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"The effect of exponentially thickness variation and thermal effect on vibration of visco-elastic non-homogeneous rectangular plate having clamped boundary condition"

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Abstract - *Vibration reduction is a major challenge better designing to these fields; especially in a nuclear power project, aerospace applications, such a reduction must be achieved with a minimal increase in weight. We have assumed Kelvin model for visco-elasticity that is the combination of the elastic and viscous elements in parallel. Frequency equation is derived by using Galerkin's technique with a two-term deflection function. A plate structure is of key interest to space craft, telephone industry, nuclear power projects, civil and mechanical engineers. The vibration of plates is a common case of the more general problem of mechanical vibrations. In this paper to analysis the effect of exponentially thickness variation and thermal effect on vibration of visco-elastic non-homogeneous rectangular plate having clamped boundary condition on two parallel edges and simply supported boundary condition on remaining two edges. Deflection and Logarithmic decrement at different points for the first two modes of vibration are calculated for various values of non-homogeneity constants, thermal gradients, aspect ratio and taper constants.*

Key Words*: *thermal gradient, visco-elastic, exponentially thickness, rectangular plate, taper constant, aspect ratio.

1. INTRODUCTION

During heating up periods the structure is exposed to high intensity, high fluxes and the material properties thus undergo significant changes, in particular the thermal effect on the modules of elasticity of the material cannot be taken as negligible. In the advance time the effect of thermal and variable thickness on vibration of non-homogenous visco-elastic plates are widely used in better designing in civil, aeronautical, mechanical, electronics and marine structures like jet engines designing, space crafts, gas turbines, nuclear power projects etc.

Fnconneau and Marangoni [1] have studied on the effect of thermal gradient on the natural frequencies of a rectangular plate. Tomar and Gupta [2] discussed on thermal effect on frequencies of an orthotropic rectangular plate of linearly varying thickness. Rao and Satyanarayana [3] analyzed on the effect of thermal gradient of frequencies of tapered rectangular plates. Tomar and Gupta [4] have studied on thermal effect on axisymmetric vibrations of an orthotropic circular plate of parabolically varying thickness.

Tomar and Gupta [5] have discussed on vibration of an orthotropic elliptic plate of non-uniform thickness and temperature. Several authors have studied the thermal effect on vibration of homogenous plates of variable thickness but none of the authors has so far considered thermal effect on vibration of non-homogenous rectangular plates of exponentially varying thickness. Bambill et al [6] discussed on the transverse vibration of an orthotropic rectangular plate of linearly varying thickness and with a free hole edge. Recently, Li and Zhou [7] have studied on nonlinear vibration and thermal bucking of heated orthotropic circular plate by shooting method.

It is well known [8] that in the presence of a thermal gradient, the elastic coefficient of homogenous materials become functions of the space variables. Pronsato et al [9] have discussed on transverse vibration of rectangular membrane with discontinuously varying density. A simple model presented here is to study the vibration of non-homogenous visco-elastic rectangular plate of exponentially varying thickness subjected to thermal gradient has been discussed in the present paper. Deflection and logarithmic decrement at different points for the first two mode of vibration are calculated for various values of thermal gradients, non-homogeneity parameter, taper constant and aspect ratio for non-homogenous visco-elastic rectangular plate with exponentially varying thickness and boundary condition of plate is clamped on two parallel edges and simply supported on remaining two edges.

1.1 ANALYSIS AND EQUATION OF MOTION

The governing differential equation of transverse motion of a visco-elastic non-homogeneous plate of variable thickness in Cartesian co-ordinates, as by Gupta and Kumar [11],

$$\widetilde{D}[D_1(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}) + 2\frac{\partial D_1}{\partial x}(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}) +$$
$$2\frac{\partial D_1}{\partial v}(\frac{\partial^3 w}{\partial v^3} + \frac{\partial^3 w}{\partial x^2 \partial v})$$

$$+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)+2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial v}\frac{\partial^2 w}{\partial x\partial y}]+\rho h\frac{\partial^2 w}{\partial t^2}=0 \quad (1)$$

The solution of equation (1) can be sought in the form of product of two functions as follows:

$$w=w(x,y,t)=W(x,y)T(t) \quad (2)$$

where W(x,y) is the deflection function and T(t) is time function.

Using equation (2) in (1) after simplification and taking both sides of are equal to a constant p^2, we have

$$D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2}\right)+2\frac{\partial D_1}{\partial v}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\partial v}\right)$$

$$+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2}\right)+2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial v}\frac{\partial^2 W}{\partial x\partial v}-\rho h p^2 W=0 \quad (3)$$

and

$$\frac{d^2 T}{dt^2}+p^2\tilde{D}T=0 \quad (4)$$

Equation (3) is differential equation of transverse motion and equation (4) is a differential equation of time function of free vibration of non-homogeneous visco-elastic plate of with varying exponentially thickness.

It is assumed that the non-homogeneous visco-elastic rectangular plate is subjected to a steady one-dimensional temperature distribution along x-direction.

$$\tau=\tau_0\left(1-\frac{x}{a}\right) \quad (5)$$

where τ denotes the temperature excess above the reference temperature at any point at distance $\frac{x}{a}$ and τ_0 denotes the temperature excess above reference temperature at the end i.e. x=a.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this form,

$$E=E_0(1-\gamma\tau) \quad (6)$$

where E_0 is the value of the Young's modulus at some reference temperature i.e. $\tau=0$ and γ is the slope of the variation of E with τ. The modulus variations, in view of expression (5) and (6) become

$$E(x)=E_0\left(1-\alpha\left(1-\frac{x}{a}\right)\right) \quad (7)$$

where $\alpha=\gamma\tau_0$ $(0\le\alpha<1)$, a parameter known as temperature gradient.

It is assumed that exponentially thickness variation and non-homogeneity varies in the x-direction only, consequently, the thickness, density and flexural rigidity of the plate become functions of x only.

Let the two opposite edges y=0 and y=b of the plate be simply supported. So that the plate when undergoing free transverse vibrations with frequency p may have levy-type solution.[11]

$$W(x,y)=W_1(x)sin\left(\frac{\pi y}{b}\right) \quad (8)$$

Substitution of equation (8) in (3)

$$D_1\left[\frac{\partial^4 W_1}{\partial x^4}-2\left(\frac{\pi}{b}\right)^2\frac{\partial^2 W_1}{\partial x^2}+\left(\frac{\pi}{b}\right)^4 W_1\right]+2\frac{\partial D_1}{\partial x}\left[\frac{\partial^3 W_1}{\partial x^3}-\left(\frac{\pi}{b}\right)^2\frac{\partial W_1}{\partial x}\right]$$

$$+\frac{\partial^2 D_1}{\partial x^2}\left[\frac{\partial^2 W_1}{\partial x^2}-\nu\left(\frac{\pi}{b}\right)^2 W_1\right]-\rho p^2 h W_1=0 \quad (9)$$

Thus (9) reduces to a form independent of y and upon introducing the non-dimensional variables

$$\bar{H}=\frac{h}{a},\quad \bar{\rho}=\frac{\rho}{a},\quad \bar{D}=\frac{D_1}{a^3},\quad \bar{W}=\frac{W_1}{a},\quad X=\frac{x}{a} \quad (10)$$

It becomes, in non-dimensional form,

$$\bar{D}\left(\frac{\partial^4\bar{W}}{\partial x^4}-2r^2\frac{\partial^2\bar{W}}{\partial x^2}+r^4\bar{W}\right)+2\frac{\partial\bar{D}}{\partial x}\left(\frac{\partial^3\bar{W}}{\partial x^3}-r^2\frac{\partial\bar{W}}{\partial x}\right)+\frac{\partial^2\bar{D}}{\partial x^2}\left(\frac{\partial^2\bar{W}}{\partial x^2}-\nu r^2\bar{W}\right)-a^3\bar{\rho}p^2\bar{H}\bar{W}=0 \quad (11)$$

where,

$$r=\frac{\pi a}{b} \quad (12)$$

In view of the previous assumption, the thickness varies exponentially in the x-direction and non-homogeneity varies linearly in the x-direction only, one assumes,

$$\bar{H}(X)=H_0\left(e^{\beta X}\right) \quad (13)$$

where β is the taper constant and $H_0=\bar{H}|_{x=0}$

and

$$\bar{\rho}=\rho_0(1-\alpha_3 X) \quad (14)$$

where is the non-homogeneity constant and $\rho_0=\bar{\rho}|_{x=0}$

The rigidity given by equation

$$\bar{D}=\frac{E_0 H_0^3\left(e^{\beta X}\right)^3[1-\alpha(1-X)]}{12(1-\nu^2)} \quad (15)$$

Using equation (13), (14) and (15) in equation (11), one obtains

$$\Phi_1\left(\frac{\partial^4\bar{W}}{\partial X^4}-2r^2\frac{\partial^2\bar{W}}{\partial X^2}+r^4\bar{W}\right)+\Phi_2\left(\frac{\partial^3\bar{W}}{\partial X^3}-r^2\frac{\partial\bar{W}}{\partial X}\right)+\Phi_3\left(\frac{\partial^2\bar{W}}{\partial X^2}-\nu r^2\bar{W}\right)$$

$$-p^2(1-\alpha_3 X)\ell\bar{W}=0 \quad (16)$$

Here,

$$\Phi_1 = (1-\alpha+\alpha X)(1-\beta X)^2,$$
$$\Phi_2 = 2(1-\beta X)(3\alpha\beta - 3\beta + \alpha - 4\alpha\beta X)$$
$$\Phi_3 = 6\beta(\beta - \alpha - \alpha\beta + 2\alpha\beta X), \quad \ell = \frac{12(1-\nu^2)\rho_0 a^3}{E_0 H_0^2}$$

and p^2 is a frequency parameter.

The deflection function $\bar{W}(X)$, of plate is assumed to be a finite sum of characteristic functions $\bar{W}_k(X)$

$$\bar{W}(X) = \sum_{k=1}^{n} C_k \bar{W}_k(X) \qquad (17)$$

where C_k's are undetermined coefficients and $\bar{W}_k$ are characteristic function chosen to satisfy the boundary conditions of plate.

For a rectangular plate clamped at both the edges X=0 and X=1 (and simply supported at the remaining two edges)

$$\bar{W}|_{X=0} = \left.\frac{\partial \bar{W}}{\partial X}\right|_{X=0} = 0, \qquad \bar{W}|_{X=1} = \left.\frac{\partial \bar{W}}{\partial X}\right|_{X=1} = 0 \qquad (18)$$

Using Galerkin's technique, one has

$$\int L[\bar{W}(X)]\bar{W}(X)dX = 0 \qquad (19)$$

where $L[\bar{W}(X)]$ is left hand side of equation (16). Taking the first two terms of the sum (17) for the function $\bar{W}(X)$ as a solution of equation (16),

$$\bar{W}(X) = C_1 X^2(1-X)^2 + C_2 X^3(1-X)^3 \qquad (20)$$

Substituting equation (20) into equation (19) and then eliminating C_1 and C_2 given the frequency equation as,

$$\begin{vmatrix} 2(A_1+B_1p^2) & (A_2+B_2p^2) \\ (A_2+B_2p^2) & 2(A_3+B_3p^2) \end{vmatrix} = 0 \qquad (21)$$

The frequency equation (21) is a quadratic equation in p^2 from which the two values of p^2 can be found.

Choosing C_1=1, than $C_2 = -\frac{A_4}{A_5}$

where $A_4=2(A_1+p^2B_1)$, $A_5=2(A_2+p^2B_2)$

Therefore

$$\bar{W} = X^2(1-X)^2 - \frac{A_4}{A_5}X^3(1-X)^3 \qquad (22)$$

1.2 TIME FUNCTIONS OF VIBRATIONS OF NON-HOMOGENEOUS VISCO-ELASTIC PLATES

Time function of free vibrations of visco-elastic plates is defined by the general ordinary differential equation (4). Their form depends on the visco-elastic operator $\tilde{D}$. For Kelvin's model, one has

$$\tilde{D} \equiv \left(1 + \frac{\eta}{G}\frac{d}{dt}\right) \qquad (24)$$

Taking temperature dependence of shear modulus G and visco-elastic constants η in the same form as that of Young's modulus,

$$G(\tau) = G_0(1-\gamma_1\tau), \qquad \eta(\tau) = \eta_0(1-\gamma_2\tau) \qquad (25)$$

where G_0 is shear modulus and η_0 is visco-elastic constant at some reference temperature i.e. at $\tau = 0, \gamma_1$ and γ_2 are slope variation of τ with G and η respectively. Using equation (5) in (25),

$$G(X) = G_0[1-\alpha_1(1-X)], \qquad \eta(X) = \eta_0[1-\alpha_2(1-X)] \qquad (26)$$

where

$$\alpha_1 = \gamma_1\tau_0, \quad 0 \le \alpha_1 < 1, \text{ and}$$
$$\alpha_2 = \gamma_2\tau_0, \quad 0 \le \alpha_2 < 1$$

Using equation (26) in (24), we get

$$\tilde{D} \equiv \left(1 + q\frac{d}{dt}\right) \qquad (27)$$

where $q = \frac{\eta_0[1-\alpha_2(1-X)]}{G_0[1-\alpha_1(1-X)]}$ (28)

Using equation (27) in equation (4), one obtains

$$\frac{d^2T}{dt^2} + p^2 q\frac{dT}{dt} + p^2 T = 0 \qquad (29)$$

and its solution comes out as

$$T(t) = e^{-\frac{p^2qt}{2}}(e_1 \cos st + e_2 \sin st) \qquad (30)$$

where $s^2 = p^2 - \frac{1}{4}p^4 q^2$, and e_1 and e_2 are constants of integration.

Let us assume that the initial conditions are $T = 1$ and $\frac{dT}{dt} = 0$ at $t = 0$, equation (30) become

$$T(t) = e^{-\frac{p^2qt}{2}}\left(\cos st + \frac{p^2q}{2s}\sin st\right) \qquad (31)$$

Thus deflection w(x,y,t) may be expressed from equation (2), (8), (22) and (31),

$$w(x,y,t) = \bar{W}(X)e^{-\frac{p^2qt}{2}}\left(\cos st + \frac{p^2q}{2s}\sin st\right)\sin\frac{\pi y}{b} \qquad (32)$$

where p is frequency given by equation (21).

Logarithmic decrement of the vibration is given by

$$\Lambda = \log_e \frac{w_2}{w_1} \qquad (33)$$

where w_1 is the deflection at any point of the plate at a time period $K=K_1$ and w_2 is the deflection at the same point at the time period succeeding K_1.

2. RESULT AND DISCUSSION

Deflection, logarithmic decrement corresponding to the first two modes of vibration at different points of C-S-C-S non-homogeneous visco-elastic rectangular plate with exponentially varying thickness have been computed for different combinations of non-homogeneity parameter, taper constant, aspect ratio and thermal constants.

Results are presented in Tabular form (1-17). For numerical computation, following materials parameters are used [10]:

$$E_0 = 7.08 \times 10^{10}\ N/M^2,$$
$$G_0 = 2.682 \times 10^{10}\ N/M^2,$$
$$\eta_0 = 1.4612 \times 10^{6}\ N.S/M^2$$
$$\rho_0 = 2.80 \times 10^{3}\ Kg/M^3, \nu = 0.345,$$ H_0=0.01 M

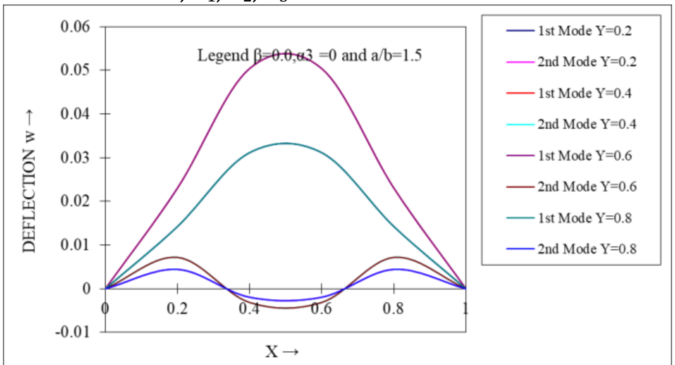
Graph (1-6) show that the deflection (w) for fixed aspect ratio ($\frac{a}{b}$=1.5) starts from zero to increase then decrease to zero for first mode of vibration but for the second mode of vibration value starts zero to increase then decease then increase and finally become to zero for fixed Y and increasing value of X for time 0.K and 5.K respectively.

shows that the deflection (w) of first two modes of vibration decease with an increase in aspect ratio ($\frac{a}{b}$).shows that the deflection (w) of first two modes of vibration decease with an increase in non-homogeneity parameter (α_3).

logarithmic decrement (Λ) of first two modes of vibration increases with an increase in the thermal constant (α). show that logarithmic decrement (Λ) of first two modes of vibration decreases with an increase in the taper constant(β). show that logarithmic decrement (Λ) of first two modes of vibration deceases with an increase in X and in the aspect ratio ($\frac{a}{b}$) respectively.

Graph- 1

Deflection w for different X, Y and β=0.0 and a/b=1.5 for all α, α_1, α_2, α_3 =0.0 at Initial time 0.K

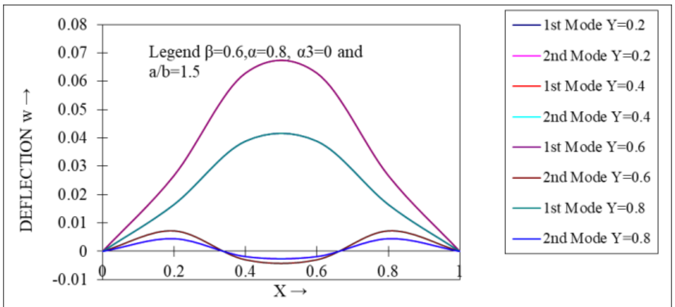


Deflection w for different X, Y and β=0.0 and a/b=1.5 for all α, α_1, α_2, α_3 =0.0 at Initial time 0.K

Fig 1

Graph 2

Deflection w for different X, Y and α =0.8, β=0.6 and a/b=1.5 for all α_1, α_2, α_3 =0.0 at Initial time 0.K

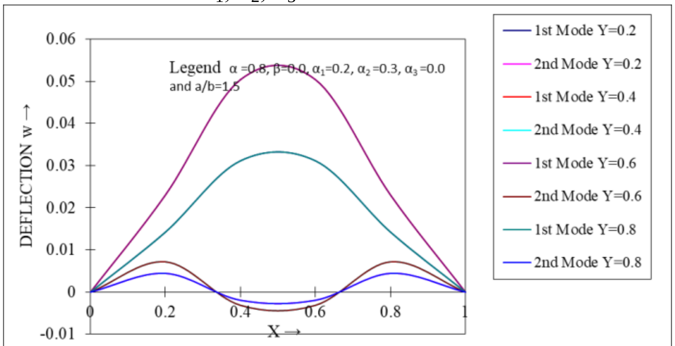


Deflection w for different X, Y and α =0.8, β=0.6 and a/b=1.5 for all α_1, α_2, α_3 =0.0 at Initial time 0.K

Fig 2

Graph 3

Deflection w for different X, Y and α =0.8, β=0.6 and a/b=1.5 for all α_1, α_2, α_3 =0.0 at Initial time 0.K

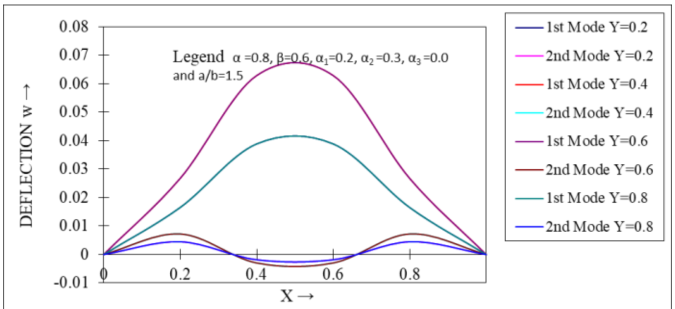


Deflection w for different X, Y and α =0.8, β=0.0, α_1=0.2, α_2 =0.3, α_3 =0.0 and a/b=1.5 at time 5.K

Fig 3

Graph 4

Deflection w for different X, Y and α =0.8, β=0.6, α_1=0.2, α_2 =0.3, α_3 =0.0 and a/b=1.5 at time 5.K

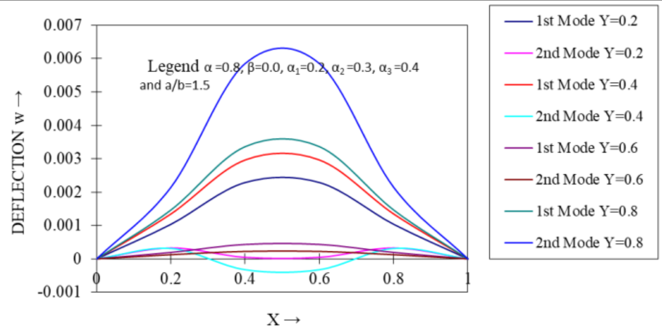


Deflection w for different X, Y and α =0.8, β=0.6, α_1=0.2, α_2 =0.3, α_3 =0.0 and a/b=1.5 at time 5.K

Fig 4

Graph 5

Deflection w for different X, Y and α =0.8, β=0.0, α_1=0.2, α_2 =0.3, α_3 =0.4 and a/b=1.5 at time 5.K

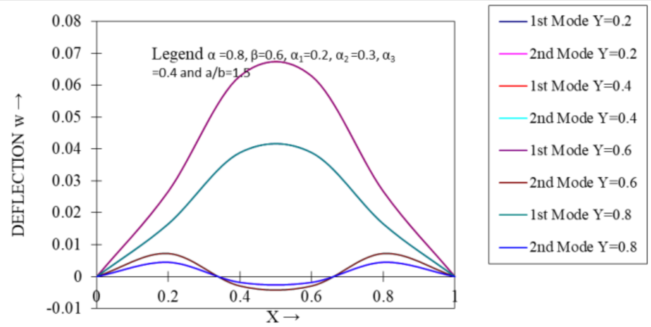


Deflection w for different X, Y and α =0.8, β=0.0, α_1=0.2, α_2 =0.3, α_3 =0.4 and a/b=1.5 at time 5.K

Fig 5

Graph 6

Deflection w for different X, Y and α =0.8, β=0.6, α_1=0.2, α_2 =0.3, α_3 =0.4 and a/b=1.5 at time 5.K



Deflection w for different X, Y and α =0.8, β=0.6, α_1=0.2, α_2 =0.3, α_3 =0.4 and a/b=1.5 at time 5.K

Fig 6

3. CONCLUSIONS

In this study, the dynamic analysis on thermal effect on deflection of non-homogeneous rectangular plates with exponentially varying thickness is analyzed. The differential equations have been solved using Galerkin method of separation and variable separation method. The obtained analytical solutions were used to examine the effects of taper, thermal, non-homogeneity parameter and aspect ratio.

From the parametric studies, the following observations were established:

1. Increase in aspect ratio decreases deflection of plate. 2. Increase in non-homogeneity decreases deflection of plate. 3. Increasing the taper increases deflection. 4. Increase in aspect ratio decreases logarithmic decrement. 5. Increase in non-homogeneity decreases logarithmic decrement.
6. Increasing the thermal increases logarithmic decrement.
7.Increase in taper decreases logarithmic decrement. Therefore, engineers can see and made the plates in that manner so these results can fulfill those requirements.

ABBREVIATIONS

x,y-coordinate in the plane of plate,

E-Young's modulus,
G-shear modulus,
ν-Poisson's ratio,
ρ-mass density per unit length of plate material,
D_1-flexural rigidity,
h-thickness of plate,
Ď-visco elastic operator,
t-time,
η-visco elastic constant,
w(x,y,t) - transverse deflection of plate at point,
a,b - length and breadth of the plate,
α, α_1, α_2-temperature constants,
β-taper constant,
α_3-non-homogeneity constant,
K- time period

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