

# Mixed Convection Flow and Heat Transfer in a Two Sided Lid Driven Cavity Using SIMPLE Algorithm

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**Abstract** - In this paper mixed convection flow and heat transfer in steady 2-D incompressible flow through a two sided lid driven cavity is studied. The upper and lower walls of cavity are moving with uniform velocity. The upper wall is at higher temperature and the lower wall is kept at lower temperature. SIMPLE algorithm is employed for solution of governing equations of the model. A staggered grid system is employed for numerical computations for velocity, pressure and temperature. Various values of under relaxation factors for velocity, pressure and temperature are used to obtain the stability of the numerical solutions. Bernoulli equation has been taken up to check the accuracy of the computed solutions. The significant findings from this study have been given under conclusion.

**Keywords** - Incompressible flow, two sided lid driven cavity, SIMPLE algorithm, staggered grid system, mixed convection flow.

## 1. INTRODUCTION

The phenomenon of motion of viscous fluid in a two-dimensional rectangular cavity was first studied by Kawaguti [1]. Since then, numerous researches have been done to study the motion of fluid in lid driven cavity under different conditions [2], [3], [4].

Numerical investigation of steady state two-dimensional mixed convection problem in a vertical two-sided lid-driven differentially heated square cavity was done by Öztop and Dağtekin [5]. They considered three cases depending on the direction of moving walls. They found that both Richardson number and direction of moving walls affect the fluid flow and heat transfer in the cavity. Muthamilselvan et al studied the effect of magnetic field on mixed convection in a two-sided lid-driven cavity filled with a fluid saturated porous medium keeping the top and bottom horizontal walls at constant but different temperatures [6]. A uniform magnetic field was applied in the vertical direction normal to the moving wall. They found that Hartmann number, Richardson number, Darcy number and direction of the moving walls have strong influence on the fluid flow and heat transfer in the

enclosure. Numerical investigation to analyze the steady state two-dimensional mixed convection in two sided lid-driven porous cavity under the combined buoyancy effects of thermal and mass diffusion, when the walls are moving in opposite direction was done by Benmalekb and Souidi [7]. Their results illustrate that heat transfer and mass transfer characteristics inside the cavity are enhanced for low values of the Richardson number due to the dominant mechanical effect induced by the moving lids. They also observed that heat and mass transfer are decreased for low values of the Darcy number. Numerical investigation of mixed convection fluid flow and heat transfer in an inclined two-sided lid-driven cavity subjected to Al<sub>2</sub>O<sub>3</sub>-water nanofluid was done by Esfe et al [8]. The effects of inclination angle, Richardson number, nanoparticle volume fraction, temperature, and nanoparticle diameter based on variable property formulations were recorded. Their results indicated that the addition of Al<sub>2</sub>O<sub>3</sub> nanoparticles has produced a remarkable enhancement on heat transfer with respect to that of the pure fluid. Esfe et al investigated mixed convection flows through a SiO<sub>2</sub>-water nanofluid in a square double lid-driven cavity with various inclination angles and sinusoidal heating on the left wall [9]. They studied the effects of variations of Richardson and Reynolds number, angle of inclination, and solid volume fraction of nanoparticles on the hydrodynamic and thermal characteristics. They found that the increases heat transfer rate with the solid volume fraction for a particular *Re*. Sivasankaran et al numerically investigated magneto-hydrodynamic mixed convection flow and heat transfer of Cu-water nanofluid in a square cavity filled with a Darcian porous medium with a partial slip [10]. They performed a parametric study and presented a set of graphical results to demonstrate interesting features of the solution. An investigation of the heat transfer performance of a two-sided lid-driven cavity filled with Cu-water nanofluid saturated porous media was done by Mukherjee et al [11]. They introduced A concept of speed ratio to analyze the effect of different moving velocity of top and bottom walls. They found that the heat transfer rate increases with the increase of speed ratio and for all the values of the speed ratio, average Nusselt number increases

Using Finite Difference Method Hussain analyzed numerically the thermal behaviour and parameters effect on heat transfer inside the 2D double lid driven cavity [12]. He analyzed the velocity and temperature profiles for a vast range of dimensionless parameters namely Reynolds number ( $Re$ ), Richardson number ( $Ri$ ) and Prandtl number ( $Pr$ ) and presented graphically and found that these flow parameters have significant effects in controlling the flow behavior inside the cavity. Manchanda and Gangawane presented numerical results for 2D, steady, laminar and incompressible flow in a square, two-sided, lid-driven cavity with a decentered heated triangular block for non-Newtonian power-law fluids [13]. It was found that mixed convection parameter have a negligible impact on fluid and thermal structure inside the cavity. Saha et al analyzed the flow and heat transfer characteristics inside a double lid-driven cavity underneath buoyancy consequences of thermal diffusion [14]. Their results revealed that the influence of the development of the velocity profiles in the chamber decreases with the augmentation of  $Re$ . Karabay numerically investigated fluid flow and heat transfer characteristics of  $Al_2O_3$ -water nanofluid in a two-sided lid-driven cavity with wavy walls [15]. The results showed that both flow and temperature distributions are sensitive to the geometric parameters. It is also found that introducing nanoparticles to the base fluid enhances heat transfer considerably for all Richardson numbers considered in this study. Benmalek et al analyzed mixed convection in a square cavity filled with Non-Newtonian fluid of Bingham model with two moving vertical walls by finite volume method [16]. Their results showed that increase in yield stress drops the heat transfer and the flow become flatter, while increasing Reynolds number augments it. The mixed convection and surface radiation effect in a two-sided lid-driven square cavity was studied by Dahani et al [17]. Their results showed significant effects of the Richardson number and surface radiation on the overall structure of the flow and heat transfer characteristics. Zaydan et al numerically investigated steady, laminar mixed convection inside a lid-driven square cavity filled with nanofluid [18]. They focused on the heat transfer enhancement when different nano-particles are incorporated separately in different base fluids. They found that the choice of the efficient binary mixture for an optimal heat transfer depends not only on the thermophysical properties of the nanofluids but also on the range of the Richardson number. Mixed convection heat transfer of Cu-water nanofluid in an arc cavity with non-uniform heating was numerically studied by Aljabair et al [19]. Their results found a good agreement

with the others works. Kumar et al studied the mixed convection phenomenon inside a two-dimensional, tall lid-driven cavity with top and bottom lids moving in opposite directions, when the cavity contains a uniformly heated equilateral triangular obstacle at its geometric center [20]. They showed that with a rise in the aspect ratio of the cavity, the flow-pattern becomes more dispersed inside the cavity. Tang et al investigated the hydrodynamics and thermal characteristics due to mixed convection in a vertical two-sided lid-driven differentially square cavity containing four hot cylinders in a diamond array [21]. They discussed the influence of different flow governing parameters, including the direction of the moving walls, the distance between neighboring cylinders and the Richardson number on the fluid flow and heat transfer with different values of Reynolds number, Grashof number and Prandtl number. Chowdhury and Alim carried out numerical investigation to analyze the impacts of internal heat source size, solid concentration of nanoparticles, magnetic field, and Richardson number on flow characteristics in an oppositely directed lid-driven wavy-shaped enclosure [22]. They showed that the overall heat transfer rate declines with the increasing length of internal heat source. They also proved that the presence and rising values of solid concentration of nanoparticles cause the augmentation of heat transfer whereas the magnetic field has a negative influence and the Richardson number has a positive influence on heat transfer.

SIMPLE algorithm proposed by Patankar and Spalding has been extensively used by researchers for the study of motion in lid driven cavity [23]-[27]. But SIMPLE algorithm has not been applied to the problem of mixed convective heat transfer in a two sided lid driven cavity. In the view of above discussion we aim to examine the mixed convection and heat transfer in a two sided lid-driven cavity using SIMPLE algorithm. The results obtained are shown through quiver plot and contour plots for various considered dimensionless parameters.

## 2. METHODOLOGY

We consider a two dimensional, incompressible, steady and laminar flow through a square cavity of height and length  $L$ . The side walls of the cavity have no-slip condition. The upper and lower walls are moving in their own plane. The cavity upper wall is moving with velocity  $U_U$  and is kept at a high temperature  $T_h$ . The bottom wall is moving with higher velocity  $U_L$  and is kept at a low temperature  $T_c$ . Following assumptions are being made

1. The left and right walls are adiabatic.

2. All thermo-physical properties of the fluid are constant except the density variation of the buoyancy term.

3. The density vary linearly with temperature as  $(\rho - \rho_c) = g\beta (T - T_c)$  where  $\rho$  is the fluid density,  $g$  is the acceleration due to gravity and  $\beta$  is the coefficient of thermal expansion.

4. The fluid is considered as Newtonian.

5. Viscous dissipation is neglected.

Under these assumptions, the governing conservations equations are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_c) \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}$$

where  $u$  and  $v$  are the fluid velocity components in the  $x$ - and  $y$  directions,  $p$  the pressure,  $T$  the temperature,  $g$  is acceleration due to gravity,  $\beta$  the volumetric coefficient of thermal expansion,  $\nu$  is kinematic viscosity,  $\rho$  the density of the fluid and  $\alpha$  the thermal diffusivity.

Introducing the non-dimensional quantities

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, \theta = \frac{T - T_c}{T_h - T_c}, P = \frac{p}{\rho U_0^2} \tag{5}$$

we can write Eqn. (1)-(4) in non-dimensional form as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{6}$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \tag{7}$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{Gr}{Re^2} \theta \tag{8}$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \tag{9}$$

where  $Re$ ,  $Pr$  and  $Gr$  are non-dimensionalised Reynolds number, Prandtl number and Grashof number respectively and are defined as

$$Re = \frac{U_0 L}{\nu}, Pr = \frac{\nu}{\alpha}, Gr = \frac{g\beta\Delta T L^3}{\nu^2}$$

The initial and boundary conditions in the non dimensionalised form are

$$\begin{aligned} U=1, V=0, \theta=1 & \quad 0 \leq X \leq 1, Y=1 \\ U=10, V=0, \theta=0 & \quad 0 \leq X \leq 1, Y=0 \\ U=V=0, \frac{\partial \theta}{\partial X}=0 & \quad 0 \leq Y \leq 1, X=0 \text{ \& } X=1 \end{aligned}$$

SIMPLE algorithm is applied to solve the governing equations along with the boundary conditions. The algorithm is a guess and correct procedure for the calculation of pressure and velocities. The computational domain is discretised employing the staggered grid arrangement. Employing finite volume method, the nonlinear governing partial differential equations are converted into a system of discretised equations. These discretised equations along with pressure correction equation are solved to obtain the velocities, pressure and temperature at all node points of the grid.

### 3. RESULTS AND DISCUSSION

The governing equations of motion are solved numerically to obtain unknown variables  $u$ ,  $v$ ,  $p$  and  $T$  for various values of  $\alpha$ ,  $\beta$ ,  $\rho$  and  $\mu$ . The SIMPLE algorithm has been implemented in MATLAB programming language. Different values of under relaxation factors for  $u$  velocity,  $v$  velocity and pressure were tested for all cases.

The effect of thermal diffusivity  $\alpha$  on  $u$  velocity is negligible. This can be seen in Figure 1a and 1b.

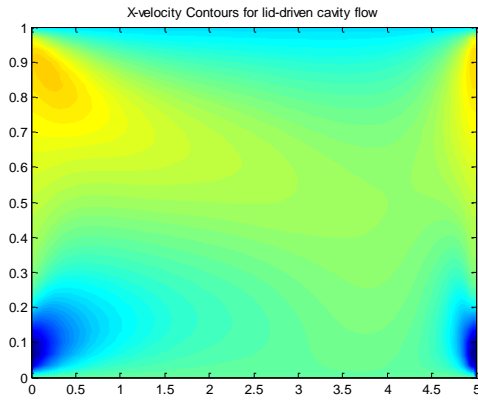


Fig. 1a. Contour plot for u velocity for  $\alpha=10$

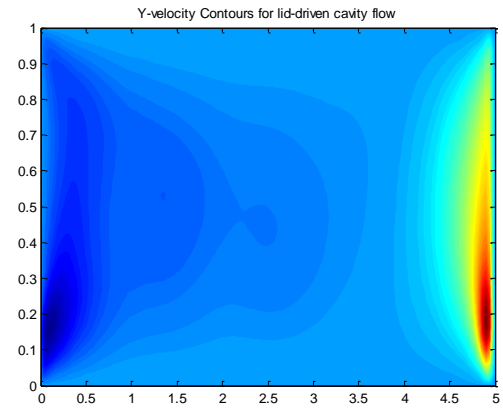


Fig. 2b. Contour plot for v velocity for  $\alpha=1$

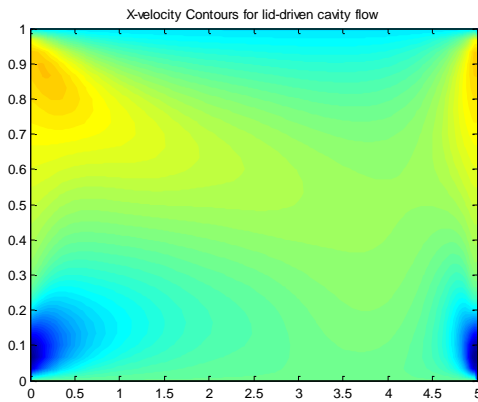


Fig. 1b. Contour plot for u velocity for  $\alpha=1$

In Figure 3a and 3b, the variation in pressure with change in  $\alpha$  can be seen. As  $\alpha$  decrease, low pressure region near north east corner, decreases in size. Also the low pressure region along west wall decreases as  $\alpha$  decrease. No significant change in temperature can be seen with change in  $\alpha$ .

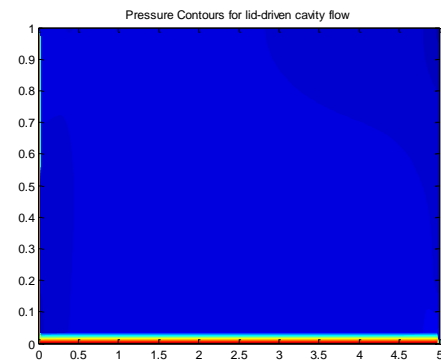


Fig. 3a. Pressure contours for  $\alpha=10$

The effect of thermal diffusivity on v velocity is shown in Figure 2a and 2b. It can be seen that increase in value of  $\alpha$  results in lower v- velocity. We can see a sudden increase in the v velocity near midpoints of east boundary for all values of  $\alpha$ .

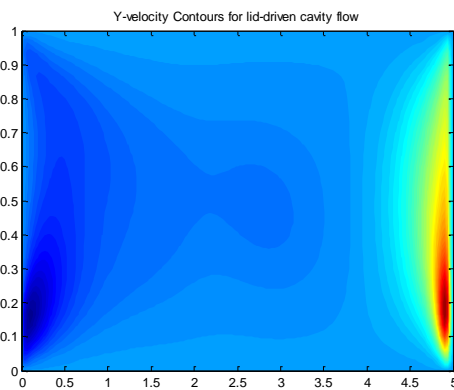


Fig. 2a. Contour plot for v velocity for  $\alpha=10$

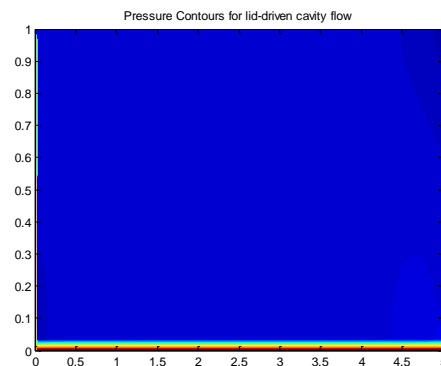


Fig.3b. Pressure contours for  $\alpha=1$



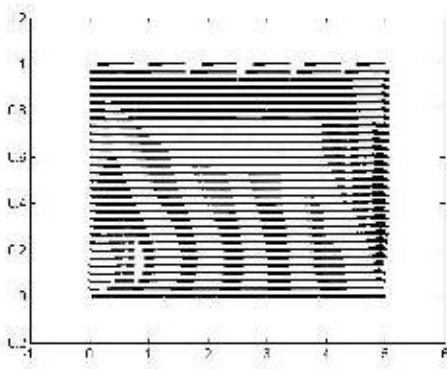


Fig 4a Direction field when  $\alpha=10$

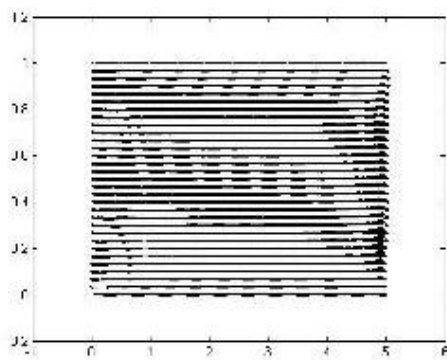


Fig 4b Direction field when  $\alpha=1$

In Figure 4a and 4b we can see the change in velocity profile as  $\alpha$  decreases.

The effect of coefficient of thermal expansion  $\beta$  on the  $u$  velocity is shown in Figure 5a, 5b and 5c and the effect of coefficient of thermal expansion  $\beta$  on the  $v$  velocity is shown in Figure 6a, 6b and 6c. It is seen that increase in value of  $\beta$  results in higher  $u$  - velocity and  $v$ - velocity at all points of the cavity. It is noticed that  $u$  velocity has highest values near south boundary which decreases towards north boundary.

The values of  $v$  are highest at west boundary and reduce towards centre of east boundary.

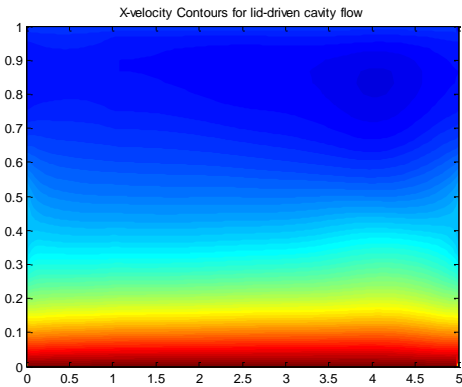


Fig. 5a Contour plot for  $u$  velocity for  $\beta=0.0001$

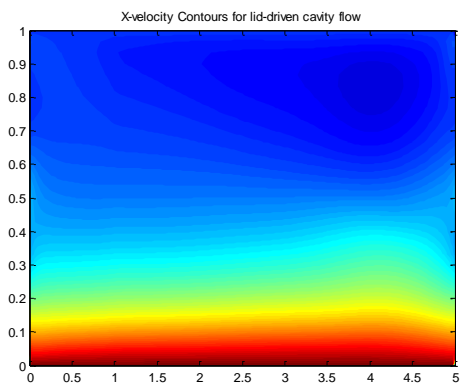


Fig. 5b Contour plot for  $u$  velocity for  $\beta=0.001$

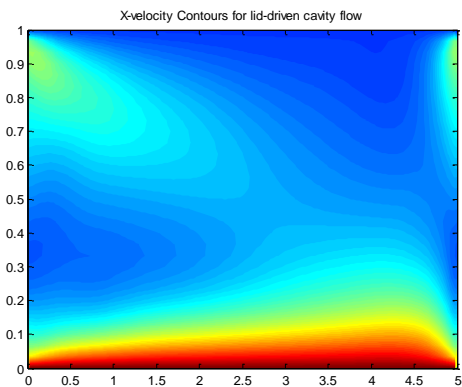


Fig. 5c Contour plot for  $u$  velocity for  $\beta=0.01$

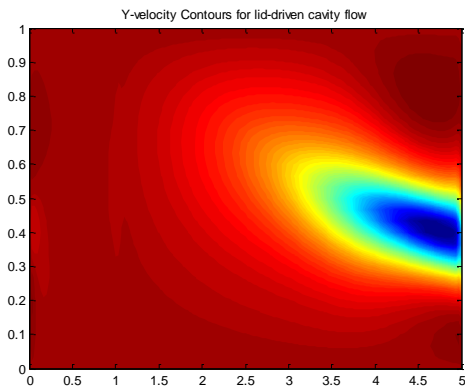


Fig. 6a Contour plot for v velocity for  $\beta=0.0001$

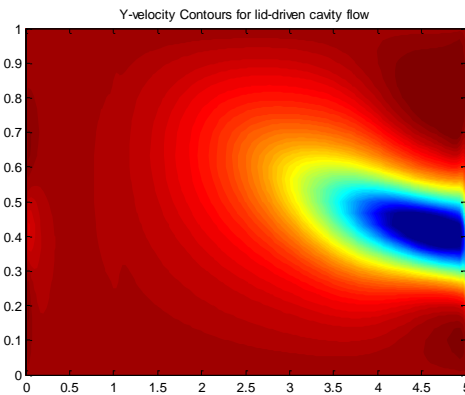


Fig. 6b Contour plot for v velocity for  $\beta=0.001$



Fig. 6c Contour plot for v velocity for  $\beta=0.01$

north boundary for all point in cavity. Note that as  $\beta$  increases, the pressure in the cavity also increases.

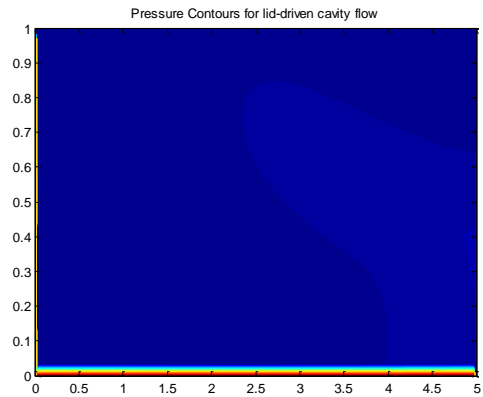


Fig. 7a Pressure Contour plot for  $\beta=0.0001$

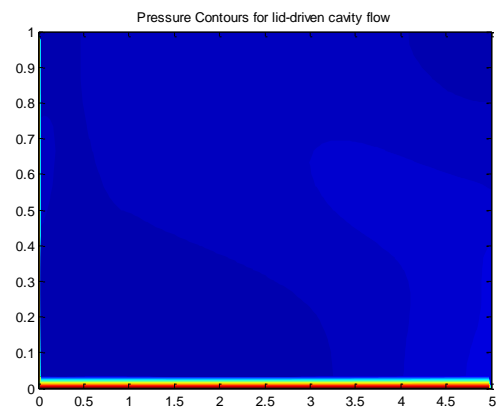


Fig. 7b Pressure Contour plot for  $\beta=0.001$

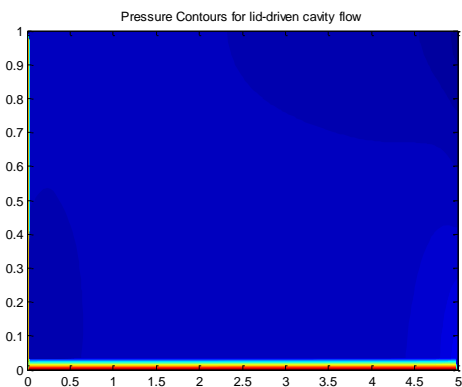


Fig. 7c Pressure Contour plot for  $\beta=0.01$

In Figure 7a, 7b and 7c the pressure contours for different values of  $\beta$  can be seen. The pressure is highest near the south boundary of the cavity and then decreases toward

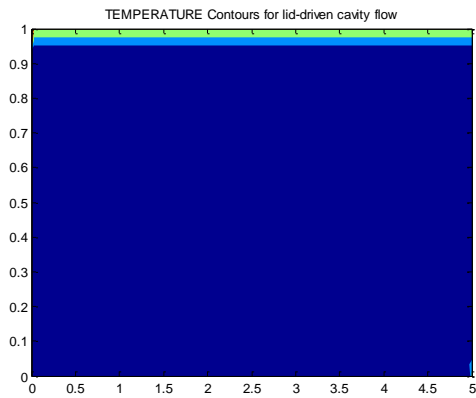


Fig 8a Temperature contour for  $\beta=0.001$

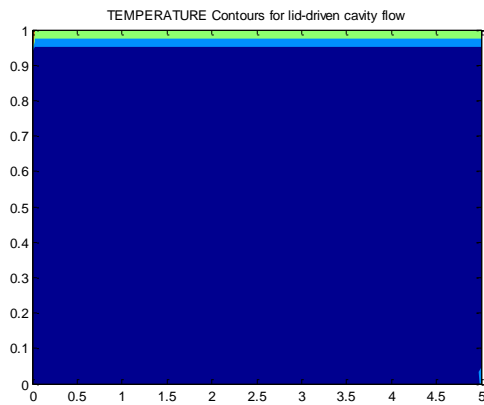


Fig 8b Temperature contour for  $\beta=0.01$

In Fig 8a and 8b we can see that no significant change is there with increase in value of  $\beta$ . In Fig 9a and 9b, remarkable change can be seen with increase in value of  $\beta$ .

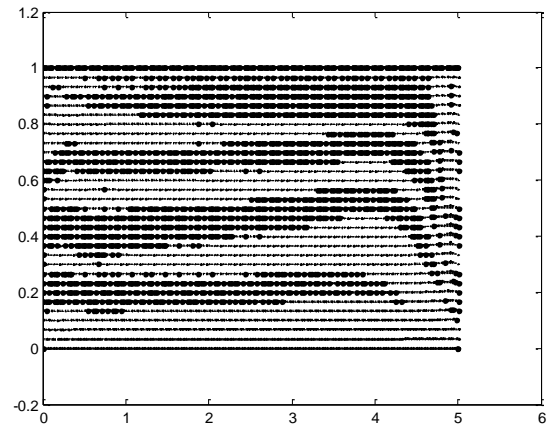


Fig 9b Direction field for  $\beta=0.01$

Figure 10a, 10b and 10c shows the effect of density of fluid  $\rho$  on the u- velocity. The effect of density on v- velocity can be seen in Figure 11a, 11b and 11c.

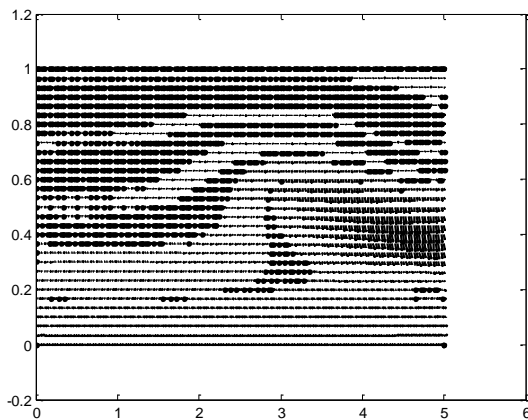


Fig 9a Direction field for  $\beta=0.001$

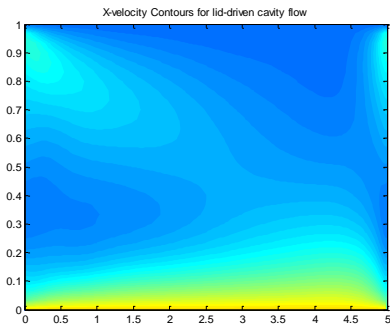


Fig. 10a Contour plot u velocity for  $\rho=1$

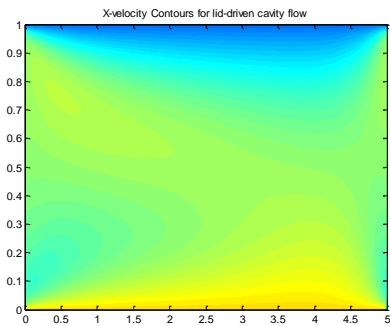


Fig. 10b Contour plot u velocity for  $\rho=0.5$

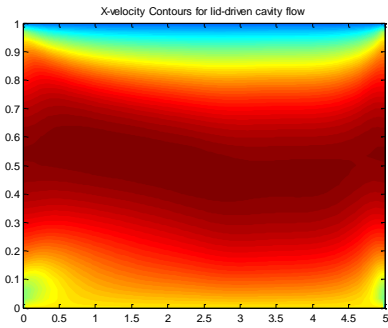


Fig. 10c Contour plot u velocity for  $\rho=0.25$

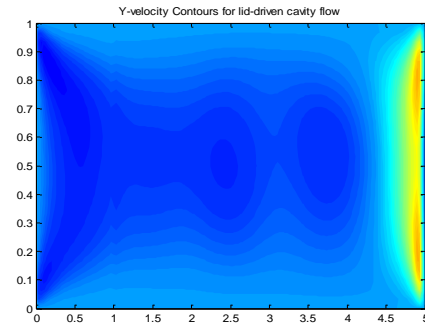


Fig. 11c Contour plot v velocity for  $\rho=0.25$

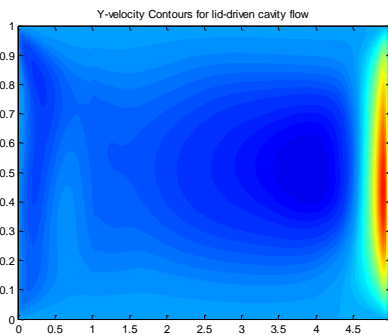


Fig. 11a Contour plot v velocity for  $\rho=1$

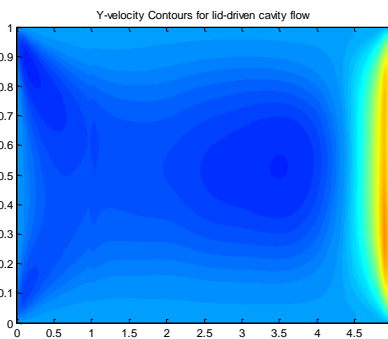


Fig. 11b Contour plot v velocity for  $\rho=0.5$

It can be seen that decrease in value of  $\rho$  results in increase in u - velocity. The u - velocity has higher values near south boundary which reduces towards north boundary. The pattern remains same for all values of  $\rho$ . The v-velocities are higher near midpoints along east boundary, which reduces as the value of  $\rho$  decreases.

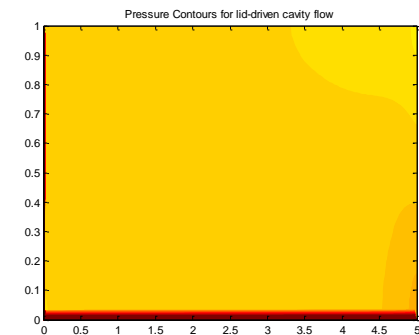


Fig 12a Pressure Contour for  $\rho=1$

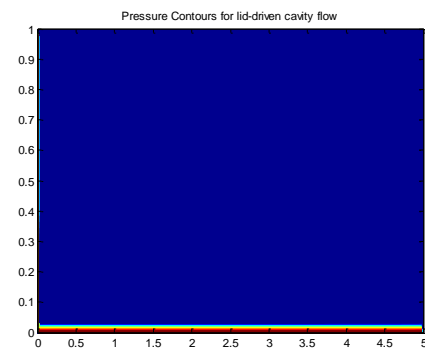


Fig 12b Pressure Contour for  $\rho=0.5$

In Figure 12a and 12b we can see, as expected, that the pressure in the cavity decreases with decreasing density. In Fig 13a and 13b, variation of temperature with pressure at all node points of the cavity can be seen.

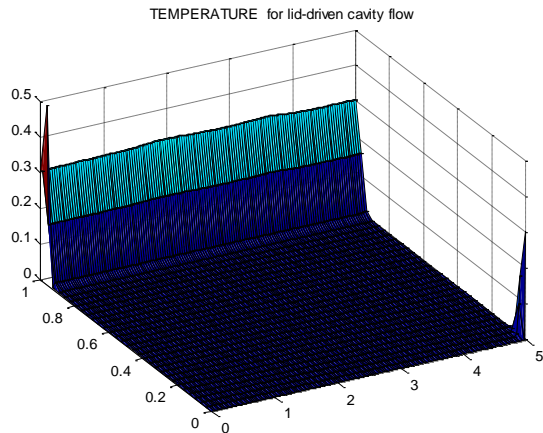


Fig. 13a. Temperature at all node points for  $\rho=1$

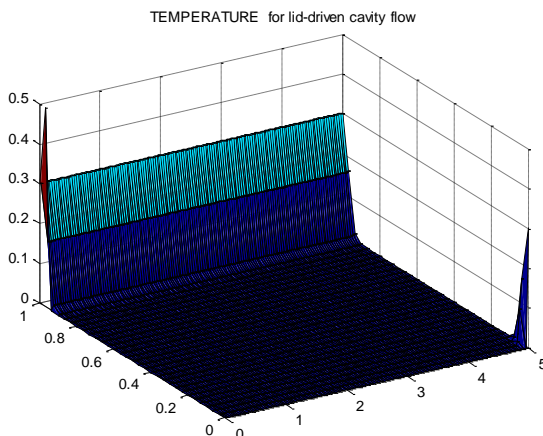


Fig 13b. Temperature at all node points for  $\rho=0.5$

Figure 14a and 14b shows the effect of density of fluid on the direction field of the flow. As the density of the fluid decreases, the direction field of the flow changes drastically

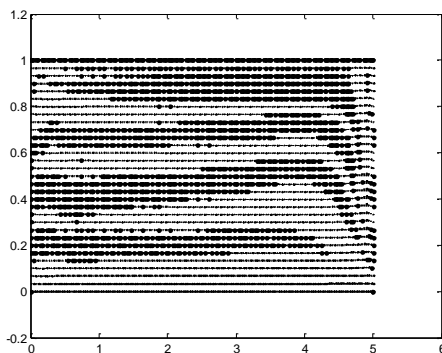


Fig 14a Direction field of the flow for  $\rho=1$

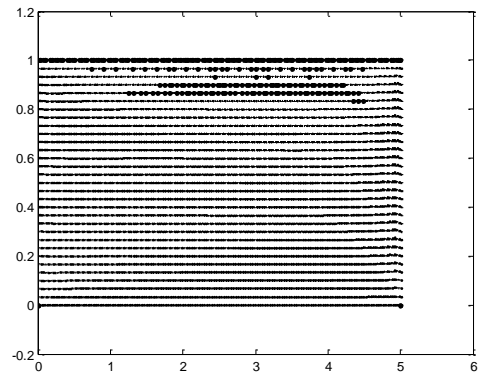


Fig 14b Direction field of the flow for  $\rho=0.5$

The effect of viscosity of the fluid on u velocity can be seen in Figure 15a, 15b and 15c. It can be seen that the pattern of decrease of u velocity from south towards north boundary remains same, but varies as the viscosity changes.

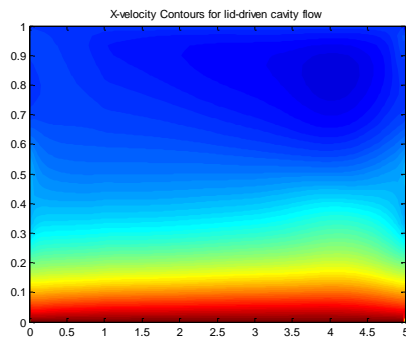


Fig 15a Contour plots for u velocity for  $\mu=0.01$

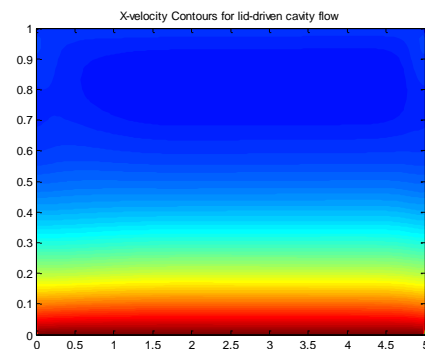


Fig 15b Contour plots for u velocity for  $\mu=0.05$



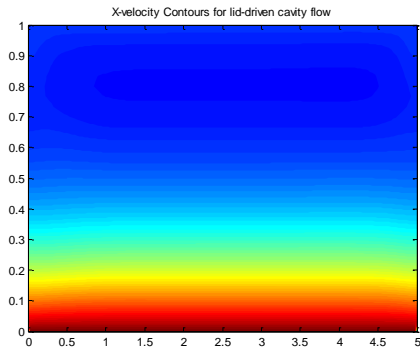


Fig 15c Contour plots for u velocity  $\mu=0.1$

The effect of viscosity of the fluid on v velocity can be seen in Figure 16a, 16b and 16c. It is seen that v velocity increases significantly with increase in viscosity.

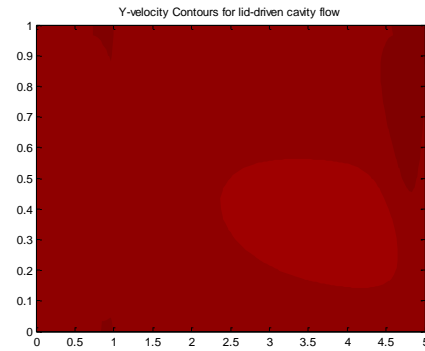


Fig 16c Contour plots for v velocity for  $\mu=0.1$

The variation of pressure inside the cavity with varying viscosity can be seen in Figure 17a, 17b and 17c. It can be clearly seen that the pressure decreases with increasing viscosity of the fluid. The change in temperature with change in viscosity is minimal.

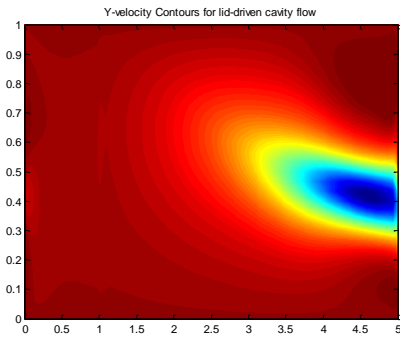


Fig 16a Contour plots for v velocity for  $\mu=0.01$ ,

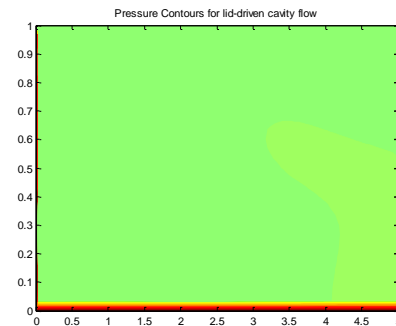


Fig 17a Pressure Contour plots for  $\mu=0.01$

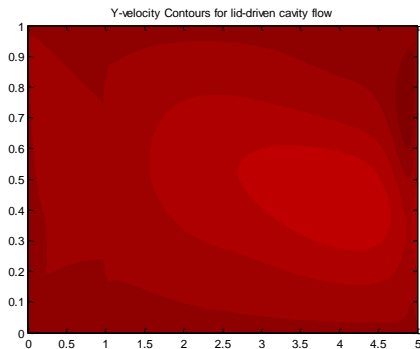


Fig 16b Contour plots for v velocity for  $\mu=0.05$

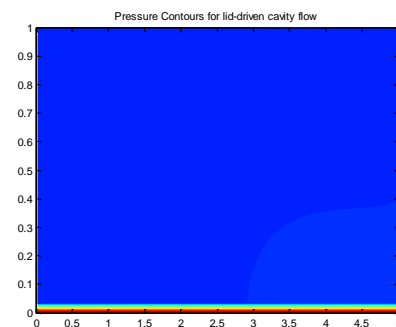


Fig 17b Pressure Contour plots for  $\mu=0.05$

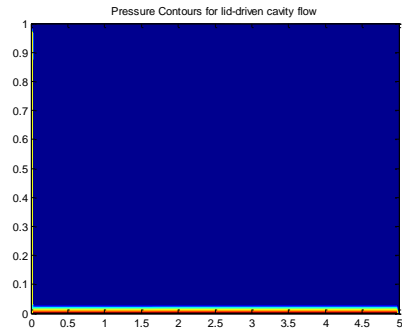


Fig 17c Pressure Contour plots for  $\mu=0.1$

The velocity profile also changes with increase in viscosity as can be seen in Figure 18a and 18b.

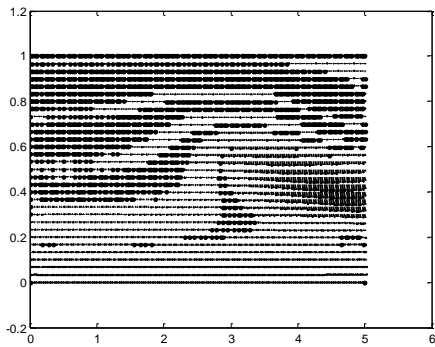


Fig 18a Velocity profile for  $\mu=0.01$

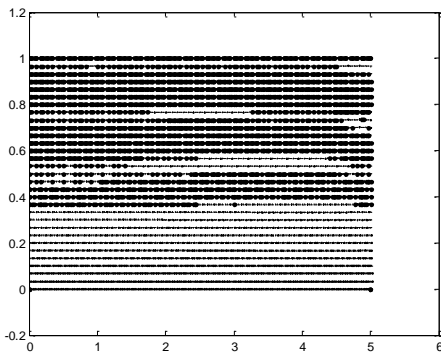


Fig 18b Velocity profile for  $\mu=0.05$

The effect of velocity of the bottom boundary on u- velocity can be seen in Figure 19a and 19b. As the velocity of bottom boundary increases, u- velocity increases, especially near the south boundary. For higher velocity of bottom boundary, the v velocity decreases throughout the cavity.

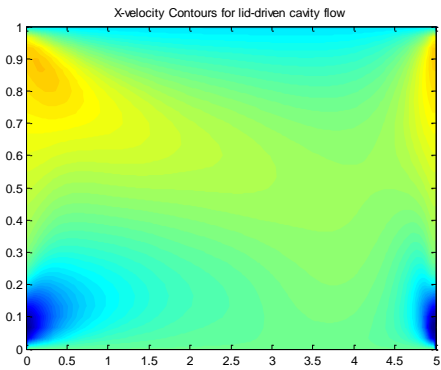


Fig 19a Contour for u velocity for wall velocity= 1

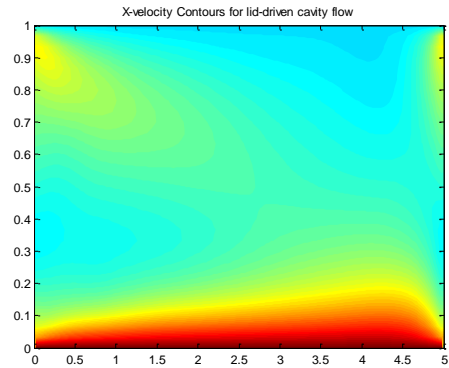


Fig 19b Contour for u velocity for wall velocity =3

The highest v velocity can be seen along the east boundary. This is shown in Figure 20a and 20b.

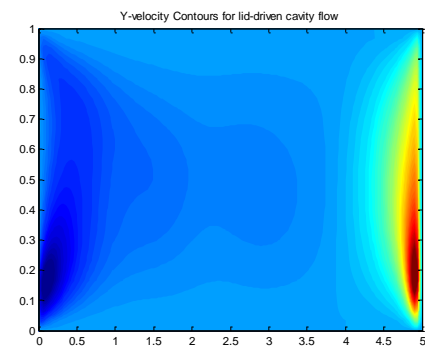


Fig 20a Contour for v velocity for wall velocity =1

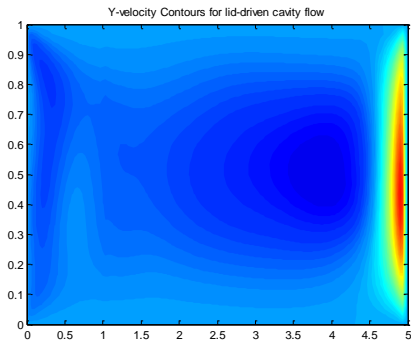


Fig 20b Contour for v velocity for wall velocity=3

In Figure 21a and 21b, we can see the change in pressure in the cavity with increase in the velocity of bottom boundary. The lower pressure regions near north east corner and south west corners increase in size with increase in velocity of bottom wall.

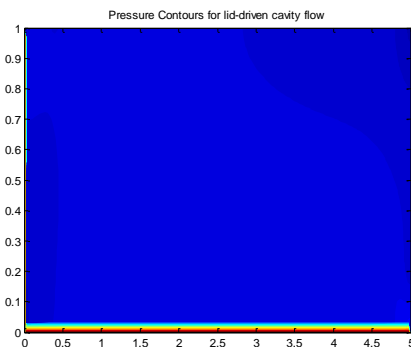


Fig 21a Pressure contour for wall velocity=1

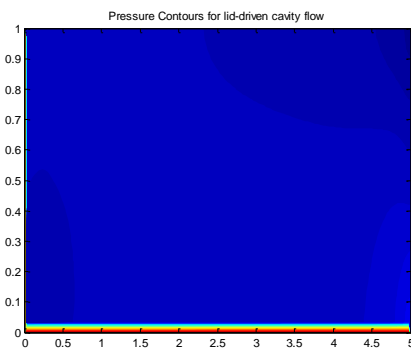


Fig 21b Pressure contour for wall velocity=3

In Figure 22a and 22b, we can see the change in direction field of the flow with increasing velocity of top boundary.

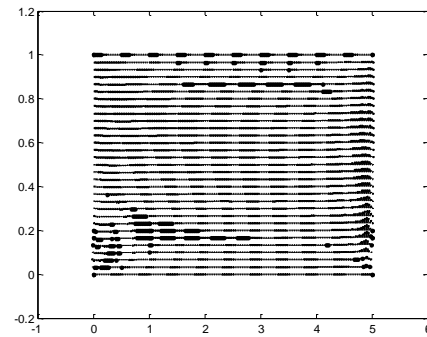


Fig 22a Direction fields for wall velocity=1

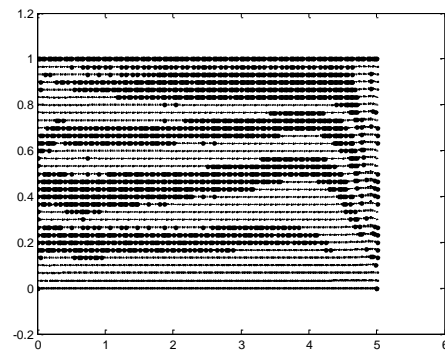


Fig 22b Direction fields for wall velocity=3

#### 4. CONCLUSION

A numerical study of mixed convection flow and heat transfer in steady 2-D incompressible flow through a two sided lid driven cavity is presented in this paper. The governing equations of motion are solved for u velocity, v velocity, pressure and temperature using SIMPLE algorithm. The numerical computations were conducted using staggered grid system. It was noted that the solution converges for only few values of under relaxation factor for pressure. Following results were obtained from the study-

1. It was seen that v velocity decreases with increase in value of  $\alpha$ , but u velocity shows no remarkable change with change in thermal diffusivity.
2. As  $\alpha$  decrease, low pressure region near north east corner, decreases in size.
3. Both u-velocity and v-velocity increases at all points of the cavity with increase in value of  $\beta$ .

4. Pressure in the cavity increases with increase in value of  $\beta$ .
5. Decrease in value of  $\rho$  results in increase in  $u$  – velocity and decrease in  $v$ -velocity. The pressure in the cavity decreases with decreasing density  $\rho$ .
6. Temperature changes along north boundary with decrease in  $\rho$ .
7. Increase in  $\mu$  significantly increase  $v$ -velocity but results in decreases in pressure.
8. The velocity profile changes drastically with change in values of  $\alpha$ ,  $\beta$  and  $\rho$ .

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