Understanding the effect of Financing Variability Using Chance-Constrained Approach in Time-Cost Tradeoff

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Abstract - *The issue of balancing trade-offs between project duration and associated costs can be effectively addressed through the implementation of the linear optimization methodology. It's important to consider that this particular approach solely focuses on the monetary aspects. During our study, we decided to analyze the influence of financing variability within the scheduling process. To achieve this, we used chance-constrained programming (CCP) allowing us to estimate the coefficient of variation for financing feasibility at a desired confidence level. Evaluating uncertainty involves calculating CV within a specified confidence level. In order to understand its effect, an objective function along with a set of constraints is used. An effective way to find direct costs. The utilization of Excel Solver helps in successfully completing the study. To better comprehend how financing variability impacts our study, two scenarios were taken during our analysis.*

***Key Words*: Financing feasibility, Linear optimization, Chance constrained programming (CCP), Coefficients of financing variability (CV_{AF}), Time-Cost Tradeoff (TCT)**

1. INTRODUCTION

In the early stages of construction projects, it is important to formulate a plan and estimate the time required for completion as well as the projected expenses. This process, commonly referred to as "time and cost estimation of the project," is crucial in order to achieve efficiency from beginning to end. By accurately estimating these factors, it becomes possible to implement effective planning strategies and maintain concise control over costs. [1]

Accomplishing a construction project successfully is truly a challenging endeavor. As it requires careful planning and precise execution. Regrettably, unforeseen circumstances frequently lead to modifications in time and cost estimates even when plans are carefully formulated.

As deviations emerge in the project, obtaining adequate financing presents a critical task, resulting in the suspension of operations until securing necessary funds- thus causing delays and hindering momentum. As such, it often proves impractical to limit potential divergences entirely; instead, ensuring ready access to supplementary funding becomes imperative for effectively handling unanticipated expenses.

Chance-constrained programming (CCP) is an optimization technique that can provide valuable insights into how to handle the variability of financing. By incorporating the probability of events into the optimization equation CCP enables a more precise estimation of potential outcomes. This programming approach serves as a powerful tool for evaluating the risks based on the desired confidence level. [1]

According to our model. The optimal solution is determined to have a risk level of 15%. Based on statistics there is an 85% probability of fulfilling the constraints and only a 15% chance of breaching them. As a result. It can be assumed that the solutions presented in this study have a commendable success rate of complying with constraints approximately 85% of the time.

This study aims to develop a comprehensive mathematical model that considers various network Precedence relationships as well as the financing variability within a project.

2. LITERATURE REVIEW

In the realm of construction project management striking an effective equilibrium between time and cost holds significant importance. To tackle conventional time cost problems associated with these projects. Various mathematical models exist alongside diverse approaches like heuristics and metaheuristics. Modern advancements in machine learning and artificial intelligence have also opened up avenues for addressing these challenges effectively.

Two broad groups of mathematical programming models are used to address the time-cost tradeoff problem in various ways. One group utilizes linear programming (LP) models, as seen in studies conducted by researchers such as **Meyer and Shaffer** [2] . Another approach involves integer programming (IP), demonstrated by **Liuet** al.'s combined LP/IP hybrid method that establishes both lower bounds and exact solutions for project time-cost relationships through LP and IP techniques respectively [3]. Butcher's dynamic programming-based approach is another perspective worth mentioning here[3]. Furthermore, **Reda and Carr** [4]present a mixed integer programming solution in their study. **Senouci and Adeli** [5] take a holistic approach by integrating resource-constrained scheduling, resource leveling, and project total cost minimization in their mathematical model. Conversely, **Yang** [1]'s chance-constrained programming method incorporates funding variability into LP techniques without considering uncertainties of project activities or duration. Additionally**, Khalaf et al.** [6] introduce a cost-effective strategy focusing on stretching non-critical activities within the available maximum budget by crashing all relevant activities simultaneously in the project network. To maximize savings from this approach, they leverage Linear Programming (LP) technique and fully utilize slack in various non-critical paths within the network. The cost savings achieved through the LP model are then subtracted from the initial cost of crashing all activities to calculate the final project cost.

Various computational optimization techniques using artificial intelligence have been presented to address time cost tradeoff problems. For instance, genetic algorithms have been employed by **Senouci and Eldin** [7] to develop a model for resource scheduling that considers precedence relationships, resource leveling, and resource-constrained scheduling. Similarly. **Zheng et al.** [8] proposed a genetic algorithm-based multi-objective approach to optimize both total time and total cost simultaneously.

In addition to genetic algorithms, **Elbeltagi et al.** [9] utilized five evolutionary-based optimization algorithms, namely Genetic Algorithm, Memetic Algorithm, Particle Swarm, Ant Colony, and Shuffled Frog Leaping. Similarly. **Abbasnia et al.** [10] applied the Fuzzy logic theory along with a genetic algorithm as an optimizer for time cost tradeoff.

Previous studies in this domain primarily aimed at minimizing cost (direct or total project cost) while considering precedence constraints between activities and other factors such as resource leveling and constrained resource scheduling [11] [5] [7]. However, Yang [1] identified funding variability as another important constraint in this context.

To address these various constraints comprehensively in the context of time cost optimization (TCO) this paper introduces a new mathematical model that incorporates all precedence relationships, and financing variability of project duration.

This paper is based on paper **Yang** [1],author used CCP for quantifying effect of cost uncertainty with some probability in funding variability

3. RESEARCH GAPS

Even though project delays, cost overruns, and other challenges are central to all types of projects. There is a noticeable gap in evaluating these issues. Despite numerous studies on construction project management. There has been limited investigation into the impact of funding fluctuations on time cost trade-offs. Considering that external financing often has a significant influence on budgets and timelines. It is crucial to have a thorough understanding of these dynamics.

4. RESEARCH METHODOLOGY

Chance constrained Programming (CCP) was originally uncovered by Charnes, Cooper, and Symmonds during the 1950s as a means of optimizing financial planning [12] [13]. In this study, the CCP approach is used to address time-cost trade-off problems. When confronted with decision-making in uncertain scenarios. The utilization of Chance constrained Programming (CCP) can offer a highly efficient solution.

CCP mathematical formulation requires the introduction of random variables and a linear program in order to achieve this goal. An objective function is developed with constraints that aim to optimize within specified maximum and minimum bounds.

This equation for linear optimization is taken from: [14]

Equation 1

Maximize $\sum_{j=1}^{n} c_{ij} x_j$

Subject to:

Equation 2

$$\Pr\left(\sum_{i=1}^{n} (a_{ij} x_j \le bi) \ge \alpha_i\right) \quad where\ j = 1, 2 \dots n\ and\ x_j \ge 0$$

Where, decision variable, i=constraint, c_{ij}= coefficient for j^{th} variable in i^{th} constraint, a_{ij}= left hand coefficient for j^{th} variable in the i^{th} constraint, b_i =right hand coefficient for the i^{th} constraint, α_i =prescribed confidence level (probability level) The level of certainty to fulfill the constraint should be beyond or equal to the specified α_i ,in some choice of x_j.

The previous equation can be rewritten as follows:

Equation 3

$$\Pr\left(\sum_{i=1}^{n} (b_i \le a_{ij} x_j) \le (1 - \alpha_i)\right) \quad where \quad j = 1, \dots \dots m$$

Estimating the mean and standard deviation for b_i :

Equation 4

$$(b - m_{b_i}) / (\sigma_{b_i}) \le \sum_{i=1}^{n} (a_{ij} x_j - m_{b_i}) / (\sigma_{b_i}) \qquad j = 1, \dots \dots m$$

The mean of b_i and standard deviation of b_i = m_{bi} and σ_{bi} respectively. It was assumed that the right-hand side of the equation and b_i both follow a standard normal distribution. Assume b_i is distributed normally, the right-hand side of inequality must follow standard normal distribution. For standard normal distribution Mean=0, standard deviation=1.

Equation 5

$(b_i - m_{bi})/\sigma_{bi} = Z_{\alpha i}$

Z_α = inverse of the cumulative standardized normal distribution.

Thus, from Eq. 5th and Eq. 4th , this equation is obtained:

Equation 6

$$Z_{\alpha_i} \leq \sum_{i=1}^{n} (a_{ij} x_j - m_{b_i})/\sigma_{b_i}$$

After converting the original stochastic constraint to the deterministic equivalent, the following equation is obtained: [14].

Equation 7

$$\sum_{i=1}^{n} (a_{ij} x_j - m_{b_i})/\sigma_b \leq Z_{1\_\alpha_i}$$

Where 1-α_i inverse of the cumulative standardized normal distribution is evaluated at probability 1-α_i. [14]

Equation 8

$$\sum_{i=1}^{n} a_{ij} x_j \leq m_{b_i} + Z_{1\_\alpha_i} * \sigma_b$$

5. MODEL FORMATION

This is the objective function that will be used for minimizing the direct cost of the project:

$$\sum_{i=1}^{t} C_i T_i$$

Constraints:

In order to restrict the possible solution range, chance constraints impose requirements on both outcome probabilities and acceptable limits for variables. This involves establishing minimum and maximum values for each relevant variable, thereby reducing computation times while ensuring that solutions fall within predetermined numerical ranges.

The financing constraint is expressed in terms of financing variability. Consider the available financing in a deterministic form.

$$\sum_{i=1}^{n} (c_i \leq AF)$$

Available financing refers to the total amount of financing that can be obtained from different sources. This assertion is supported by the central limit theorem, which states that "when multiple random variables are combined, the resulting distribution will be approximately normal, regardless of whether the individual distributions of the contributing variables are normally distributed or not." This principle holds true for both continuous and discrete random variables, as well as for distributions that are skewed or symmetric. [12]

Thus, the financing constraint can be stated as follows:

$$\Pr(\sum_{i=1}^{n} (c_i \leq AF) \geq \alpha_i)$$

Rather than using the standard deviation, utilize the coefficient of variation (CV_{AF}), the equation will be as follows:-

$$\sum_{i=1}^{n} c_i \leq m_{AF} + Z_{1\_\alpha_i} \times \sigma_{AF_i}$$

Where m_{AF} and σ_{AF} are the mean and standard deviation of available financing for a project, respectively:-

$$\sum_{i=1}^{n} c_i \leq m_{AF} + Z_{1\_\alpha_i} \times (CV_{AF} \times m_{AF})$$

It was assumed that at the Confidence level (α= 85%) the financial constraint must be satisfied.

$$Z_{1-\alpha i} = Z_{1-0.85} = Z_{0.15} = -1.44$$

It was carefully observed by the project planner that variability in financing will be 10% so he assigned CV_{AF} to be 10% :-

$$\sum_{i=1}^{n} c_i \leq m_{AF} + (-1.44) \times (10\% \times m_{AF})$$

$$\frac{\sum_{i=1}^{n} Ci}{0.856} \leq m_{AF}$$

The presence of instability or unpredictability in financing indicates that there may be a direct correlation with reduced output on the right-hand side of the mentioned Equation. Consequently, it is highly likely that project timelines may become longer, and there could be an overall increase in measures aimed at minimizing exposure to potential economic hazards.

Similarly for 15 % CV_{AF} (15% financing variability)

$$\frac{\sum_{i=1}^{n} Ci}{0.784} \leq m_{AF}$$

Solver constraints:

To achieve cost reduction when there is variability in financing or time, or both, it is essential to optimize the cost towards the desired point within a specific time duration.

Crashed duration (weeks crashed) <= maximum crashing allowed The activity cannot be exceeded beyond its crashing limit, thereby preventing any further crashing. In order to manage the duration of the project, an additional constraint has been imposed. As a result, the crashed duration can now be controlled by manually inputting the desired project duration.
Due date < = project duration
The due date is the duration till the project to be crashed.

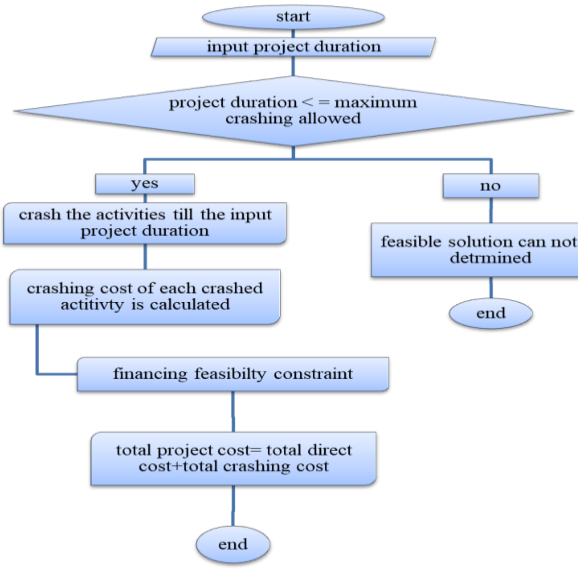
Start and finish relationship:
Early Start = Early finish value from last predecessor, Early Finish = Early start + Activity duration
LS = LF – Activity duration, LF = minimum value of LS from immediate successor
Slack/float = It can be defined as the difference between the earliest and latest, start or finish time.
Slack/float= EF – ES or LF – LS

6. Solving Procedure:

The optimization is done by means of Excel solver; four scenarios are adopted: -

- Traditional time-cost trade-off
- Traditional time-cost trade off with 10% and 15% Financing variability

Figure 1: Flowchart of working



7. Numerical Example

A numerical example is taken to analyze the working and performance of this model. The case study consists of eight activities, three-time estimates under normal and crash conditions, and the most likely time estimates which are given for the direct cost of the activity. An indirect cost of INR 2000 per week is considered in this example [12].

Figure 2: A Numerical Example

			Normal conditon					Crash Conditions						
Activity	Predecessor	successor	optimistic time	most likely time	pessimistic time	Te(estimated time)	normal cost	optimistic time	most likely time	pessimistic time	Te(estimated time)	crash cost	Maximum crashing allowed	cost per week
A		C,E	4	5	12	6	14000	2	3	4	3	50000	3	12000
B		D	3	3	3	3	9000	1	2	3	2	12000	1	3000
C	A	D	2	4	6	4	16000	1	2	3	2	36000	2	10000
D	B,C	F,H	3	4	11	5	11000	2	3	4	3	41000	2	15000
E	A	G	1	2	3	2	8000	1	1	1	1	12000	1	4000
F	D	G	2	4	6	4	10000	2	3	4	3	26000	1	16000
G	E,F		1	3	5	3	13000	1	1	1	1	41000	2	14000
H	D		2	5	8	5	12000	2	3	4	3	39000	2	13500

Minimize $\sum_{i=1}^{n} C_i T_i$

Minimize $\sum_{i=1}^{i} C_i T_i$ = project cost without crashing + 12000T_{EA}+ 3000T_{EB}+ 10000T_{EC}+ 15000T_{ED}+ 4000T_{EE}- +16000T_{EF}+14000T_{EG}+ 13500T_{EH}

8. Solving Scenario

Two scenarios have been investigated with the objective of fully comprehending the impacts of financial fluctuation:

A. In the first scenario, the applied traditional time cost trade-off method resulted in a direct cost of INR93000 for a duration of 20 weeks.

B. In the second scenario two alternatives were pursued. The initial one (2a) involved a 10% variability in financing (CV_{AF}) generating a direct cost of INR108645. Alternative (2b) introduced a higher degree of uncertainty at 15% variability in financing (CV_{AF}) resulting in a direct cost of INR118623 for both scenarios with duration being maintained at 20 weeks.

9. Analysis of Result

Table 1: Direct cost of considered scenarios

Scenario	1	2a	2b
Weeeks			
20	93000	108644	118623
19	103000	120327	131377
18	113000	132010	144132
17	127000	148364	161989
16	141000	164720	179846
15	159000	185747	202806
14	177000	206775	225765

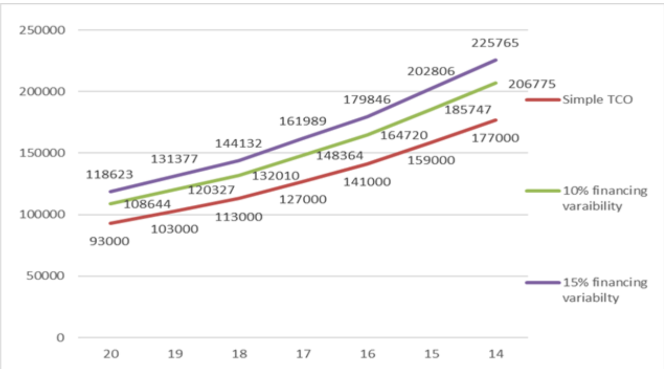
For comparing scenario 2a and 2b, which correspond to 10% and 15% financing variability respectively, let's take an example cost of INR132010. This cost is associated with scenario 2a, and it takes 18 weeks of time to complete. When the same cost is considered for scenario 2b, which is

15% financing variability, it takes 18.95 weeks of time to complete. It has been observed that alterations in funding can impact the duration of finishing a project despite fixed expenditures. Take scenario 2a; for example: with a time limit of 16 weeks and costs standing at INR164720. Conversely, in scenario 2b with an identical deadline of 16 weeks; expenses total up to INR179846.It can be stated that even a slight escalation of financing variability (by as little as 5%) can create significant results such as a lengthier duration (a rise of approximately 5.27%) for completing the project and an increased overall expense (about 9%) if kept on schedule.

Time-Cost Curve

- Now fix the project deadline at 17 weeks the necessary direct cost will be INR 127000, INR148364, and INR161989, for scenario 1, scenario 2a, and scenario 2b, respectively. The overall percentage increase from traditional time-cost scenarios amounts to 16.82%, 27.55%,
- At direct cost INR177000, the project can be completed in 14 weeks, 15.42 weeks, 16.15 weeks, for scenario 1, scenario 2a, and 2b, the overall amount of percentages that indicate an increase in relation to the 1st scenario are 10.14%, 15.35%.

Figure 3: Time-direct cost curve



Total project calculations:

Table 2: Cost after including Indirect Cost

Weeks	Indirect cost	Total project cost for Simple TCO	Total project cost for 10% Financing Variability	Total project cost for 15% Financing Variability
20	40000	133000	148644	158623
19	38000	141000	158327	169377
18	36000	149000	168010	180132
17	34000	161000	182364	195989
16	32000	173000	196720	211846
15	30000	189000	215747	232806
14	28000	205000	234775	253765

Table 3: comparing a Total Cost

Weeks	Total project cost for Simple TCO	Total project cost for 10% Financing Variability	Total project cost for 15% Financing Variability
20			158623
19.78			161000
19		158327	169377
18.72		161000	
18		168010	180132
17.44			189000
17	161000	182364	195989
16.53		189000	
16		196720	
15	189000		
14			

For example, the total expense of INR205000 is estimated from the traditional time cost scenario, if financing variability of 10% occurs in the project, then the estimated completion time will be 15.56 weeks instead of 14 weeks. Similarly evaluating all scenarios, it can be concluded that there are various timelines for completing of project available within an allocated budget.

10. Conclusion

The scope of this research involves constructing a time-cost trade-off model that takes into consideration the fluctuations in financing. By presenting this simplified model, decision-makers can gain valuable.

In order to grasp the interrelation between cost and time effectively, it is imperative to investigate the time cost curve for each scenario. A prime example would be that raising the coefficient of variation for funding by approximately 10 to 15% results in a corresponding increment in direct costs by roughly about 9%.

Subsequently drawing diverse conclusions becomes possible through this analysis.

This model suggests that if the cost remains fixed, there will be an increase in time. Similarly, if time is fixed, there will be an increase in cost.

Now, this model gives choice to the client for the completion of the project with respect to time and finance.

This particular model offers the option to address different confidence levels, such as 99%, 95%, 90%, 80%, and 75%. It also allows for a range of variations, including 2.5%, 5%, 10%, 15%, and 20%. These variations are significant in understanding and evaluating the impact on both delays and expenses.

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