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On Extending Goldbach's Conjecture: The Patterns of Isolated and Twin Primes into The Even Sums

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Abstract - *Goldbach's conjecture states that every even integer (greater or equal to four) can be expressed as the sum of two primes. Mathematicians have tried to prove it analytically, from the centuries. Unfortunately, all such attempts couldn't prove it mathematically. We have utilized the patterns of even sums to extend the actual conjecture. In this study, we've extended the conjecture by imparting two hypotheses. Our objective relies upon the necessity of 'unveiling the isolated and twin prime pattern' within even integers. In this article, we have illustrated two hypotheses, based on our intriguing observations. The former insists that, in the sum of two primes that represent an even integer (greater or equal to six), at least one of these primes must belong to a twin prime pair. Due to further investigation, we have presented the latter, which strengthened the former. It says that there exists a subset of the set {E= 12m: m is positive integer}, where each member can be expressible simultaneously as the sum of two primes (both are part of a twin prime), the sum of two primes (both are part of the same twin prime pair), the sum of two primes (one twin prime, and isolated prime), and the sum of two primes (both are isolated primes). To support our claim, we've performed extensive computational verification using Python software. The former is verified up to 250000 and the latter confirms its validity up to 5000. We expect our future works on their larger verification will make a new avenue in the course of number theory.*

Key Words: Prime numbers, Twin prime pairs, Isolated primes, Goldbach Conjecture

1. INTRODUCTION

The history of number theory is a rich tapestry woven across millennia, showcasing humanity's enduring fascination with the properties and patterns of numbers. The journey begins with ancient civilizations like Babylonia and Egypt, where mathematical insights were first recorded [10]. The Goldbach conjecture is one of the intriguing conjectures in "Number Theory". It states that every even number (greater or equal to four) can be represented as the sum of two prime numbers [9]. The claim is satisfied up to a very massive range. Mathematicians are still trying to prove it for countless even integers [8]. Proving the converse is an easy task and could be done through elementary learning. Unfortunately, till now, no such analytical proof exists to claim its validity for all the even integers. In this paper, we have concentrated on the patterns of twins and isolated primes into even sums. Here the presented hypothesis connects the even integers with twin prime pairs [7]. Apart from all of them, we are going to explore the patterns of isolated primes together with twin primes in a special case of even integers. This rudiment and ideas regarding primes would play a pivotal role in the advancement of Goldbach's conjecture.

2. Essential preliminaries:

For general readers, we discuss here some basic elementary concepts, which are relevant to our work.

2.1 Primes:

A prime number cannot be represented by a product of two smaller natural numbers than itself. Those positive integers are divisible by a unit and itself only, known as 'prime'. There are infinitely many primes, as established by Euclid around 300 BC. No known simple formula separates prime numbers from composite numbers except their basic property

2.2 Twin prime pairs:

The prime numbers are defined to be worthy individually but when we refer to a 'twin prime' then it is to be understood they exist in pairs. A twin prime is a pair of numbers difference of which is two. Sometimes the term 'twin prime' is called 'twin prime pair. There are a few examples of twin prime pairs as (3, 5), (5, 7), (11, 13), (17, 19) etc.

2.3 Isolated primes:

Isolated primes are such primes, which has no twin within the gap of two. The non-twin primes can be categorized as isolated primes. There are some isolated primes given as 23, 37, 47, 53,67, 79, 83, 89, 97, etc.

2.4 Goldbach's conjecture:

In 1742, Christian Goldbach proposed an interesting hypothesis in a letter to Leonhard Euler. Later it is known to us by the name 'Goldbach's conjecture'. This simply relates even numbers with the primes [4]. Mathematicians have wrestled with this conundrum for centuries to prove it analytically. It states that every even number greater than 4 can be expressed as the sum of two prime numbers.

For convenience, we've considered a few numerical examples as follows:

4= (2+2), 6= (3+3), 8= (3+5), 10= (5+5), 12= (5+7), 14= (7+7), 16= (5+11), 18= (7+11), 20= (3 +17), 22= (5+17), 24= (11+13), 26= (13+13), 28= (11+17), 30= (7+23), 32= (13+19) etc.

3. Research Objectives:

In this article, we've utilized Goldbach's conjecture to claim new patterns into even sums. We've tried to extend the conjecture beyond its criteria. Hence, our approach added new features (i.e. the patterns of twin and isolated primes into the even integers). Apart from dealing with only primes, we gave priority to the isolated primes, which play a pivotal role in 'even sum' mysteries. The former hypothesis insists on claiming, the existence of 'at least' one twin prime in the even sums expressible as the sum of two prime numbers. The latter tries to reveal such a special kind of even integers whose, additive representations could be categorized in four distinct ways simultaneously.

4. Methodology:

We don't have any kind of rigorous proof for the claims. But we can show the truthfulness of our innovative idea, by splitting the even numbers up to 250000 into two prime numbers (in the case of the former). For the next hypothesis, we've performed computational verification up to 5000 (using Python software).

5. Hypotheses:

We can infer hypotheses from our discussion, which can be stated as follows

5.1 In the sum of two primes that represent an even integer, at least one (greater or equal to six) of these primes must belong to a twin prime pair.

5.2 There exists a subset of the set {E= 12m: m is positive integer}, where each member can be expressible simultaneously as the following:

I. As the sum of two twin primes (not necessarily from the same pair).
II. As the sum of two twin primes from the same pair.
III. As the sum of one twin prime and one isolated prime.
IV. As the sum of two isolated primes.

6. Discussion and Analysis:

Considering a few random even integers with all possible representations as the sum of two primes are given below

- 30 = (**7 + 23**) = (11 + 19) = (13 + 17)
- 82 = (3 + 79) = (**11 + 71**) = (23 + 59) = (29 + 53) = (41 + 41)
- 120 = (7 + 113) = (11 + 109) = (13 + 107) = (17 + 103) = (19 + 101) = (23 + 97) = (31 + 89) = (37 + 83) = (41 + 79) = (47 + 73) = (53 + 67) = **(59 + 61)**
- 210= (11 + 199) = (13 + 197) = (17 + 193) = (19 + 191) = (29 + 181) = (31 + 179) = (37 + 173) = (43 + 167) = (47 + 163) = (53 + 157) = (59 + 151) = (61 + 149) = (71 + 139) = **(73 + 137)** = (79 + 131) = (83 + 127) = (97 + 113) = (101 + 109) = (103 + 107)
- 216 = (5 + 211) = (17 + 199) = (19 + 197) = (23 + 193) = (37 + 179) = (43 + 173) = (53 + 163) = (59 + 157) = (67 + 149) = (79 + 137) = (89 + 127) = (103 + 113) = **(107 + 109)**
- 350 = (3 + 347) = (13 + 337) = **(19 + 331)** = (37 + 313) = (43 + 307) = (67 + 283) = (73 + 277) = (79 + 271) = (109 + 241) = (127 + 223) = (139 + 211) = (151 + 199) = (157 + 193) (337 + 13)

We've taken 30, 82, 120, 210, 216, 350 etc. By expressing them into the sum of two primes in all possible formats, three distinct categories are given below (we have used the representations, which are shown with 'bold' previously)

1. An even integer is expressible as the sum of two primes (each of them belongs to the same twin prime pair)
   e.g. 120 (= 59 + 61); 216 (= 107 + 109)
2. An even integer is expressible as the sum of two primes (both belong to the distinct twin prime pair)
   e.g. 82 (= 11 + 71); 210 (= 73 + 137)
3. An even integer is expressible as the sum of two primes (one of them is part of a twin prime pair, another is an isolated prime)
   e.g. 30 (= 7 + 23); 350 (=19 + 331)

From the above discussion, it is clear that in the additive representation of even integers (as the sum of exactly two primes), there exists at least one twin prime. On the other hand, taking the twin prime pairs can be classified as (6n-1, 6n+1) for a suitable choice of positive integer n. When we add both the quantities of a twin pair, the resulting quantity is divisible by 12. This ensures that, for adding such pairs we can construct a set that is a subset of {E= 12m: m is positive integer}. Now we claim that each member of the subset can be represented as the sum of two primes in 4 distinct ways. Such ways are categorized into two basic parts, excluding the isolated primes, and including the isolated primes.

For convenience, the following numerical illustrations are considered

Let's have the twin prime pair, (149, 151) [with n= 25 in the (6n-1, 6n+1)] then adding both of them, 149+151= 300 and 12|300 (i.e. 300 divided by 12 and belongs to the subset).

Using the Python software, we've computed all such representations of it as the sum of two primes. We observed that the presentations can be categorized in 4 distinct ways as follows (but the last category, as the sum of two isolated primes could be found in even integers after 44)

300
= (19+281) [as the sum of two twin primes (not necessarily from the same pair).]
= (149+151) [as the sum of two twin primes (from the same pair).]
= (7+293) [as the sum of one twin prime and one isolated prime.]
= (127+173) [as the sum of two isolated primes]

From the above, it can be noticed Goldbach's conjecture is utilized and presented in the form of new hypotheses.

7. Tables:

The list of verifications (due to limited space, we've selected a few of the representations) can be found in this part (the first and second hypotheses are in '**Table -1**' and '**Table- 2**' respectively).

Table -1:

Even numbers split into two primes, including one part of the twin prime		
8 = 3 + 5	806 = 19 + 787	7330 =101 + 7229
56 = 3 + 53	1000= 3 + 997	7600 =11+7589
94 = 5 + 89	1006 =29 + 977	7756 =3 +7753
174 = 7 + 167	1358 =31+ 1327	7756 =3 + 7753
200 = 3 + 197	1532 =43+ 1489	7938 =5 +7933
208 = 11 + 197	1660 =3+ 1657	7998 =5 + 7993
326 = 13 + 313	1822 =11+ 1811	8026 =17+ 8009
64146=109+64037	100000=11+99991	214568=7 + 214561
66062=181+65881	129526=39509+90017	224560=59 + 224501
67208 =19+67189	129670=7349+122321	236890=11 + 236879
67400 =31+67369	129718=41759+87959	249986=13 + 249973
69142=149+68993	129856=13757+116099	250000=11 + 249989
69248 =139+69109	130042=84221+45821	250002=13 + 249989

Table -2:

Even Integer	The distinct categories of even sum representations			
	Excluding Isolated Primes		Including Isolated Primes	
	Both primes individually constitute a twin prime pair	Both of them are part of the same twin prime pair	One twin prime and one isolated prime	Both are isolated primes (greater or equal to 46)
36	*5+31*	*17+19*	*13+23*	*NA*
60	17+43	29+31	13+47	23+37
456	*107+349*	*227+229*	*433+23*	*89+36*
480	*17+463*	*239+241*	*107+373*	*83+397*
624	*7+617*	*311+313*	*601+23*	*37+587*
696	*347+349*	*347+349*	*659+37*	*89+607*
924	*101+823*	*461+463*	*5+919*	*331+593*
1044	*11+1033*	*521+523*	*5+1039*	*53+991*
1140	*107+1033*	*569+571*	*1093+47*	*127+1013*
1200	*107+1093*	*599+601*	*431+769*	*499+701*
1644	*821+823*	*(821+823)*	*1621+23*	*733+911*
1656	*229+1427*	*827+829*	*1609+47*	*443+1213*
1716	*17+1699*	*857+859*	*1669+47*	*89+1627*
1764	*41+1723*	*881+883*	*1697+67*	*23+1741*
3396	*139+3257*	*1697+1699*	*3329+67*	*53+3343*
3444	*193+3251*	*1721+1723*	*3391+53*	*173+3271*
3576	*107+3469*	*1787+1789*	*3529+47*	*733+2843*
3744	*283+3461*	*1871+1873*	*2143+1601*	*67+3677*
3756	*199+3557*	*1877+1879*	*3673+83*	*409+3347*
3864	*43+3821*	*1931+1933*	*3767+97*	*853+3011*
3900	*229+3671*	*1949+1951*	*3853+47*	*23+3877*
3996	*829+3167*	*1997+1999*	*107+3889*	*929+3067*

8. Conclusion:

Our research work is based on the overlooked pattern. To verify that all the even numbers satisfy the hypothesis is our main interest. We have performed Python programming to observe the truthfulness of our hypothesis. We claim that our observation presents the extended version of Goldbach's conjecture.

9. Future scope of current research:

We know that mathematics is all about finding 'why'.
Thus, there are many opportunities to be explored further. We have designed some basic scopes of in future to explore as follows:

1. Extending Goldbach's Conjecture.
2. Proving Goldbach's conjecture harnessing the twin prime pairs.
3. Establish a bridge among the twin, isolated primes and even integers
4. Our used methodology can utilize twin prime conjecture.

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