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Free Convective Flow of Immiscible Permeable Fluids in a Vertical Channel with First Order Chemical Reaction

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Abstract — *In this article, the effect of first order chemical reaction on free convective flow of immiscible permeable fluids in a vertical channel is studied. The flow in the porous medium is modeled using Brinkman equation. The channel walls are maintained at two different constant temperatures. Viscous and Darcy dissipation terms are included in the energy equation. The coupled ordinary nonlinear differential equations governing the heat and mass transfer are solved analytically by using perturbation method and numerically by using finite difference method. Separate solutions for the porous medium in both the regions are obtained and these solutions are matched at the interface using suitable matching conditions. The solutions are evaluated numerically and the results are presented graphically for various values of flow governing parameters such as thermal Grashof number, mass Grashof number, porous parameter, viscosity ratio, width ratio and conductivity ratio. In addition, closed form expressions for volumetric flow rate, Nusselt number, species concentration and total energy added to the flow are also derived. It is also found that both analytical and numerical solutions agree very well for small values of perturbation parameter.*

Key Words: Chemical reaction, porous medium, perturbation method, finite difference method.

1. INTRODUCTION

Convection in porous media is applied in utilization of geothermal energy, the control of pollutant spread in groundwater, the design of nuclear reactors, compact heat exchangers, solar power collectors, heat transfer associated with the deep storage of nuclear waste and high performance insulators for buildings. Considerable progress in this area was made by Nield and Bejan [1], Kaviany [2] and Vafai and Tien [3] also analyzed the effects of a solid boundary and the inertial forces on flow and heat transfer in porous media. The coupled fluid flow and heat transfer problem in a fully developed composite region of two parallel plates filled with Brinkman-Darcy porous medium was analytically investigated by Kaviany [4]. Rudraiah and Nagraj [5] studied the fully developed free-convection flow of a viscous fluid through a porous medium bounded by two heated vertical plates. Beckerman [6] studied natural convection in vertical enclosure containing simultaneously fluid and porous layers. Singh et al. [7] analyzed heat and mass transfer phenomena due to natural convection in a composite cavity containing a fluid layer overlying a porous layer saturated with the same fluid, in which the flow in the porous region was modeled using Brinkman-Forchheimer extended Darcy model that includes both the effect of macroscopic shear (Brinkman effect) and flow inertia (Forchheimer effect).

Forced convection in composite channel is a subject of intensive investigation. This is due to the rapid development of technology and numerous modern thermal applications relevant to this area such as cooling of microelectronic devices. Poulikakos and Kazmierczak [8] presented analytical solutions for forced convection flow in ducts where the central part is occupied by clear fluid and the peripheral part is occupied by a Brinkman-Darcy fluid-saturated porous medium. The results of Poulikakos and and Kazmierczak [8] were extended by Kuznetsov [9] to account for the Forchheimer (quadratic drag) effects. Prasad [10] have made an excellent review for composite systems. Alzami and Vafai [11] reviewed different types of interface conditions between a porous medium and fluid layer.

Some novel designs of heat sinks for cooling microelectronic devices utilize highly porous materials such as aluminum foam (Paek et al. [12]). Nield and Kuznetsov [13] considered a forced convection problem in a channel whose center is occupied by a layer of isotropic porous medium (porous layer 1) and whose peripheral part is occupied by another layer of isotropic porous medium (porous layer 2), each of the layers with its own permeability and thermal conductivity. They utilized the Darcy law for the flow in porous layers. Malashetty et al. [14-16] studied two-fluid flow and heat transfer in an inclined channel containing a porous-fluid layer and composite porous medium. Recently, Umavathi et al. [17-23], Umavathi and Manjula [24], Umavathi [25] and Prathap Kumar et al. [26, 27] studied mixed convection in a vertical porous channel.

Combined heat and mass transfer problems with a chemical reaction are of importance in many processes and have received a considerable amount of attention in recent years. Such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occurs simultaneously. Natural convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agriculture drying and in many industrial applications, such as the curing of plastics, cleaning and chemical processing of materials relevant to the manufacture of the printed circuitry and the manufacture of pulp-insulated cables. In many chemical engineering processes, chemical reactions take place between a foreign mass and a working fluid mass which moves due to the stretch of a surface. The order of the chemical reactions depends on several factors. One of the simplest chemical reactions is the first order reaction in which the rate of the reaction is directly proportional to the species concentration. Chamkha [28] studied the analytical solutions for heat and mass transfer by the laminar flow of a Newtonian, viscous, electrically conducting and heat generating /absorbing fluid on a continuously moving vertical permeable surface in the presence of a magnetic field and the first order chemical reaction. Muthucumaraswamy and Ganeshan [29] studied the numerical solution for the transient natural convection flow of an incompressible viscous fluid past an impulsively started semi-infinite isothermal vertical plate with the mass diffusion, taking into account a homogeneous chemical reaction of the first order. The analytical solution of the free convection heat and mass transfer from a vertical plate embedded in a fluid-saturated porous medium with the constant wall temperature and concentration was obtained by Singh and Queeny [30]. The heat and mass transfer characteristics of the natural convection about a vertical surface embedded in a saturated porous medium subjected to a chemical reaction taking into account the Soret and Dufour effects was analyzed by Postelnicu [31]. Prathap Kumar et al. [32-34] studied the effect of homogenous and heterogeneous reaction on the dispersion of a solute for an immiscible fluid. Keeping in view the wide area of practical applications on multi fluid flow and effects of chemical reaction as mentioned, the objective of this study is to investigate the heat and mass transfer of two immiscible permeable fluids between vertical parallel plates.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The geometry under consideration illustrated in figure 1, consists of two infinite parallel plates maintained at equal or constant temperature, taking X axis along the midsection of channel and Y axis perpendicular to walls. The region-I $(0 \le Y \le h_1)$ is filled with a homogeneous isotropic porous material having permeability κ_1, density ρ_1, viscosity μ_1, thermal conductivity K_1, thermal expansion coefficient β_{T1}, concentration expansion coefficient β_{C1} and diffusion coefficient D_1. The region-II $(-h_2 \le Y \le 0)$ is filled with another homogeneous isotropic porous material having permeability κ_2. This region is saturated with different viscous fluid of density ρ_2, viscosity μ_2, thermal conductivity K_2, thermal expansion coefficient β_{T2}, concentration expansion coefficient β_{C2} and diffusion coefficient D_2. The fluids are assumed to have constant property except the density in the buoyancy term in the momentum equation. A fluid rises in the channel driven by buoyancy force. The temperature properties of both the fluids are assumed to be constant. We consider the fluids to be incompressible; flow is steady, laminar and fully developed. It is assumed that the fluid viscosity and Brinkman viscosity (i. e effective viscosity) are same. The flow in both the regions is assumed to be driven by a common constant pressure gradient $\left(dp/dX\right)$ and temperature gradient $\Delta T = \left(T_{w1} - T_{w2}\right)$. It is also assumed that at any given instant, the temperature of the fluid and the temperature of solid are same. The temperature and concentration of boundary at $Y = h_1$ is T_{w1} and C_{w1}, while at $Y = -h_2$ is T_{w2} and C_{w2} respectively.

Under these assumptions, the governing equations of motion, energy and concentration for incompressible fluids yields

Region I:

$$g\beta_{T1}(T_1 - T_{w2}) - \frac{1}{\rho_1}\frac{dp}{dX} + \upsilon_1 \frac{d^2U_1}{dY^2} + g\beta_{C1}(C_1 - \overline{C_2}) - \frac{\upsilon_1 U_1}{\kappa_1} = 0 \quad (1)$$

$$\mathrm{K}_1 \frac{d^2T_1}{dY^2} + \mu_1\left(\frac{dU_1}{dY}\right)^2 + \frac{\mu_1}{\kappa_1}U_1^{\,2} = 0 \quad (2)$$

$$D_1 \frac{dC_1^{\,2}}{dY^2} - K_1 C_1 = 0 \quad (3)$$

Region II:

$$g\beta_{T2}(T_2 - T_{w2}) - \frac{1}{\rho_2}\frac{dp}{dX} + \upsilon_2 \frac{d^2U_2}{dY^2} + g\beta_{C2}(C_2 - \overline{C_2}) - \frac{\upsilon_2}{\kappa_2}U_2 = 0 \quad (4)$$

$$\mathrm{K}_2 \frac{d^2T_2}{dY^2} + \mu_2\left(\frac{dU_2}{dY}\right)^2 + \frac{\mu_2}{\kappa_2}U_2^{\,2} = 0 \quad (5)$$

$$D_2 \frac{dC_2^{\,2}}{dY^2} - K_2 C_2 = 0 \quad (6)$$

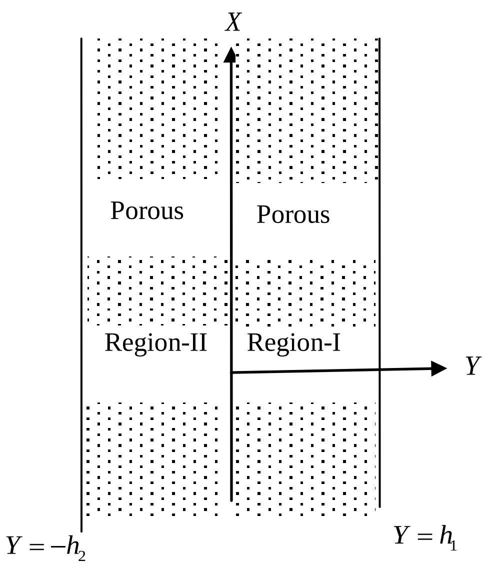

Fig-1: Physical configuration

The boundary conditions on velocity are no-slip conditions and the two boundaries are held at constant different temperatures. In addition, continuity of velocity, shear stress, temperature, heat flux, concentration and mass flux at the interface are assumed.

$$U_1(h_1) = 0,\; U_2(-h_2) = 0,\; U_1(0) = U_2(0),$$

$$\mu_1 \frac{dU_1}{dY}(0) = \mu_2 \frac{dU_2}{dY}(0),\; T_1(h_1) = Tw_1,\; T_2(-h_2) = Tw_2$$

$$T_1(0) = T_2(0),\quad \mathrm{K}_1 \frac{dT_1}{dY}(0) = \mathrm{K}_2 \frac{dT_2}{dY}(0)$$

$$C_1(h_1) = \overline{C_1},\; C_2(-h_2) = \overline{C_2},\; C_1(0) = C_2(0),$$

$$D_1 \frac{dC_1}{dY}(0) = D_2 \frac{dC_2}{dY}(0) \quad (7)$$

The non-dimensional parameters are

$$u_i = \frac{U_i}{\overline{U_1}},\quad y_i = \frac{Y_i}{h_i},\quad \theta_1 = \frac{T_1 - T_{w2}}{T_{w1} - T_{w2}},\quad \theta_2 = \frac{T_2 - T_{w2}}{T_{w1} - T_{w2}},$$

$$\phi_1 = \frac{C_1 - \overline{C_2}}{\overline{C_1} - \overline{C_2}},\quad \phi_2 = \frac{C_2 - \overline{C_2}}{\overline{C_1} - \overline{C_2}},\quad Br = \frac{\overline{U_1}^2 \mu_1}{K_1(T_{w1} - T_{w2})},$$

$$Gr = \frac{g\beta_{T1} h_1^3 (T_{w1} - T_{w2})}{\upsilon_1^{\,2}},\quad Gc = \frac{g\beta_{C1} h_1^3 \left(\overline{C_1} - \overline{C_2}\right)}{\upsilon_1^{\,2}},$$

$$\mathrm{Re} = \frac{\overline{U_1} h_1}{\upsilon_1},\; \sigma_1^{\,2} = \frac{h_1^2}{\kappa_1},\; \sigma_2^{\,2} = \frac{h_2^2}{\kappa_2},\; p = \frac{h_1^2}{\mu_1 \overline{U_1}}\frac{dp}{dX},$$

$$\alpha_1^{\,2} = \frac{\mathrm{K}_1 h_1^2}{D_1},\quad \alpha_2^{\,2} = \frac{\mathrm{K}_2 h_2^2}{D_2} \quad (8)$$

The governing equations (1) to (6) can be written in a dimensionless form by employing the dimensionless quantities (8)

Region I:

$$\frac{d^2u_1}{dy^2} + GR_T\,\theta_1 + GR_C\,\phi_1 - p - \sigma_1^{\,2}\,u_1 = 0 \quad (9)$$

$$\frac{d^2\theta_1}{dy^2} + Br\left[\left(\frac{du_1}{dy}\right)^2 + \sigma_1^{\,2}\,u_1^{\,2}\right] = 0 \quad (10)$$

$$\frac{d^2\phi_1}{dy^2} - \alpha_1^{\,2}\,\phi_1 = 0 \quad (11)$$

Region II:

$$\frac{d^2u_2}{dy^2} + GR_T\, m\, r\, h^2\, b_t\, \theta_2 + GR_C\, m\, r\, h^2\, b_c\, \phi_2 - m\, h^2\, p - \sigma_2^{\,2}\, u_2 = 0 \quad (12)$$

$$\frac{d^2\theta_2}{dy^2} + Br\frac{k}{m}\left[\left(\frac{du_2}{dy}\right)^2 + \sigma_2^{\,2} u_2^{\,2}\right] = 0 \quad (13)$$

$$\frac{d^2\phi_2}{dy^2} - \alpha_2^{\,2}\phi_2 = 0 \quad (14)$$

where,

$$GR_T = \frac{Gr}{\mathrm{Re}},\; GR_C = \frac{Gc}{\mathrm{Re}},\; h = \frac{h_2}{h_1},\; m = \frac{\mu_1}{\mu_2},$$

$$b_t = \frac{\beta_{T2}}{\beta_{T1}},\; \theta_1 = \frac{T_1 - T_{w2}}{T_{w1} - T_{w2}},\; r = \frac{\rho_2}{\rho_1},\; b_c = \frac{\beta_{C2}}{\beta_{C1}},\; d = \frac{\mathrm{D}_2}{\mathrm{D}_1},$$

$$k = \frac{\mathrm{K}_1}{\mathrm{K}_2},\quad \alpha = \frac{\alpha_2}{\alpha_1}$$

The boundary and interface conditions in non-dimensional form become

$$u_1(1)=0\,,\ u_2(-1)=0\,,\ u_1(0)=u_2(0)\,,\ \frac{du_1}{dy}(0)=\frac{1}{mh}\frac{du_2}{dy}(0)$$

$$\theta_1(1)=1\,,\ \theta_2(-1)=0\,,\ \theta_1(0)=\theta_2(0)\,,\ \frac{d\theta_1}{dy}(0)=\frac{1}{kh}\frac{d\theta_2}{dy}(0)$$

$$\phi_1(1)=1\,,\ \phi_2(-1)=0\,,\ \phi_1(0)=\phi_2(0)\,,\ \frac{d\phi_1}{dy}(0)=\frac{d}{h}\frac{d\phi_2}{dy}(0) \quad (15)$$

3. METHOD OF SOLUTIONS

3.1 Perturbation Method

Equations (9) to (14) are coupled and highly non-linear equations because of viscous and Darcy dissipation terms, hence exact solutions cannot be found. The approximate analytical solutions can be found using regular perturbation method. The Brinkman number can be exploited as the perturbation parameter. Therefore the solutions are assumed in the form

$$u_i(y)=u_{i0}(y)+Br\,u_{i1}(y)+Br^2\,u_{i2}(y)+... \quad (16)$$

$$\theta_i(y)=\theta_{i0}(y)+Br\ \theta_{i1}(y)+Br^2\ \theta_{i2}(y)+... \quad (17)$$

Using equations (16) and (17) in equations (9), (10), (12) and (13) and equating the coefficients of like powers of Br to zero and one we determine zeroth and first order equations as follows

Region I:

Zeroth order equations:

$$\frac{d^2\theta_{10}}{dy^2}=0 \quad (18)$$

$$\frac{d^2u_{10}}{dy^2}+GR_T\ \theta_{10}+GR_C\ \phi_1-p-u_{10}\ \sigma_1^{\,2}=0 \quad (19)$$

First order equations:

$$\frac{d^2\theta_{11}}{dy^2}+\left(\frac{du_{10}}{dy}\right)^2+\sigma_1^{\,2}\ u_{10}^{\,2}=0 \quad (20)$$

$$\frac{d^2u_{11}}{dy^2}+GR_T\ \theta_{11}-u_{11}\ \sigma_1^{\,2}=0 \quad (21)$$

Region II:

Zeroth order equations:

$$\frac{d^2\theta_{20}}{dy^2}=0 \quad (22)$$

$$\frac{d^2u_{20}}{dy^2}+GR_T\ m\,r\,h^2\,b_t\ \theta_{20}+GR_C m\,r\,h^2\,b_c\ \phi_2-m\,h^2\,p-\sigma_2^{\,2}\,u_{20}=0 \quad (23)$$

First order equations:

$$\frac{d^2\theta_{21}}{dy^2}+\frac{k}{m}\left[\left(\frac{du_{20}}{dy}\right)^2+\sigma_2^{\,2}\,u_{20}^{\,2}\right]=0 \quad (24)$$

$$\frac{d^2u_{21}}{dy^2}+GR_T\ m\ r\,h^2\,b_t\ \theta_{21}-\sigma_2^{\,2}\,u_{21}=0 \quad (25)$$

The corresponding boundary and interface conditions as given in equation (15) can be written as,

Zeroth order boundary and interface conditions

$$u_{10}(1)=0\,,\ u_{20}(-1)=0\,,\ u_{10}(0)=u_{20}(0)\,,$$

$$\frac{du_{10}}{dy}(0)=\frac{1}{mh}\frac{du_{20}}{dy}(0)$$

$$\theta_{10}(1)=1\,,\ \theta_{20}(-1)=0\,,\ \theta_{10}(0)=\theta_{20}(0)\,,$$

$$\frac{d\theta_{10}}{dy}(0)=\frac{1}{kh}\frac{d\theta_{20}}{dy}(0) \quad (26)$$

First order boundary and interface conditions

$$u_{11}(1)=0\,,\ u_{21}(-1)=0\,,\ u_{11}(0)=u_{21}(0)\,,$$

$$\frac{du_{11}}{dy}(0)=\frac{1}{mh}\frac{du_{21}}{dy}(0)\ ,\ \theta_{11}(1)=0\,,\ \theta_{21}(-1)=0\,,$$

$$\theta_{11}(0)=\theta_{21}(0)\,,\ \frac{d\theta_{11}}{dy}(0)=\frac{1}{kh}\frac{d\theta_{21}}{dy}(0) \quad (27)$$

The solutions for equations (11) and (14) are obtained directly

$$\phi_1=B_1Cosh(\alpha_1 y)+B_2Sinh(\alpha_1 y) \quad (28)$$

$$\phi_2=B_3Cosh(\alpha_2 y)+B_4Sinh(\alpha_2 y) \quad (29)$$

The solutions of zeroth and first order equations (18) to (25) are obtained by using boundary and interface conditions as defined in equations (26) and (27) respectively and are given by

$$\theta_{10}=c_1y+c_2 \quad (30)$$

$$\theta_{20}=c_3y+c_4 \quad (31)$$

$$u_{10}(y)=A_1Cosh(\sigma_1 y)+A_2Sinh(\sigma_1 y)+r_1+r_2y+r_3Cosh(\alpha_1 y)+r_4Sinh(\alpha_1 y) \quad (32)$$

$$u_{20}(y)=A_3Cosh(\sigma_2 y)+A_4Sinh(\sigma_2 y)+r_5+r_6y+r_7Cosh(\alpha_2 y)+r_8Sinh(\alpha_2 y) \quad (33)$$

$$\theta_{11}(y) = E_2 + E_1 y + q_1 y^2 + q_2 y^3 + q_3 y^4 + q_4 Cosh(\alpha_1 y) + q_5 Sinh(\alpha_1 y) + q_6 Cosh(2\alpha_1 y) + q_7 Sinh(2\alpha_1 y) + q_8 Cosh(2\sigma_1 y) + q_9 Sinh(2\sigma_1 y) + q_{10} Cosh(\sigma_1 y) + q_{11} Sinh(\sigma_1 y) + q_{12} y Cosh(\sigma_1 y) + q_{13} y Sinh(\sigma_1 y) + q_{14} y Cosh(\alpha_1 y) + q_{15} Sinh(\alpha_1 y) + q_{16} Cosh(\alpha_1 + \sigma_1) y + q_{17} Cosh(\alpha_1 - \sigma_1) y + q_{18} Sinh(\alpha_1 + \sigma_1) y + q_{19} Sinh(\alpha_1 - \sigma_1) y \quad (34)$$

$$\theta_{21}(y) = E_4 + E_3 y + F_1 y^2 + F_2 y^3 + F_3 y^4 + F_4 Cosh(\alpha_2 y) + F_5 Sinh(\alpha_2 y) + F_6 Cosh(2\alpha_2 y) + F_7 Sinh(2\alpha_2 y) + F_8 Cosh(2\sigma_2 y) + F_9 Sinh(2\sigma_2 y) + F_{10} Cosh(\sigma_2 y) + F_{11} Sinh(\sigma_2 y) + F_{12} y Cosh(\sigma_2 y) + F_{13} y Sinh(\sigma_2 y) + F_{14} y Cosh(\alpha_2 y) + F_{15} Sinh(\alpha_2 y) + F_{16} Cosh(\alpha_2 + \sigma_2) y + F_{17} Cosh(\alpha_2 - \sigma_2) y + F_{18} Sinh(\alpha_2 + \sigma_2) y + F_{19} Sinh(\alpha_2 - \sigma_2) y \quad (35)$$

$$u_{11}(y) = E_5 Cosh(\sigma_1 y) + E_6 Sinh(\sigma_1 y) + H_1 + H_2 y + H_3 y^2 + H_4 y^3 + H_5 y^4 + H_6 Cosh(\alpha_1 y) + H_7 Sinh(\alpha_1 y) + H_8 Cosh(\alpha_1 y) + H_9 Sinh(\alpha_1 y) + H_{10} Cosh(2\sigma_1 y) + H_{11} Sinh(2\sigma_1 y) + H_{12} y Cosh(\sigma_1 y) + H_{13} y Sinh(\sigma_1 y) + H_{14} y \cosh(\alpha_1 y) + H_{15} Sinh(\alpha_1 y) + H_{16} Cosh(\alpha_1 + \sigma_1) y + H_{17} Cosh(\alpha_1 - \sigma_1) y + H_{18} Sinh(\alpha_1 + \sigma_1) y + H_{19} Sinh(\alpha_1 - \sigma_1) y + H_{20} y^2 Cosh(\sigma_1 y) + H_{21} y^2 Sinh(\sigma_1 y) \quad (36)$$

$$u_{21}(y) = E_7 Cosh(\sigma_2 y) + E_8 Sinh(\sigma_2 y) + H_{22} + H_{23} y + H_{24} y^2 + H_{25} y^3 + H_{26} y^4 + H_{27} Cosh(\alpha_2 y) + H_{28} Sinh(\alpha_2 y) + H_{29} Cosh(\alpha_2 y) + H_{30} Sinh(\alpha_2 y) + H_{31} Cosh(2\sigma_2 y) + H_{32} Sinh(2\sigma_2 y) + H_{33} y Cosh(\sigma_2 y) + H_{34} y Sinh(\sigma_2 y) + H_{35} y Cosh(\alpha_2 y) + H_{36} Sinh(\alpha_2 y) + H_{37} Cosh(\alpha_2 + \sigma_2) y + H_{38} Cosh(\alpha_2 - \sigma_2) y + H_{39} Sinh(\alpha_2 + \sigma_2) y + H_{40} Sinh(\alpha_2 - \sigma_2) y + H_{41} y^2 Cosh(\sigma_2 y) + H_{42} y^2 Sinh(\sigma_2 y) \quad (37)$$

Heat Transfer

The wall heat transfer expression in terms of the Nusselt number becomes

$$Nu_+ = (1+h)\frac{d\theta_1}{dy} \quad \text{at} \quad y = 1$$

$$Nu\_ = \left(1 + \frac{1}{h}\right)\frac{d\theta_2}{dy} \quad \text{at} \quad y = -1$$

$$Nu_+ = (1+h)\big(c_1 + Br\big(E_1 + 2q_1 + 3q_2 + 4q_3 + q_4\alpha_1 Sinh(\alpha_1) + q_5\alpha_1 Cosh(\alpha_1) + 2\alpha_1 q_6 Sinh(2\alpha_1) + 2\alpha_1 q_7 Cosh(2\alpha_1) + 2\sigma_1 q_8 Sinh(2\sigma_1) + 2\sigma_1 q_9 Cosh(2\sigma_1) + q_{10}\sigma_1 Sinh(\sigma_1) + q_{11}\sigma_1 Cosh(\sigma_1) + q_{12}\left(\sigma_1 Sinh(\sigma_1) + Cosh(\sigma_1)\right) + q_{13}\left(\sigma_1 Cosh(\sigma_1) + Sinh(\sigma_1)\right) + q_{14}\left(\alpha_1 Sinh(\alpha_1) + Cosh(\alpha_1)\right) + q_{15}\left(\alpha_1 Cosh(\alpha_1) + Sinh(\alpha_1)\right) + q_{16}(\alpha_1+\sigma_1) Sinh(\alpha_1+\sigma_1) + q_{17}(\alpha_1-\sigma_1) Sinh(\alpha_1-\sigma_1) + q_{18}(\alpha_1+\sigma_1) Cosh(\alpha_1+\sigma_1) + q_{19}(\alpha_1-\sigma_1) Cosh(\alpha_1-\sigma_1)\big)\big) \quad (38)$$

$$Nu\_ = (1+1/h)\big(c_3 + Br\big(E_3 - 2F_1 + 3F_2 - 4F_3 - F_4\alpha_2 Sinh(\alpha_2) + F_5\alpha_2 Cosh(\alpha_2) - 2\alpha_2 F_6 Sinh(2\alpha_2) + 2\alpha_2 F_7 Cosh(2\alpha_2) - 2\sigma_2 F_8 Sinh(2\sigma_2) + 2\sigma_2 F_9 Cosh(2\sigma_2) - F_{10}\sigma_2 Sinh(\sigma_2) + F_{11}\sigma_2 Cosh(\sigma_2) + F_{12}\left(\sigma_2 Sinh(\sigma_2) + Cosh(\sigma_2)\right) - F_{13}\left(\sigma_2 Cosh(\sigma_2) + Sinh(\sigma_2)\right) + F_{14}\left(\alpha_2 Sinh(\alpha_2) + Cosh(\alpha_2)\right) - F_{15}\left(\alpha_2 Cosh(\alpha_2) + Sinh(\alpha_2)\right) - F_{16}(\alpha_2+\sigma_2) Sinh(\alpha_2+\sigma_2) - F_{17}(\alpha_2-\sigma_2) Sinh(\alpha_2-\sigma_2) + F_{18}(\alpha_2+\sigma_2) Cosh(\alpha_2+\sigma_2) + F_{19}(\alpha_2-\sigma_2) Cosh(\alpha_2-\sigma_2)\big)\big) \quad (39)$$

The constants appeared in the solutions are not presented as they can be obtained while finding the solutions.

The dimensionless total volume flow rate is given by

$$Qv = Qv_1 + Qv_2 \quad (40)$$

where

$$Qv_1 = \int_0^1 u_1\, dy, \; Qv_2 = \int_{-1}^0 u_2\, dy$$

The dimensionless total heat rate added to the fluid is given by

$$E = E_1 + E_2 \quad (41)$$

where

$$E_1 = \int_0^1 u_1\, \theta_1 dy, E_2 = \int_{-1}^0 u_2\, \theta_2 dy$$

The dimensionless total species rate added to the fluid is given by

$$Cs = Cs_1 + Cs_2 \quad (42)$$

where

$$Cs_1 = \int_0^1 u_1\, \phi_1\, dy, \;\; Cs_2 = \int_{-1}^0 u_2\, \phi_2\, dy$$

Equations (29) to (42) are evaluated for different values of the governing parameters and the results are presented graphically.

3.2 Finite Difference Method

The approximate analytical solutions obtained in the subsection are valid for values of Brinkman number less then one. However in many practical problems especially when viscous dissipation dominates, the Brinkman number takes the values greater than one. In such situations it is required to find the approximate solutions numerically. The governing equations (1) to (6) with the boundary and interface conditions (15) are solved using FDM. In numerical iterations, computation domain is divided into a uniform grid system. The second derivative and the squared – first derivatives terms are discritized with central difference of second order accuracy. By replacing the derivatives with the corresponding finite difference approximation, we obtain a set of n algebraic equations, where n is the number of divisions from -1 to 1.To validate the present numerical method, computed solutions are compared with analytical solutions. The numerical and analytical solutions agree very well in the absence of Brinkman number and as the Brinkman number increases, error between FDM and PM also increases. The solutions obtained by FDM and PM are depicted in Table1 and percentage error between FDM and PM is also evaluated.

4. RESULTS AND DISCUSSION

The problem concerned is with the heat and mass transfer in a vertical channel for composite porous medium in the presence of homogeneous first order chemical reaction. The flow is modeled with Darcy-Lapwood-Brinkman equation. The viscous and Darcy dissipation terms are included in the energy equation. The continuity of velocity, temperature, shear stress, heat flux, concentration and mass flux at the interface is assumed. The equations governing the flow which are highly nonlinear and coupled are solved analytically using perturbation method (PM) and numerically using finite difference method (FDM).The perturbation solutions are valid for small values of Brinkman number and numerical solutions are valid for all values of Brinkman number.

The effect of thermal Grashof number GR_T on the velocity and temperature fields is shown in figures 2a and 2b respectively, in the presence $(\alpha = 1)$ and in the absence $(\alpha = 0)$ of first order chemical reaction. As GR_T increases the flow increases in both the regions. Physically an increase in the value of Grashof number means an increase of the buoyancy force which supports the motion. Further figures 2a and 2b also reveal that the magnitude of velocity and temperature is large in the absence of chemical reaction when compared with values in the presence of the chemical reaction.

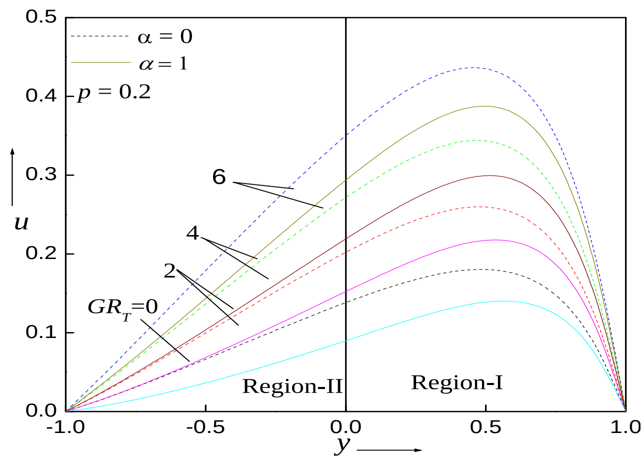

Fig-2a: Velocity profiles for different values of thermal Grashof number GR_T.

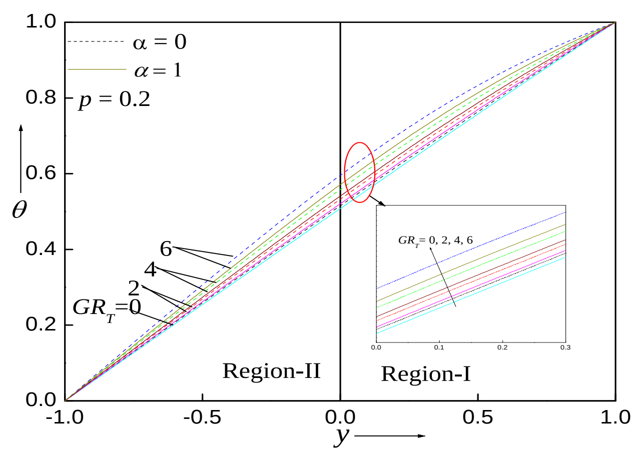

Fig-2b: Tempareture profiles for different values of thermal Grashof number GR_T.

The effect of mass Grashof number GR_C on the velocity and temperature fields shows the similar effect as that of thermal Grashof number, as shown in figures 3a and 3b respectively. That is to say that as GR_C increases, flow increases in both the regions. The mass Grashof number is the ratio of species buoyancy force to the viscous force. As expected, the fluid velocity and temperature increases due

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to the increase in the species buoyancy force. The effects of GR_T and GR_C on the flow were the similar results observed by Shivaiah and Anand Rao [35] for the flow past a vertical porous plate and Malashetty et al. [14-16] in the absence of chemical reaction.

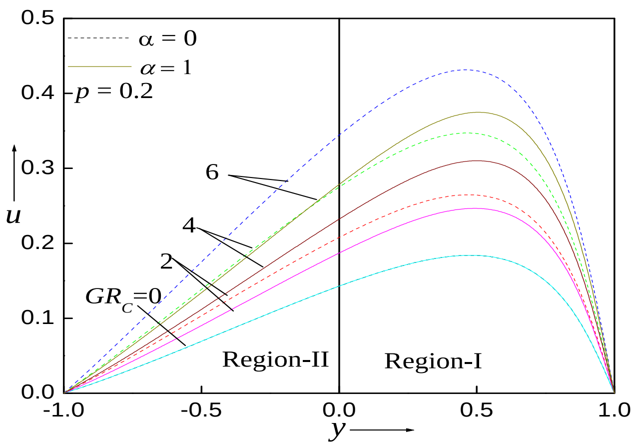

Fig-3a: Velocity profiles for different values of mass Grashof number GR_C .

The variation of velocity and temperature for different values of porous parameter σ $(=\sigma_1=\sigma_2)$ is shown in figures 4a and 4b respectively.

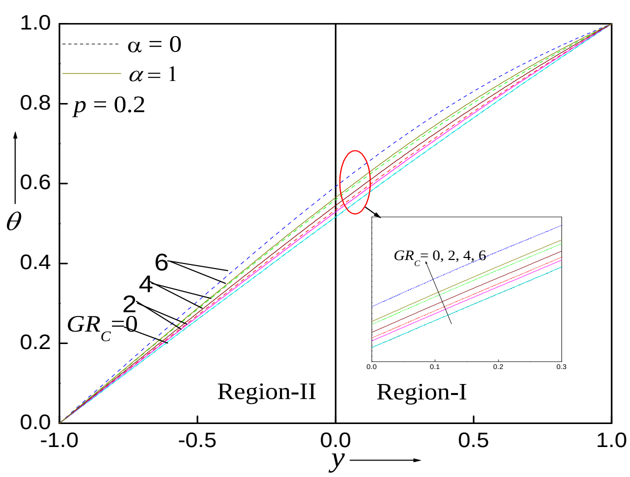

Fig-3b: Temperature profiles for different values of mass Grashof number GR_C .

As the porous parameter increases the velocity, temperature decreases in both the regions. For large values of σ the frictional drag resistance against the flow motion is pronounced and as a result velocity is reduced in both the regions.

The effect of viscosity ratio m (μ_1/μ_2) is to increase the velocity and temperature fields in both the regions as shown in figures 5a and 5b respectively. The viscosity ratio m is defined as the viscosity of the fluid in region-I to the viscosity of the fluid in region-II. It is observed from figure 5b that the effect of viscosity ratio on the temperature field is not very significant.

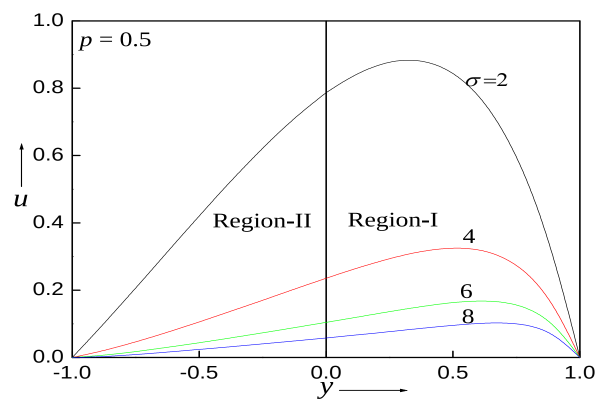

Fig-4a: Velocity profiles for different values of porous parameter σ .

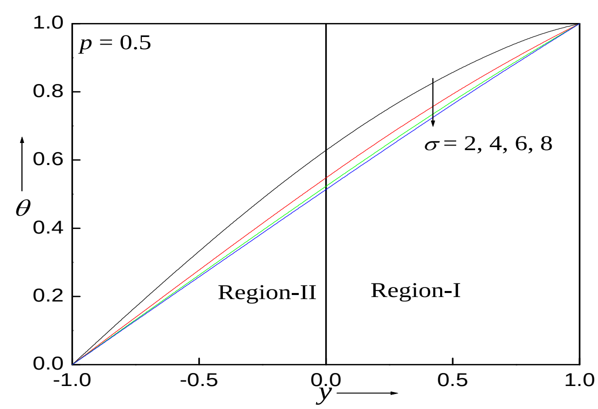

Fig-4b: Temperature profiles for different values of porous parameter σ .

The effect of width ratio h (h_2/h_1) is to enhance velocity and temperature field in both the regions as displayed in figures 6a and 6b respectively. The width ratio h is defined as the ratio of width of the fluid layer in region-II to the width of the fluid in region-I. It is well known that as h increases, velocity increases which intern enhances the dissipation and hence results in enhancement of temperature field also.

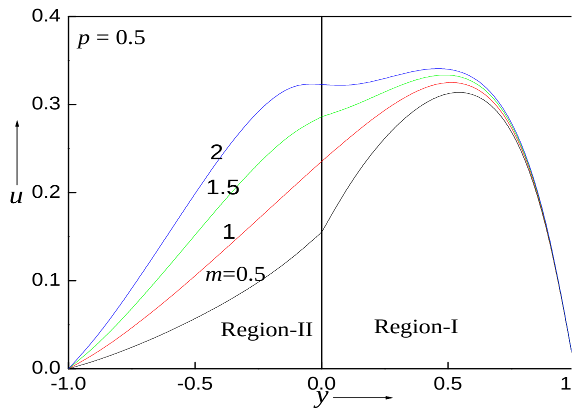

Fig-5a: Velocity profiles for different values of viscosity ratio m.

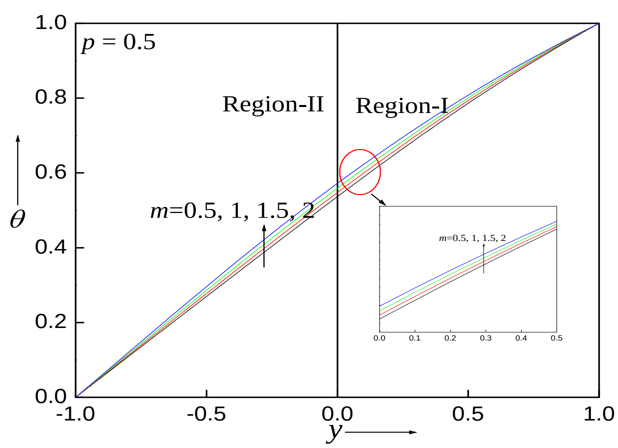

Fig-5b: Temperature profiles for different values of viscosity ratio m.

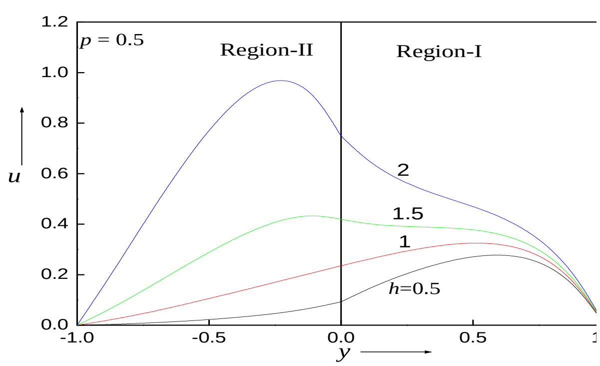

Fig-6a: Velocity profiles for different values of width ratio h.

The effect of conductivity ratio $k\ (K_1/K_2)$ on the flow is similar to the effects on viscosity ratio and width ratio, as seen in figures 7a and 7b. The conductivity of the permeable fluid layer in region-I is large compared to the conductivity of fluid layer in region-II, larger the amount of heat transfer and hence velocity also increases.

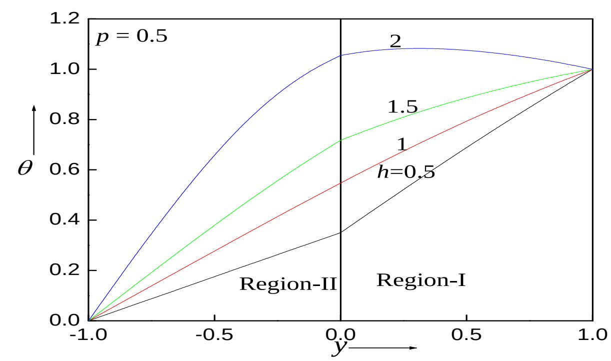

Fig-6b: Temperature profiles for different values of width ratio h.

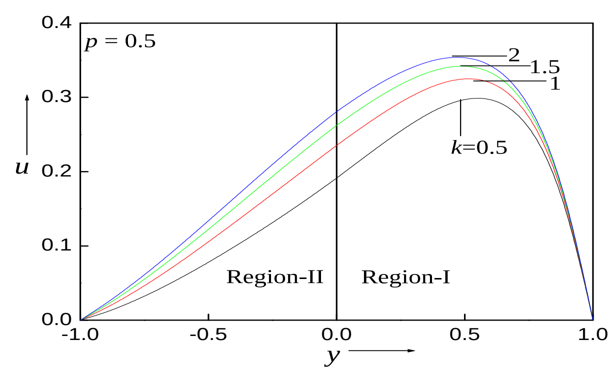

Fig-7a: Velocity profiles for different values of thermal conductivity ratio k.

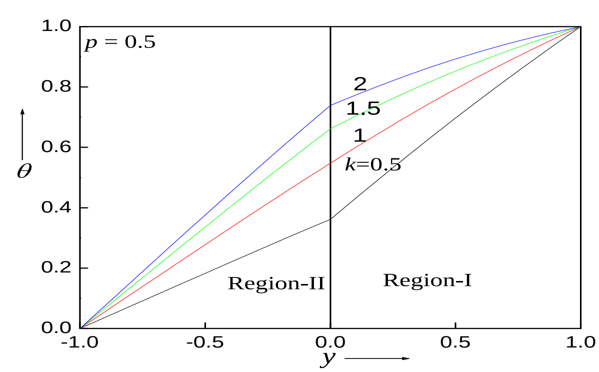

Fig-7b: Temperature profiles for different values of thermal conductivity ratio k.

The effects of h and k in the presence of first order chemical reaction was the similar results observed by Malashetty [14] in the absence of first order chemical reaction.

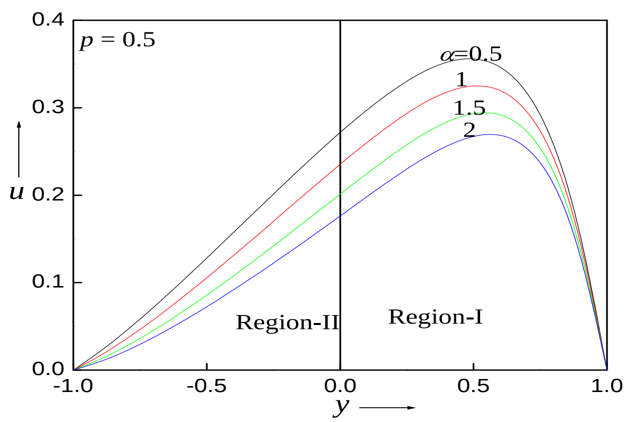

Fig-8a: Velocity profiles for different values chemical reaction parameter α .

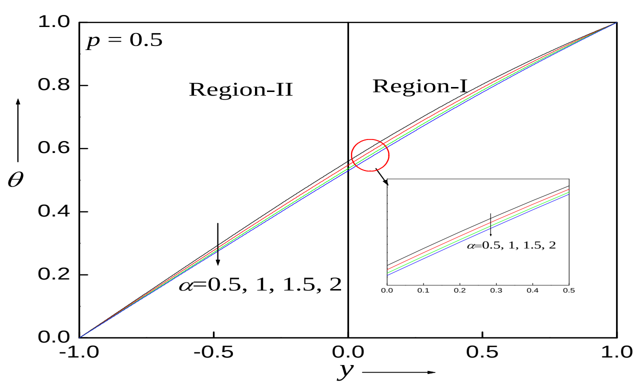

Fig-8b: Temperature profiles for different values chemical reaction parameter α .

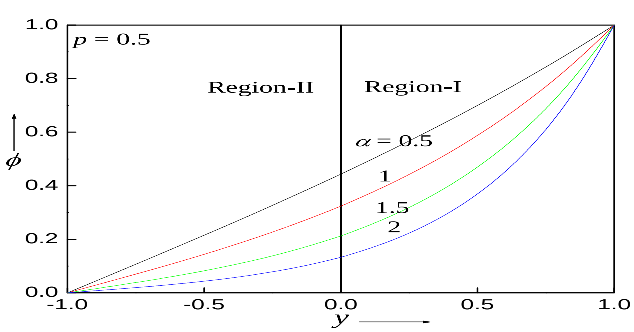

Fig-8c: Concentration profiles for different values chemical reaction parameter α .

The effect of first order chemical reaction parameter α on velocity, temperature and concentration fields is depicted in figures 8a, 8b and 8c respectively. It is evident from these figures that as α increases the velocity, temperature and concentration are reduced in both the regions. Physically an increase in the values of α increases in number of solute molecules that undergoing chemical reaction resulting in decrease in the fluid flow.

This was the similar results observed by Damesh and Shannak [36] for viscoelastic fluid and Krishnendu Bhattacharyya [37] for viscous fluid.

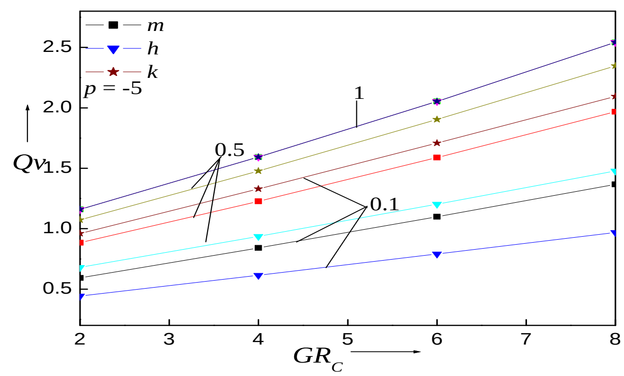

Fig-9: Effect of mass Grashof number, viscosity ratio, width ratio and conductivity ratio on the volume flow rate.

Further, one can also come to the conclusion from figures 9, 10 and 11 that, as m, h and k increases the total volumetric flow rate, species concentration and heat rate also increases. The values of total volumetric flow rate, species concentration and heat rate remains the same when $m = h = k = 1$.

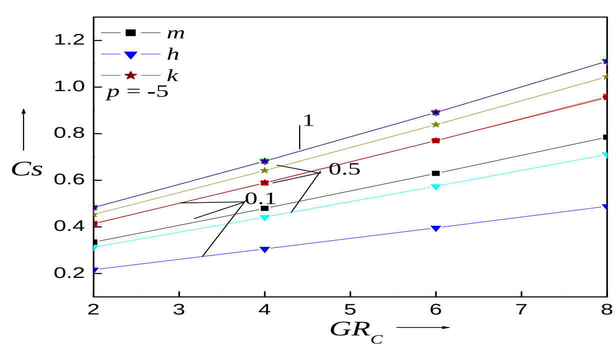

Fig-10: Effect of mass Grashof number, viscosity ratio, width ratio and conductivity ratio on total species rate added to the fluid.

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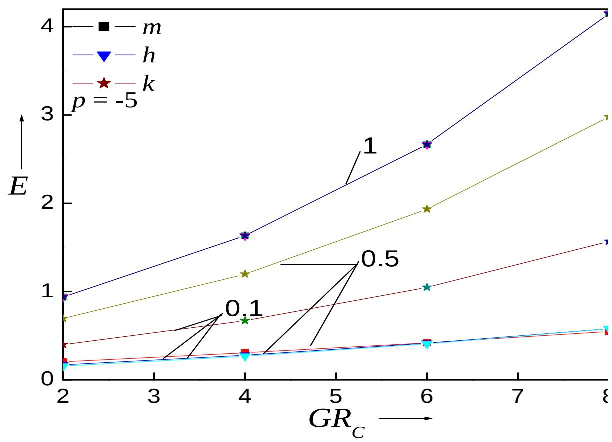

Fig-11: Effect of mass Grashof number, viscosity ratio, width ratio and conductivity ratio on total heat rate added to the fluid.

This is the valid result because considering all the ratios to be equal to one implies the channel is filled with same porous fluids in both the regions. However, variation of m, h and k for values not equal to one shows the different profiles for total volumetric flow rate, species concentration and heat rate. In all the three graphs, the magnitude of volumetric flow rate, species concentration and heat rate is large for k, when compared with m and h. The magnitude of volumetric flow rate, species concentration and heat rate is optimal for m when compared with h.

The Nusselt number at the cold (Nu_-) and hot walls (Nu_+) is shown in figure 12 for variations of mass Grashof number GR_C. It is seen that as GR_C increases Nu_- and Nu_+ increases in magnitude.

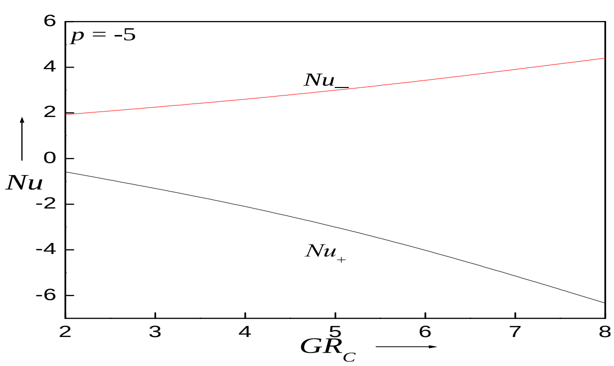

Fig-12: Effect of mass Grashof number on the Nusselt number.

The effect of Brinkman number on the velocity and temperature field is shown in table 1. It is seen that, as the Brinkman number increases the velocity and temperature increase in both the regions. An increase in Brinkman number results in increase of dissipation effects which result in an increase of temperature and as a consequence velocity increase for the increase in buoyancy force in the momentum equation. This table also shows a comparison of numerical and analytical solutions. It is seen that analytical and numerical solutions are exact to the order of 10^{-4} in the absence of Brinkman number and the difference increases as the Brinkman number increases. Further the percentage of error is also calculated and shown in table1.

5. CONCLUSIONS

The problem of heat and mass transfer in a vertical channel filled with porous immiscible fluids was analyzed analytically by using regular perturbation method and numerically by finite difference method. The following conclusions are drawn

1. The effect of thermal Grashof number and mass Grashof number was to enhance the velocity and temperature fields.
2. The effect of porous parameter σ is to suppress the flow in both regions.
3. The larger the values of viscosity ratio, width ratio, conductivity ratio, the larger the flow field.
4. The flow field was found to be less in the presence of first order chemical reaction parameter when compared in the absence of chemical reaction parameter. Further as the chemical reaction rate parameter increases heat and mass transfer decreases.
5. The volumetric flow rate, species concentration and heat rate added to the flow was to increase for increasing values of mass Grashof number, viscosity ratio, width ratio and conductivity ratio.
6. Nusselt number at the hot and cold wall increases in magnitude for increasing values of mass Grashof number.
7. The percentage of error between analytical and numerical solutions increases as the Brinkman number increases.

Table-1: Velocity and temperature values for different values of Brinkman number with $GR_C=1, \sigma_1=\sigma_2=4$, $p=-1, GR_T=1$.

Velocity									
	$Br=0$			$Br=0.5$			$Br=1.5$		
y	FDT	PM	%Error	FDT	PM	%Error	FDT	PM	%Error
-1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.6	0.0696	0.0696	0.0000	0.071	0.0710	0.0020	0.0743	0.0737	0.0610
-0.2	0.0997	0.0997	0.0000	0.1021	0.1020	0.0110	0.1075	0.1066	0.0940
0	0.1107	0.1107	0.0000	0.1133	0.1132	0.0120	0.1192	0.1181	0.1060
0.2	0.1194	0.1194	0.0000	0.122	0.1219	0.0130	0.1278	0.1268	0.1030
0.6	0.1174	0.1174	0.0000	0.1191	0.1190	0.0090	0.1229	0.1223	0.0620
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Temperature									
-1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.6	0.2000	0.2000	0.0000	0.2254	0.2243	0.1060	0.2835	0.2730	1.0490
-0.2	0.4000	0.4000	0.0000	0.4432	0.4414	0.1790	0.5419	0.5242	1.7670
0	0.5000	0.5000	0.0000	0.5473	0.5454	0.1860	0.6554	0.6363	1.9070
0.2	0.6000	0.6000	0.0000	0.6474	0.6455	0.1890	0.7552	0.7365	1.8660
0.6	0.8000	0.8000	0.0000	0.8329	0.8317	0.1210	0.9072	0.8951	1.2120
1	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	1.0000	0.0000

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