explored, and it was found that the laminate thickness and accuracy of ply angles were highly influential on the ability of the FEM to predict accurate deformations [7].

2. Statement of the problem

In this paper, study is done by taking 3 layer laminate composite, and calculating strains and stresses for different type of laminates with different orientation of lamina are stacked.

Mathematical relationship is developed for 3 layered symmetric and unsymmetric laminates by applying classical lamination theory, two cases are considered.

In case 1: unsymmetric 0°/30°/-45° graphite /epoxy laminate under biaxial loading (mechanical loading) without considering thermal loading condition is taken in to validation by classical lamination theory.

In case 2: symmetric 0°/90°/0° graphite/epoxy laminate under uniaxial loading along with thermal loading conditions under proper curing and room temperature is validated by finding strains and stresses through classical lamination theory. Now both cases are taken in to analytical validation through ANSYS. Results obtained from classical lamination theory and ANSYS mechanical APDL (ANSYS Parametric Design Language) are compared.

3. Mathematical model

3.1 Stress-Strain Relations for an Orthotropic Material

The Generalized Hooke's Law of stress and strain of any material is,

$$\sigma_i = C_{ij} \epsilon_j \tag{3.1}$$

In terms of strain,

$$\epsilon_j = S_{ij} \sigma_i \tag{3.2}$$

Where C_{ij} is the stiffness matrix, S_{ij} is the compliance matrix, σ_i are stress components, ϵ_j are strain components

The Generalized Hooke's Law for an Orthotropic Material reduces to

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} \tag{3.3}$$

3.2 Hooke's Law for Lamina under plane stress state

A thin plate is a prismatic member having a small thickness, and it is the case for a typical lamina. If a plate is thin and there are no out-of-plane loads, it can be considered to be under plane stress. If the upper and lower surfaces of the plate are free from external loads, then $\sigma_3 = 0, \tau_{31} = 0$ and $\tau_{23} = 0$. This assumption then reduces the three-dimensional stress-strain equations to two-dimensional stress-strain equations.

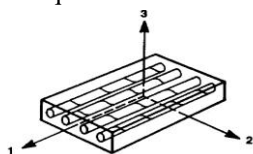


Fig -3: Unidirectional lamina

A unidirectional lamina falls under the orthotropic material category. If the lamina is thin and does not carry any out-of-plane loads, one can assume plane stress

conditions for the lamina. Therefore, taking $\sigma_3 = 0, \tau_{31} = 0$ and $\tau_{23} = 0$.

Equation (3.3) for an orthotropic plane stress problem can then be written as

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \tag{3.4}$$

Inverting Equation (3.4) stress-strain relationship is,

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} \tag{3.5}$$

Where Q_{ij} are the reduced stiffness coefficients, which are related to the Compliance coefficients as Stiffness coefficients in terms of engineering or technical constants is

$$Q_{11} = \frac{E_1}{1 - \nu_{12} \nu_{21}}, Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{12} \nu_{21}}, Q_{22} = \frac{E_2}{1 - \nu_{12} \nu_{21}}$$

$$Q_{66} = G_{12} \tag{3.6}$$

E_1 = longitudinal Young's modulus (in direction 1)

E_2 = transverse Young's modulus (in direction 2)

ν_{12} = major Poisson's ratio, where the general Poisson's

G_{12} = in-plane shear modulus (in plane 1-2)

3.3 transformed reduced stiffness matrix for angle lamina:

The coordinate system used for showing an angle lamina is as given in Figure 4. The angle between the two axes is denoted by an angle θ .

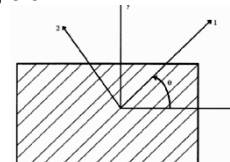


Fig -4: Angle lamina showing local and global axes

Relation between stress and strain in global axis is,

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{3.7}$$

Where \bar{Q}_{ij} are called the elements of the transformed reduced stiffness matrix,

$$\bar{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \tag{3.8}$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^4)$$

$$\bar{Q}_{22} = Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})c^3s - (Q_{22} - Q_{12} - 2Q_{66})s^3c$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})s^3c - (Q_{22} - Q_{12} - 2Q_{66})c^3s$$

$$\bar{Q}_{66} = (Q_{11} - Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4)$$

Where, $c = \cos(\theta)$, $s = \sin(\theta)$

From above equations, it can be seen that they are just functions of the four stiffness elements Q_{11} , Q_{12} , Q_{22} , and Q_{66} , and the angle of the lamina, θ .

3.4 Classical lamination theory:

The relationships are developed for a plate under in-plane loads such as shear and axial forces, bending and twisting moments. The classical lamination theory is used to develop these relationships. The following assumptions are made in the classical lamination theory to develop the relationship. Each lamina is orthotropic, and homogeneous.

Laminate strains can be written as

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \end{bmatrix} \quad (3.9)$$

Above equation shows the linear relationship of the strains in a laminate to the curvatures of the laminate.

Strain and Stress in a Laminate:

If the strains are known at any point along the thickness of the laminate, putting equation (3.9) in equation (3.7),

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \end{bmatrix} \quad (3.10)$$

From Equation (3.10), the strains vary linearly only through the thickness of each lamina (Figure 5).

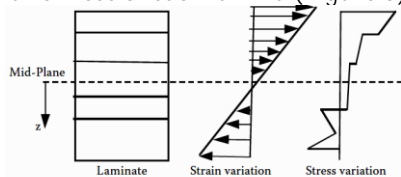


Fig -5: Laminate thickness with stress and strain variation

Equation (3.10) gives the stresses in each lamina in terms of these unknowns. The stresses in each lamina can be integrated through the laminate thickness to give resultant forces and moments (or applied forces and moments). Consider a laminate made of n plies shown in Figure 6

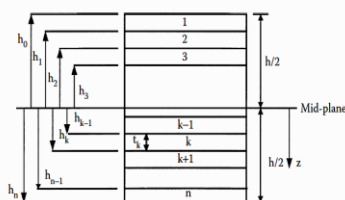


Fig -6: Locations of plies in laminate

The resultant forces and moments can be written in terms of the midplane strains and curvatures:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \end{bmatrix} \quad (3.11)$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (3.12)$$

Where,

$$A_{ij} = \sum_{k=1}^n [\bar{Q}_{ij}]_k (h_k - h_{k-1}) \quad (3.13)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n [\bar{Q}_{ij}]_k (h_k^2 - h_{k-1}^2) \quad (3.14)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n [\bar{Q}_{ij}]_k (h_k^3 - h_{k-1}^3) \quad (3.15)$$

i=1, 2, 6; j=1, 2, 6

The [A], [B], and [D] matrices are called the extensional, coupling, and bending stiffness matrices, respectively.

Combining Equation (3.11) and Equation (3.12) gives six simultaneous linear equations and six unknowns as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (3.16)$$

Thermal and mechanical stresses:

The 3 dimensional strain-stress relations are given by

$$\sigma_i = C_{ij}(\varepsilon_j - \alpha_j \Delta T) \quad i, j = 1, 2, \dots, 6$$

ε_i is total strain

$\alpha_i \Delta T$ are 6 free thermal strains due to temperature ΔT

The stresses in laminate coordinates for the k^{th} layer is,

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \gamma_{xy} - \alpha_{xy} \Delta T \end{bmatrix}_k \quad (3.17)$$

The resultant forces are,

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \\ B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (3.18)$$

Thermal forces are obtained as,

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \int \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix}_k \Delta T dz \quad (3.19)$$

Forces are rearranged as

$$\begin{bmatrix} \bar{N}_x \\ \bar{N}_y \\ \bar{N}_{xy} \end{bmatrix} = \begin{bmatrix} N_x + N_x^T \\ N_y + N_y^T \\ N_{xy} + N_{xy}^T \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (3.20)$$

Writing equation (3.20) in reduced matrix form,

$$[\bar{N}] = [A \ B] [\varepsilon^0] \quad (3.21)$$

Inverting equation (3.21),

$$[\epsilon^0] = [A' \ B'] [\bar{N}] \tag{3.22}$$

4. Theoretical and analytical calculation

4.1 CLT theory on case 1:

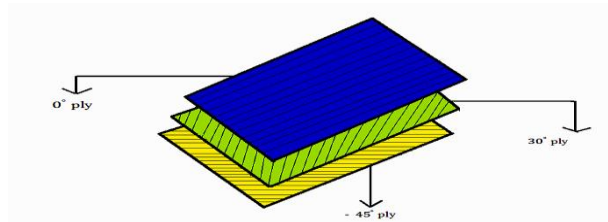


Fig -7: Stacking sequence of graphite/epoxy laminate

In case one glass/epoxy laminate with 3 layered unsymmetrical 0, 30,-45 degree lamina stacked on one other, staking sequence is shown in figure 7.

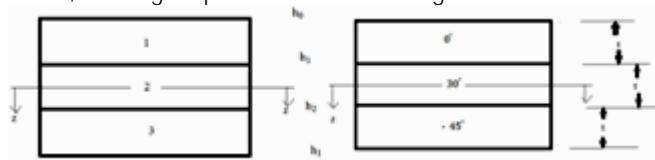


Fig -8: Ply surface position and thickness

Stresses and strains are found by applying classical laminate theory on this laminate and obtained strains and stresses, and calculated strains and stresses for same model in ansys and compared these analytical values with theoretical readings obtained by classical laminate theory.

Table 4.1 Mechanical properties for graphite epoxy lamina [8]

Property	symbol	Units	Graphite/epoxy
Fiber volume fraction	V_f		0.70
Longitudinal elastic modulus	E_1	GPa	181
Transverse elastic modulus	E_2	GPa	10.30
Major Poisson's ratio	ν_1		0.28
Shear modulus	G_{12}	GPa	7.17
Longitudinal coefficient of thermal expansion	α_1	$\mu\text{m}/\text{m}/^\circ\text{C}$	0.02
Transverse coefficient of thermal expansion	α_2	$\mu\text{m}/\text{m}/^\circ\text{C}$	22.5

Each lamina are having thickness $t=5\text{mm}$

Using equations from mathematical model, stresses are given as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{0^\circ, \text{top}} = \begin{bmatrix} 33537.22 \\ 61945.28 \\ -27532.8 \end{bmatrix} \quad \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{0^\circ, \text{bottom}} = \begin{bmatrix} 55763.16 \\ 45329.03 \\ -12798.4 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{30^\circ, \text{top}} = \begin{bmatrix} 69300 \\ 73910 \\ 33810 \end{bmatrix} \quad \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{30^\circ, \text{bottom}} = \begin{bmatrix} 143344.26 \\ 81015.80 \\ 84234.15 \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{-45^\circ, \text{top}} = \begin{bmatrix} 123521.458 \\ 156265.414 \\ -118671.456 \end{bmatrix}$$

Similarly,

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{-45^\circ, \text{bottom}} = \begin{bmatrix} -25452.6 \\ -18380.84 \\ 40881.63 \end{bmatrix}$$

4.2 Validating case 1 in ANSYS:

For 3 layered unsymmetrical 0,30,-45 degree graphite/epoxy laminate shell 181 is used as element. By using shell element laminate can be easily modeled. Where each shell thickness in case 1 is 0.005 m, Model is created by taking square cross section of each side of 1m and meshing is done by element edge length of 0.5, meshing is done for entire laminate at a time, 3 lamina are stacked on one other to get single laminate, each lamina consisting of matrix and fiber elements.

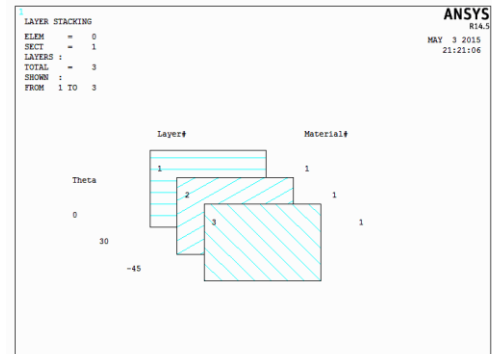


Fig -9: Lamina stacking sequence with fiber orientation representation

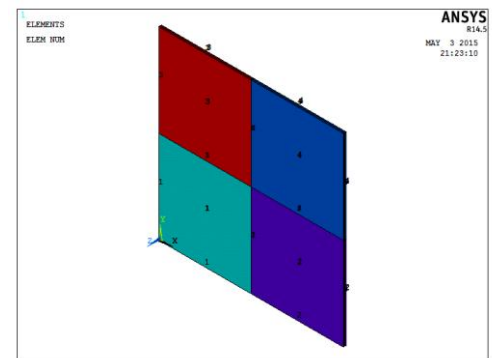


Fig -10: Model showing element numbers of laminate

Now to apply constraints, node at the centre of laminate is selected and all degrees of freedom set to zero. Loads of 1000 N/m is applied on laminate in both x and y directions.

4.3 Stresses obtained in ANSYS:

For 0° ply,

Stresses in x direction

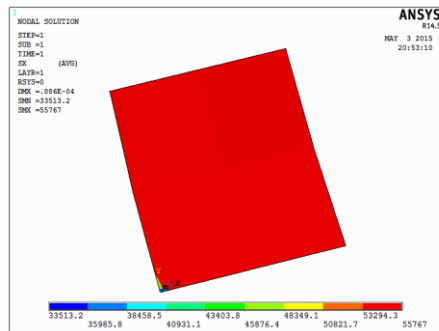


Fig -11: 0° ply, stress in x direction

Stresses in y direction

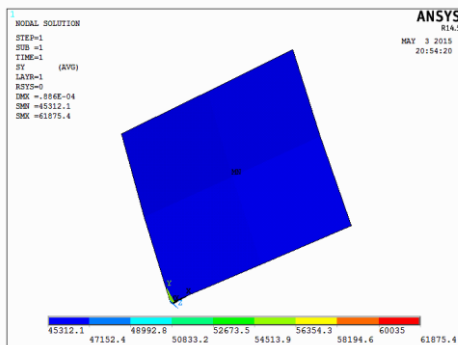


Fig -12: 0° ply, stress in y direction

Shear stress in x-y plane

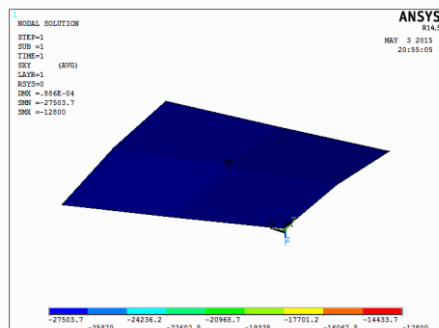


Fig -13: 0° ply, shear stress in x-y direction

For 30° lamina,

Stresses in x direction

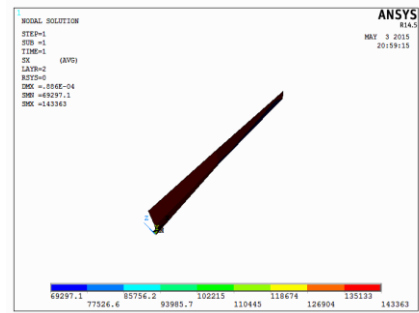


Fig -14: 30° ply, stress in x direction

Stresses in y direction

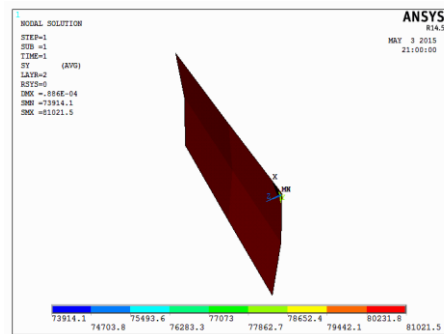


Fig -15: 30° ply, stress in y direction

Shear stresses in x-y plane

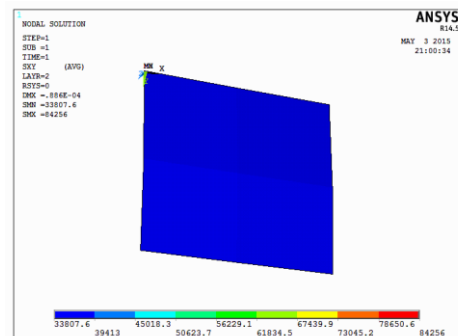


Fig -16: 30° ply shear stress in x-y direction

For -45° lamina,

Stress in x and y direction are plotted as

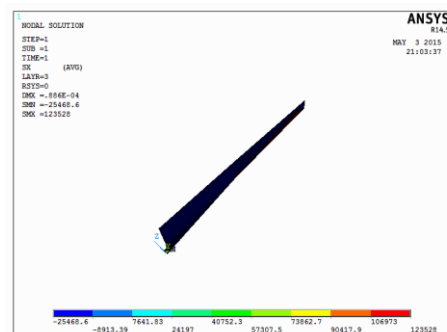


Fig -17: -45° ply stress in x direction

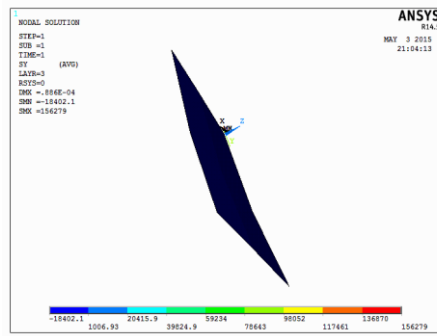


Fig -18: -45° ply stress in y direction

Shear stress in x-y plane

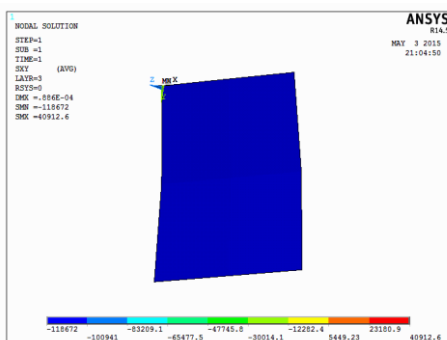


Fig -19: -45° ply shear stress in x- y direction

Stresses for entire laminate,
Stresses in x direction

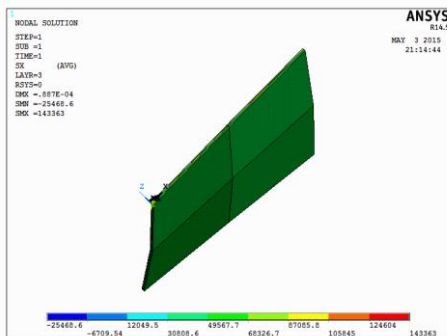


Fig -20: Stresses in x direction for laminate

Stresses in y direction

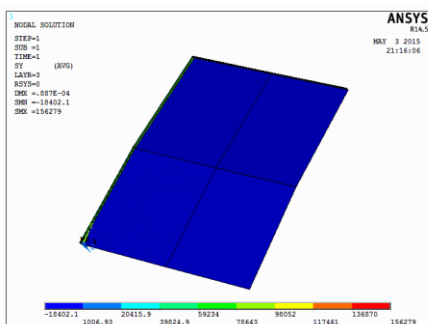


Fig -21: Stresses in y direction for laminate

Shear stress in x-y plane of laminate

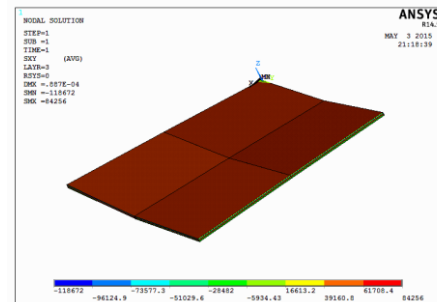


Fig -22: Shear stress in x-y plane for laminate

4.4 CLT theory on case 2:

In this case symmetric 0/90/0 degree graphite epoxy laminate is taken in to consideration and thermal loads are included under certain curing and room temperature along with_mechanical_loads .

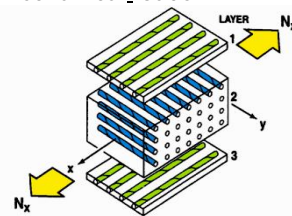


Fig -23: 0/90/0 degree graphite epoxy laminate with fiber orientation

Layers are stacked on one other with green colour of fiber indicates 0 degree ply, blue colour indicates 90 degree ply. Since this cross-ply laminate having 3 layers, it is **symmetric about it's middle surface. This is the reason why, there is no coupling between bending and extension.** Layer 1 is outer layer and fibers are in x direction, layer 2 is inner layer and fibers are in y direction, due to symmetry layer 3 is similar to layer 1. Thickness of both outer layers is equal to 0.127mm, Thickness of inner layer is equal to 10 times, thickness of outer layer 1.27 mm, Total laminate thickness is equal to 1.524mm.

Table 4.2 Mechanical properties of this graphite epoxy laminate [9]

Property	symbol	Units	Graphite/ epoxy
Longitudinal elastic modulus	E_1	GPa	290
Transverse elastic modulus	E_2	GPa	6.9
Major Poisson's ratio	γ_1		0.25
Shear modulus	G_{12}	GPa	4.8
Longitudinal coefficient of thermal expansion	α_1	$\mu\text{m}/\text{m}/^\circ\text{C}$	-1.04
Transverse coefficient of thermal expansion	α_2	$\mu\text{m}/\text{m}/^\circ\text{C}$	29.7

This composite was cured at 370 °F but is sitting in a room at 70 °F . Thus we must consider the residual stresses resulting from the temperature difference.

Curing temperature= 370 °F = 187.8°C

Room temperature = 70 °F = 21.11°C

Change in temperature $\Delta T = 21.11 - 187.8 = -166.7^\circ\text{C}$

Stresses in each lamina is given as

For 0° lamina

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{0^\circ} = \begin{bmatrix} 3330.376 \\ 54.2118 \\ 0 \end{bmatrix}$$

For 90° lamina

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_{90^\circ} = \begin{bmatrix} 114.193 \\ -11.18 \\ 0 \end{bmatrix}$$

4.5 Validating case 2 in ANSYS:

Same size of laminate is modelled as taken in case 1 ,mechanical properties are also mentioned along with thermal coefficient of expansions and both curing and room temperature also specified in this case. Meshing is done in same way as in case one. Mechanical loading and uniform temperature are also specified to get final solution.

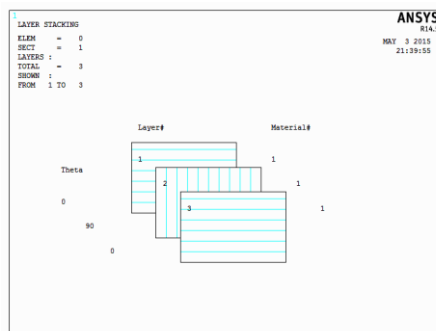


Fig-24: Stacking sequence of 0°/90°/0° graphite/epoxy

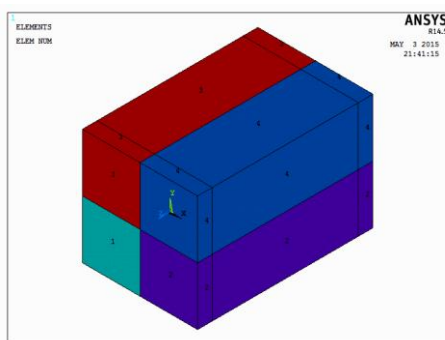


Fig -25: Model of case 2 with element numbers

Stresses in case 2:

For lamina 1, Stress in x direction

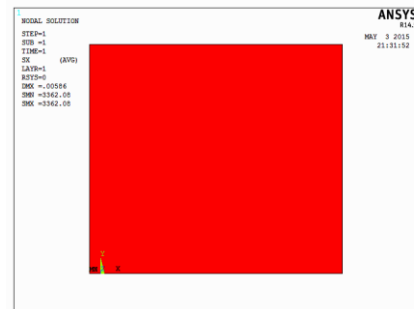


Fig -26: stresses in x direction

Stresses in y direction

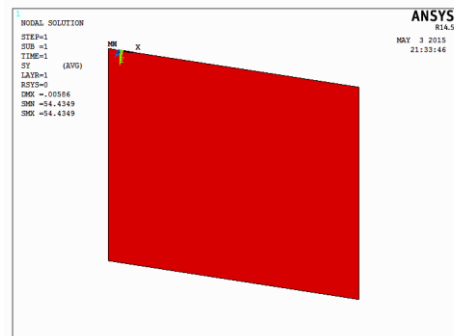


Fig -27: For lamina 2, stresses in x direction

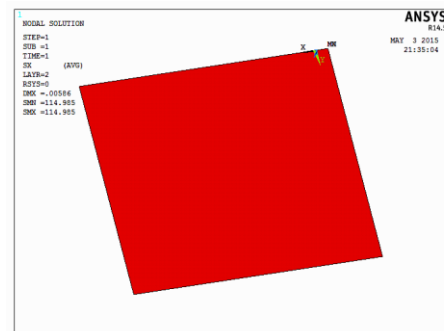


Fig -28: Stresses in x direction

Stresses in y direction

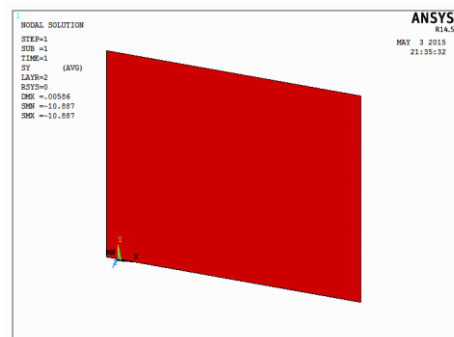


Fig -29: Stresses in y direction

Stresses for entire laminate,
Stresses in x direction

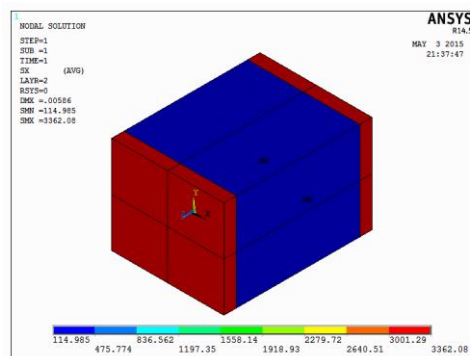


Fig -30: stresses in x direction

Stresses in y direction

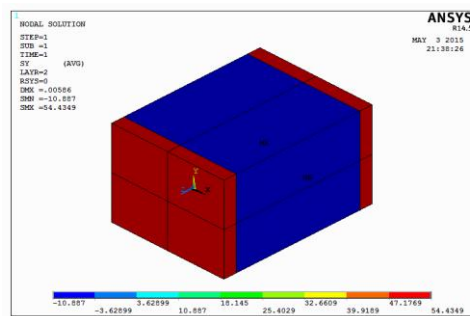


Fig -31: stresses in y direction

5. RESULT

Stresses calculated in both symmetric and unsymmetrical graphite/epoxy laminate with different mechanical and thermal loading condition by applying classical lamination theory and analysed in ANSYS. Thus obtained theoretical and analytical results for both are listed and calculated percentage error to compare the results obtained from CLT and ANSYS.

5.1 for case 1, 0°/30°/-45° graphite/ epoxy laminate

Lamina 1 0°

stresses	By CLT		ANSYS		% Error	
	UL	LL	UL	LL	UL	LL
σ_x	33537.22	55763.16	33513.2	55767	0.07	0.01
σ_y	61945.28	45329.03	61875.4	45312.1	0.11	0.04
τ_{xy}	-27532.8	-12798.4	-27503	-12800	0.11	0.01

Lamina 2 30°

stresses	By CLT		ANSYS		% Error	
	UL	LL	UL	LL	UL	LL
σ_x	69300	143344.26	69297.1	143363	0.00	0.01
σ_y	73910	81015.80	73914.1	81021.5	0.01	0.01
τ_{xy}	33810	84234.15	33807.6	84256	0.01	0.03

Lamina 3 -45°

stresses	By CLT		ANSYS		% Error	
	UL	LL	UL	LL	UL	LL
σ_x	123521.	-25452.6	123528	25468.6	0.01	0.06
σ_y	156265.	-18380.8	156279	-18402	0.01	0.12
τ_{xy}	-11867	40881.6	-118672	40912.6	0.00	0.08

5.2 for case 2, 0°/90°/0° graphite epoxy laminate

stresses	By CLT		ANSYS		% Error	
	0°	90°	0°	90°	0°	90°
σ_x	3330.376	114.13	3362.08	114.99	0.95	0.69
σ_y	54.21	-11.18	54.43	-10.9	0.41	2.62
τ_{xy}	0	0	0	0	0	0

6. CONCLUSION

The results obtained by CLT agree very closely with ANSYS results. The results are valid for symmetric and unsymmetric laminates.

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