Analysis of EEG Signal using nonextensive statistics

Pragati Patel, Ramesh Naidu Annavarapu

Department of Physics, School of Physical, Chemical, and Applied Sciences, Pondicherry University, Puducherry, 605014,

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Abstract - *Electroencephalogram (EEG) signal is the most effective, quick, and abundant source of information in understanding the brain related phenomenon. New avenues for EEG-based research in non-medical streams can also be seen with the growing number of qualitative and affordable wearable EEG headsets. But it is extremely hard to assess the information from EEG signal. However, information-theoretical approaches have appeared as a potentially beneficial means to gauge variations in the EEG datasets. This article discusses one such approach: the 'measure of Tsallis entropy (TsEn)' to explore and investigate the available natural data. This study set out to critically review the renowned research papers on Tsallis entropy-based EEG signal processing to understand the trends in EEG signal processing research. It attempts to provide practitioners and researchers with insights and future directions for applicability of Tsallis entropy for EEG signal processing and with an emphasis on the suitability of EEG research for clinical studies. It reviews about 35 published papers dividing into medical and non-medical domains and discusses the crucial role of Tsallis parameter 'q' in studying complex EEG systems. The result shows Tsallis's non-extensive initiatives seem to be more discriminatory than its Shannon counterpart and all other entropy variants and hence, can preferably be used to study the brain. The paper also concludes that Tsallis entropy offers a comprehensive test of any theory and it proves the efficacy of EEG research in clinical detection and therefore is highly significant in biomedical signal processing.*

***Key Words*: Tsallis Statistics, Non-Extensive Entropy, EEG, EEG data-sets, Biomedical Signal Processing.**

1.INTRODUCTION

In 1803, a mathematician Lazare Carnot developed the entropy theory when he saw that vitality is reduced due to friction and scattering[1, 2]. This entropy concept was mainly used in two branches of physics; statistical mechanics and thermodynamics, in the earlier days of its invention. Later in 1948, this thermodynamic entropy was introduced as data entropy into the world of data analysis by Shannon[3]. Shannon entropy is precisely developed from the Boltzmann-Gibbs (BG) statistical mechanics and standard thermodynamics and was proved to be efficient in the study of the complexity of systems[4, 5]. Despite their colossal effectiveness, this BG concept-led entropy discusses only extensive structures with short-range interactions and fails for nonextensive systems. In 1988, Tsallis proposed an entropic expression with an index q that results in non-extensive statistics. Tsallis entropy, E_q, builds the foundation of non-extensive statistical mechanics. Numerous phenomena have been studied using non-extensive (Tsallis) statistics in a variety of fields, including physics, chemistry, biology, medicine, economics, geophysics, etc. This article focuses on application and significance of Tsallis entropy in EEG data analysis [6][7].

The study of complex structures has drawn significant interest lately, and the brain being the most complex among them. There are various neuroimaging or brain scanning techniques to directly or indirectly image the brain's structure, function, or pharmacology. Owing to its excellent temporal resolution, electroencephalography (EEG) seems to be the most advisable method for studying the temporal variation of brain activity[8]. The brain's electrical activity characterized by EEG is indeed very complicated. Retrieving the right characteristics from this time series is crucial in brain-related research. There is a range of linear and non-linear approaches available to study EEG time series[9]. However, information-theoretical techniques, precisely the nonextensive entropy-based method, recently appeared as a most promising approach for retrieving reliable information from EEG. Tsallis entropy being the most effective and robust information theoretic technique, can lay the foundation for a real-time decision-making aid in many fields, considering its arithmetic coding is quick[10–12]. Tsallis entropy has proven useful in characterizing systems with long-range interactions in the past thirty years[13–20]. This study attempts to review this nonextensive entropy's applicability to EEG analysis for different purposes.

1.1 Tsallis entropy in Biomedical Signal Processing

Often, biomedical signals are disruptive and inconsistent. They generate databases that are high-dimensional and complicated. The outcomes of conventional biomedical data analysis techniques may be affected by the existing interference or intrusion in the data. Considering this problem, nonextensive information entropy has emerged as a reliable estimator of complexity or uncertainty in the signal with various implementation scope. Bock et al. have formulated an early detection strategy for

Alzheimer disorder that takes Tsallis entropy as an attribute and results in an accuracy of 77%[12]. Zhang et al. provided a feasible study to determine the occurrence of bursts and suppressions after cardiopulmonary infarction in rats via Tsallis entropy application to EEG. In his research, he discovered that Tsallis Entropy Area (TsEnA) correlates well with neurological outcomes, and with the value of q set to 3, it can contribute to preliminary diagnostic estimates about the retrieval of cerebral function quickly and efficiently[21]. As in [22], authors have employed the Tsallis entropy feature to estimate rhythm change in EEG following the brain ischemia. The study was carried out using q = 1.5, 3, and 5 with four EEG recordings to discriminate asphyxia and early recovery period from the base-line EEG. Results suggest the entropy used can quantitatively detect changes produced by variations of experimental settings. Liang et al. published an extensive study on entropy measures to monitor the depth of anesthesia. The method employed twelve indices along with Tsallis entropy, and outcomes suggest Tsallis and Renyi permutation entropy performed better than Shannon's entropy in evaluating the effect of anesthesia[23]. S. Kar et al. used Tsallis entropy for EEG signal analysis to assess and quantify driver's fatigue, indicating that the described approach can be applied onboard to determine the fatigue level in drivers of any field[24]. Robert Richer et al. carried out a study on mental state identification in real-time using Tsallis entropy. Authors found that Tsallis had the maximum sensitivity whereas Renyi had maximum specificity. As a result, the focused and relaxed state is best determined by Tsallis and Renyi based entropy measures respectively[25].

1.2 Brain Mapping With EEG

An EEG plays a role in diagnosing brain disorders and also for the advancement of EEG based brain computer interface systems. Electroencephalography offers a way to investigate the workings of the brain and to map links of one region of the central nervous system to other. EEG is utilized in various research protocols, medical and non-medical. It has been proven highly favorable as a diagnostic aid in patients with severe brain injuries, brain tumors, encephalitis, memory problems, stroke, cerebral infections, sleep disorders, epilepsy, and multiple degenerative diseases of the nervous system. It is also effective in the evaluation of patients with possible brain death. When in a drug induced coma, EEG is advantageous to estimate the proper depth of anesthesia in patients. EEG is conventionally conducted in well-controlled laboratory environments, but with recent technological developments, portable monitoring and long-term monitoring EEG have also become widespread. The present paper also explores the significance of EEG research in studying the brain.

1.3 Mathematical Background

A system's complexity can be defined using entropy. Since 1948, Shannon's entropy is the fundamental and most widely used entropy to evaluate system complexity. Mathematically, Shannon's Entropy is[3],

$$E_{Sh} = -\sum_{i=1}^{N} P_i \ln P_i \tag{1}$$

Where N are the microscopic configurations of the system and P_i is the probability of occurrence of the i^{th} configuration. The sum of the probabilities should be unity, i.e., $\sum_i P_i = 1$. E_{Sh} is based on standard thermodynamics and Boltzmann-Gibbs statistical mechanics, and therefore, the microscopic memories and interaction are of short range[26]. E_{Sh} is additive, meaning,

$$E_{Sh}(X \cup Y) = E_{Sh}(X) + E_{Sh}(Y) \tag{2}$$

The system X and Y are independent.

$$P(X \cap Y) = P(X)P(Y)$$

Shannon's entropy has had remarkable achievement in the interpretation of the extensive systems. Despite that, it cannot properly describe abrupt changes and long-range interactions[27]. To get over this disadvantage, a non-additive statistic was proposed[6, 26], the Tsallis entropy. It was defined as

$$E_{ts} = \frac{1-\sum_{i=1}^{N} P_i^q}{q-1} \tag{3}$$

When $q \rightarrow 1$, E_{ts} reduces to the definition of E_{Sh} as:

$$E_{ts} = \frac{1-\sum_{i=1}^{N} P_i^q}{q-1} = \sum_{i=1}^{N} P_i \frac{P_i^{q-1}-1}{1-q}$$

$$= \sum_{i=1}^{N} P_i \frac{\epsilon^{(q-1)\ln P_i}-1}{1-q}$$

$$\approx \sum_{i=1}^{N} P_i \frac{[1+(q-1)\ln P_i]-1}{1-q}$$

$$= \sum_{i=1}^{N} P_i \ln P_i \quad (4)$$

E_{ts} is nonextensive and follows the following rule of pseudo additivity[26]

$$E_{ts}(X \cup Y) = E_{ts}(X) + E_{ts}(Y) + (1-q)E_{ts}(X)E_{ts}(Y) \quad (5)$$

In the above equations, q is a parameter that measures the degree of nonextensivity[28]. q = 1 corresponds to extensivity, that is, Shannon's entropy. On the other hand, q < 1 corresponds to superextensive, $[E_{ts}(X \cup Y) > E_{ts}(X) + E_{ts}(Y)$ and q > 1 corresponds to subextensive $[E_{ts}(X \cup Y) < E_{ts}(X) + E_{ts}(Y)]$ statistics. Tsallis' Entropy is extreme at equiprobability. This extremum is described as[26]

$$E_{ts}^{ext} = \frac{N^{1-q}-1}{1-q} \quad (6)$$

In the limit q →1 this reduces to the extremum of Shannon's Entropy

$$E_{sh}^{ext} = \ln N \quad (7)$$

The work of Tsallis provides a generalization of Boltzmann-Gibbs statistics, which can appropriately explain systems with long-range interaction[26]. Though this generalization has been known from former centuries in the thermodynamic context, it is now gaining broader applicability. Tsallis Entropy has recently been extensively adopted in biomedical fields like ECG[29] and EEG[5, 30] signal processing. Research findings have shown that Tsallis entropy can yield the system's enhanced technicalities than the traditional Shannon entropy[5, 29, 31]. EEG offers information about large scale interaction of billions of neurons. Due to these long-range correlations, nonextensivity is instinctive in EEG[32]. Thus, using a nonextensive approach rather than the conventional Shannon entropy approach is reasonable and essential to perspective the long-range consequences in the EEG dataset[33]. Moreover, since there is associative information between the various neural populations, it is fair to classify EEG as a sub extensive framework[5, 30, 34]. For instance, consider two neuronal clusters X and Y, then entropy relationship of the two systems is stated as

$$E_{ts}(X \cup Y) < E_{ts}(X) + E_{ts}(Y) \quad (8)$$

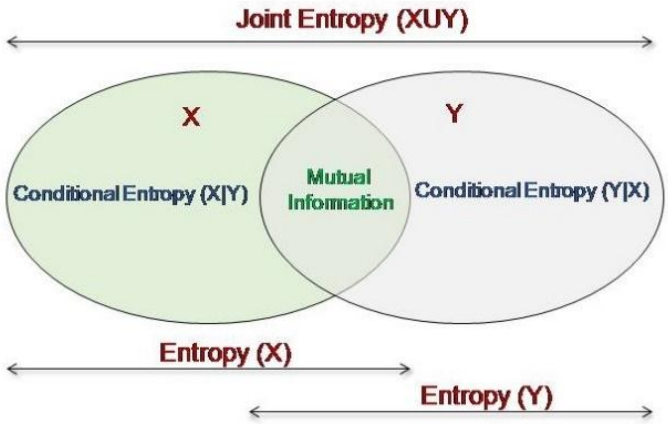

Fig-1: Relationship between the entropy of the systems X and Y.

1.5 Main Contributions of The Paper

During our literature review, we observed that articles on Tsallis entropy cut across various kinds of EEG processing. Hence, conducting an analysis of articles comprising of EEG processing using Tsallis entropy could be of great use for literature reviews to researchers. In short, contributions of this article are:

i. All possible research works that have employed Tsallis entropy feature to analyze EEG signals to study efficacy of Tsallis entropy and EEG in medical and non-medical domain have been taken into account in this review.
ii. From analysis, it's been discovered that the non-extensive Tsallis entropy indeed performs better than its Shannon counterpart in many of its applications.
iii. This analysis will assist researchers as to why Tsallis entropy should be prioritized over other entropy features when dealing with any non-extensive systems.
iv. This will also assist learners in better comprehension of all EEG datasets available for medical and non-medical research.
v. Finally, through the present review and analysis, certain findings and opinions have been listed for further studies in this field.

The following sections illustrate our review framework for Tsallis entropy based on all possible published research articles. Section 2 presents our framework for review analysis. Results and discussion are discussed in Section 3. For future studies, Section 4 offers an insight for EEG research as well as Tsallis entropy application. Finally, Section 5 presents the study's conclusion by discussing the challenges and trends in Tsallis entropy-based EEG signal processing.

2. METHODOLOGY

Tsallis entropy for EEG processing has its vast application in both medical and non-medical fields and articles on the same can be found in different multidisciplinary journals namely: Physica A, Cluster Computing, Healthcare Engineering, Physics Letters, Biomedical Engineering, Frontiers in Computational Neuroscience, Medical Imaging and Health Informatics, Transportation Research, Pattern Recognition Letters, Clinical Neurophysiology, Neurocomputing, Entropy, Biomedical Signal Processing and Control. Therefore, classifying, analysing and summarizing the research at one platform will provide insights to its future aspects in EEG signal processing using Tsallis entropy. A generalized block diagram of the proposed methodology to review the `Tsallis entropy-based EEG signal processing' is given in Figure 2.

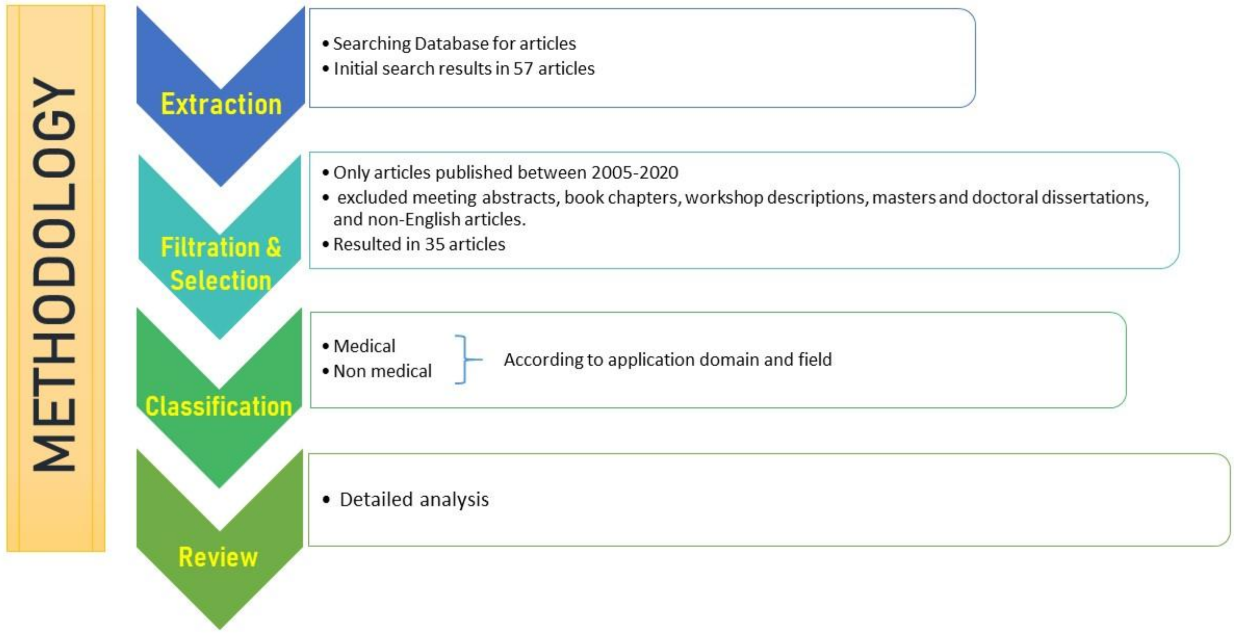

Fig- 2: Block diagram showing the review methodology used.

First step is extracting the relevant articles from various sources. We searched for papers in our selected repository across a period of twenty years, from 2000 to 2020. Second step is the filtration and selection which had strict inclusion and exclusion criteria, as follows

Inclusion Norms: Publications for getting selected for the review need to be part of the relevant conference proceedings or must be searchable through Google Scholar search engine. The first 200 articles featuring `EEG data processing' and `Tsallis entropy' in it, are only selected from the search engine method. All the selected articles are from years 2000 to 2020.

Exclusion Norms: Articles that are not relevant to the Tsallis entropy are ruled out, as defined by the prescribed sequence of manual analysis; Initially, publications that specifically do not involve Tsallis entropy related research work are disqualified. Then, all conference proceedings and Google Scholar publications are eliminated on the basis of their title and, lastly, their entire content.

Result: In total, 88 articles were selected from Google Scholar search engine and conference proceedings using the described selection norm and then further filtered down according to exclusion rule, resulting in 35 articles. Next step is the classification of the article based on their application domain which makes the further analysis easier and effective. All 35 articles are classified into two domains; medical and non-medical. Details of the classification scheme is illustrated in figure 3.

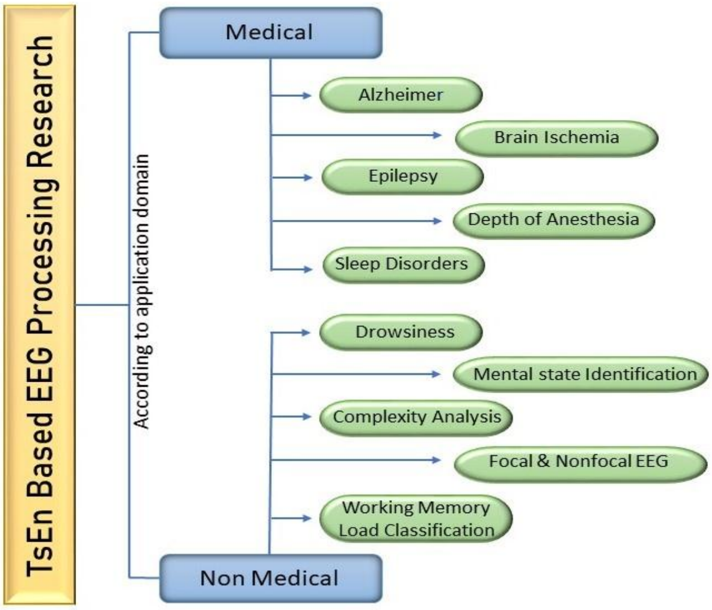

Fig- 3: Classification scheme

After classification, final step is to analyze and review it, which is discussed in the next section.

Table -1: Description of all the EEG datasets analyzed using Tsallis entropy in various studies.

Dataset	Description	References
Brain Ischemia	Dataset to explore neuronal activity in three different stages related to cardiac arrest; normal, hypoxic, and asphyxic. 15 adults male Wistar rats (300-350g, mean = 330g) have been arbitrarily given 7 minutes (10 rats) or 9 minutes (5 rats) of asphyxial attacks, as per the experiment decorum sanctioned by the Animal Care and Use Committee of John Hopkins Medical Institutions. To guarantee its no major impacts on EEG dataset, 4.5% of halothane was injected to each rat. Baseline EEG was recorded for 10 min followed by 5 min washout, finally followed by 7 or 9 mins of global asphyxia. Subjects were maintained at a temperature of 36.5-37.5°C. Two channels of EEG were recorded in the left and right parietal areas with a ground electrode in mid line. Advanced CODAS data acquisition software was used to digitize the signals obtained from EEG electrodes, at a sampling rate of 250hz and A/D conversion of 12 bit. The EEG datasets recorded were then filtered with a band pass filter of 0.5-70hz and ECG artifacts were removed. The data can be collected directly from the authors of original articles.	[22, 30, 35–37]

Epilepsy	**BERN BARCELONA DATABASE:** Dataset includes long-term intracranial EEG data recorded from five longstanding pharmacoresistant temporal lobe epilepsy patients at the Department of Neurology, the University of Bern. Multichannel EEG device was used with an extra cranial reference electrode placed between 10/20 positions to record data. Data was then down sampled at sampling frequency of 512Hz. Brain areas with seizures were localized for all five patients. At last, fourth-order Butterworth filter was applied to filter data of frequency 0.5-150Hz. Data available to download at: https://www.upf.edu/web/ntsa/downloads **BONN UNIVERSITY DATABASE:** This database includes data from three stages of epilepsy; the normal (Set A (eyes open) and Set B (eyes closed)), the pre-ictal (Set C and Set D) and the one with seizures (Set E), which have 200, 200 and 100 data points, respectively. Each data point is 23.6 seconds long and is recorded from a single EEG electrode. The complete data was recorded with a 128 electrode EEG device at a sampling frequency of 173.61 Hz, and digitized with a 12 bit A/D resolution. It was then filtered with a 0.5-85Hz band pass filter. Data and its details available to download at: http://epileptologie-bonn.de/cms/upload/workgroup /lehnertz/eegdata.html	[38–41]
Alzheimer	**DERRIFORD HOSPITAL DATABASE:** It comprises of two datasets recorded at Derriford Hospital (Datasets A and B). Dataset A has EEG recordings from 3 patients with Alzheimer's disease and 8 healthy individuals (over 65 years old), all of them from similar age groups. Dataset B comprises of 24 healthy individuals and 17 individuals with a probability of having Alzheimer's, and are not from the same age group. The mean age in normal groups is 69.4$\pm$11.5, with a minimum of 40, maximum of 84, and 42% of male in them. The mean age in the Alzheimer category is 77.6$\pm$10.0, with the minimum of 50 and maximum of is 93, of which 53% are male. Dataset A was recorded using the traditional 10-20 system in a Common Reference Montage and converted to Common Average and Bipolar Montages in software. Dataset B was recorded using the modified Maudsley system that is similar to the traditional 10-20 system. The EEG recordings were sampled at 128Hz in both datasets and include different states like awake, hyperventilation, drowsy and alert, with period of closed and open eye. Data can be accessed from hospital with strict protocol.	[10, 42–44]
Sleep State Separation	Data consists of EEG recordings from 20 healthy newborn infants (10 boys and 10 girls) aged between 282 $\pm$ 9 days, during four behavioral states; quiet sleep (QS), active sleep (AS), quiet wakefulness (QW), and active wakefulness (AW). The EEG was recorded using 19 electrodes of EEG for 108.4 ± 9.6 minutes (mean ± SD) of duration. EOG and ECG were also recorded for artifact reduction from data. The EEG signal was amplified using a REFA-72 amplifier (TMS International B.V.), and recorded at 256 Hz sampling rate with 69 Hz bandwidth.	[45]
Depth of anesthesia	**EEG dataset during sevoflurane-induced anesthesia:** The first dataset comprises of 19 patients aged between 18 to 63 years who fasted and did not undergo premedication for at least 6 hours before anesthesia. These subjects were to undergo for elective gynecologic, general, or orthopedic surgery. Prior to have EEG recordings, an electrode-skin impedance of less than 7.5 kΩ was maintained. A three electrode EEG device was used to record signals from FpZ (active), Fp1 (earth), and F8 (reference). The readings for sevoflurane concentration was taken at the mouth at 100/s[46]. **EEG data set during isoflurane-induced anesthesia:** The second dataset comprises 29 patients (9 men and 20 women, aged between 33 and 77 years) undergoing elective abdominal surgery during combined isoflurane general anesthesia and epidural anesthesia. Five EEG electrodes (A1, A2, FP1, FP2, and	[23]

	FPz(ground electrode)) were placed on the patients according to the International 10-20 System prior to anesthesia induction. The EEG data was sampled at sample frequency of 512 Hz. Isoflurane concentrations were intentionally kept at fixed levels (1.5, 1.3, 1.1, 0.9, and 0.7%) for 30 minutes at each level. The EEG recordings at 0.3 and 0.5% isoflurane were recorded immediately after the operation[47]. The two sets of data described above can be collected directly from authors of the respective articles	
Fatigue/Drowsiness	It consists of EEG data from multiple participants during a set of three actual and simulated scenario experiments, which were recorded using a 32-channel EEG device. **Experiment 1** consists of 21 healthy male participants (professional drivers) aged between 25-35 years. These participants were asked to drive for 1 hour in a busy traffic then underwent a computerized subjective and psycho-motor test. The EEG data recorded before and after the experiment are labeled as 'Level 1' \& 'Level 2' respectively. **Experiment 2** consists of twelve healthy male participants aged between 20–35 years for simulated driving tasks with sleep deprivation. Participants underwent physical exercise on a treadmill for 2–5 min to generate physical fatigue; simulated driving for about 30 min to generate physical, visual, and mental fatigue; auditory and visual tasks for 15 min to generate mental and visual fatigue; finally, the computerized game related to driving for about 20 min. A single phase of experiment lasted for a period of 3 hours and the complete experiment ran for approximately 36 hours. Three minutes of EEG data was recorded at start and end of each phase of experiment. **Experiment 3** consists of 7 subjects who performed actual driving tests for validation. The data was sampled at 256 Hz with 16-bit A/D conversion. Data can be accessed by asking the authors.	[24, 48]
Mental State Recognition	EEG data were recorded using a Muse Headband having 4 active electrodes and a common mode reference electrode. The recorded EEG was then separated into five different frequency bands. Data was originally over-sampled at 12 kHz and then down-sampled at 220 Hz. Complete data was recorded in an office space from eleven participants aged between 28.1$\pm$4.6 years (M$\pm$SD). The study procedure consisted of multiple tasks like Mental Arithmetic, Dictation, Where's Waldo. Each task is associated with one of three states of mind: Neutral, Focus, Relax. The Neutral phases are reference measurements with no specific instructions given, except not to close their eyes. For Focus phases, the participants performed different tasks to generate high levels of mental processing and binding different senses. During the Relax phases, participants relaxed themselves. Data can be accessed by asking the authors.	[25]
CAP(Cyclic Alternating pattern)	The dataset is recorded from four normal individual and four patients with sleep-disordered breathing. The EEG data is sampled at a sampling frequency of 512Hz. The dataset can be accessed from online data repository Physionet at https://physionet.org/content/capslpdb/1.0.0/	[49]
Working Memory Load	It contains data from 12 healthy male subjects aged between 24-30 years while they did an arithmetic task of varying difficulty level. The difficulty level was manipulated by varying the n-digit numbers used and carries required to calculate the addition. In the baseline rest condition data taken from relaxed subjects with their eyes closed. To minimize any muscle movement artifact (EMG) any unnecessary physical movements were avoided with their hand placed in a fixed position. EEG signals were recorded using 32 EEG channels device with a linked earlobe reference and impedance kept under 5kΩ. The EEG signals were then filtered using band-pass filter of cut-off frequency of 0.1-100 Hz and downsampled at a frequency of 256 Hz.	[50]

Parkinson's disease	Data consists of EEG recording from 62 subjects; 46 persons with Parkinson's disorder labelled as "PD" and 16 persons without Parkinson's disorder, matched to patients according to sex, age, and education, and labelled as "HC". Parkinson's disorder was diagnosed according to the United Kingdom Parkinson's Disease Society Brain Bank criteria. The patients who had dementia, history of stroke, epilepsy, multiple sclerosis and surgical interventions to the brain, or/and insufficient knowledge of German language, were excluded.	[51]

3. RESULTS AND DISCUSSIONS

We collected several articles on Tsallis entropy-based EEG signal processing via numerous web-based sources. Every selected article was analyzed and categorized as per the classification method discussed above. Though the range of the article collection was narrow, it provides a detailed overview of Tsallis entropy-based EEG signal processing research. All the articles selected after filtering process were further sorted based on their research themes.

Articles based on their area of implementation, can be broadly categorized and described into two categories: Medical and Non-medical.

Medical Context: Article is clinically contextual if it is intended to provide aid, improvement, tracking, evaluation and diagnosis of human mental and neurological disorders.

Non-Medical Context: Comprises of articles that are intended to amuse, instruct, train, track, or enhance gaming experience and e-learning through Tsallis entropy analysis of EEG database.

In order to identify the publications in the medical field, the following criteria were used:

a. If the article addressed a health condition, such as a mental or neural disorder;
b. If article either enrolled real patients as the subjects of the study, or involved two groups: one comprising healthy individuals, others of patients;
c. If the study was performed in a clinical environment;
d. and/or the article was oriented to provide new method of aid to any health conditions.

23.51% of all the articles selected were found as medical articles, after reviewing it all as per the medical classification criteria mentioned above. The majority, 76.5%, were non-medical. Figure 4 demonstrates the number of medical and non-medical publications with their year of publication. Also, all possible data used have been described under table 1.

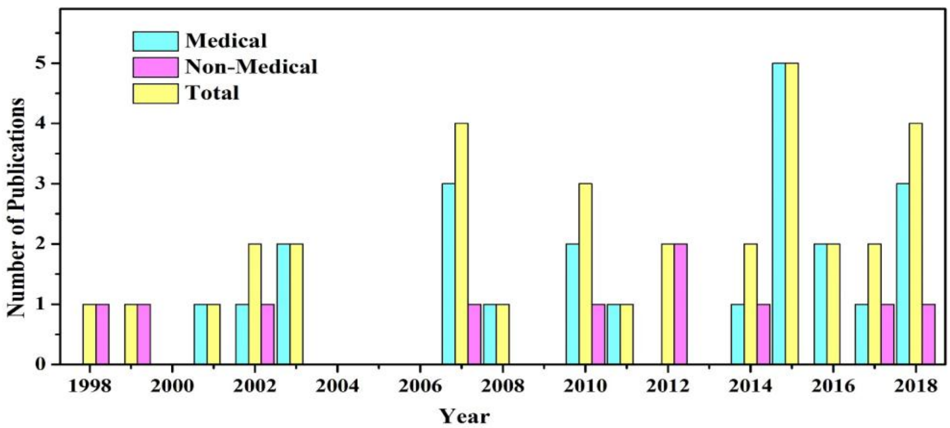

Fig- 4: Number of medical and non-medical articles with their year of publication.

3.1 Medical context Articles

3.1.1 Brain Ischemia

Approximately 60% of all individuals who safely revived a cardiac arrest eventually die each year from traumatic brain damage. The real time surveillance of the brain asphyxia or hypoxia condition after heart failure survival is still a crucial health concern. There aren't any standard scientific real-time objective evaluations being used to track these brain injuries.

Zhang et al.[37] in his work used Tsallis entropy to quantify the level of brain ischemia after cardiac arrest based of presence of burst suppression. In order to construct a standardized TsEn dependent measure, named TsEn area (TsEnA), Zhang combined three discriminatory Burst Suppression (BS) characteristics, i.e., burst frequency, BS ratio of amplitude (BSR), and BS length (duration). EEG bursts and suppression are a representation of cortical neuron excitability and refractory time, respectively[52–54]. Previous findings show that: 1) the burst frequency is greater for participants with positive neurological outcomes[55, 56], and 2) the EEG shows suppression phase i.e., it is flat or steady for negative neurological outcomes[37, 57]. Therefore, the quicker the normalization of cortical neuronal activity, the shorter the length of BS in EEG. With rise in BSR, TsEn falls monotonically and significantly and this reduction of TsEn is more prevalent in any living body with impaired neurological performance. Findings from Zhang's study shows that TsEnA correlates with neuronal outcomes around q value of 3 and can therefore offer early prognostic information on cerebral functional recovery accurately and effectively.

S. Tong et al. employed Tsallis and Renyi entropy approaches in his research to analyze brain electrical activity after hypoxic-ischemic (HI) injury[30]. The study revealed that Tsallis entropy was capable of recognizing the variations in local irregularity generated by the "spiky" burst behavior during initial asphyxic recovery, whereas Renyi entropy has been the most concise and predictive measure of physiological changes in the brain during recovery, and was shown to be sensitive to assess the rate of recovery.

Bezerianos et al. in his study of EEG rhythm changes used Shannon and Tsallis entropy ($q = 1.5,3,5$) with window size $w = 128$ data points, sliding steps $\Delta = 1$ and amplitude partition $L=10$.

In his work, four EEG recordings were analyzed and significant differences between base-line EEG and EEG after asphyxia up to the initial phase of late recovery was observed statistically. It was found that Tsallis entropy can discriminate the different injury levels and different segments of recovery; the asphyxia period~silent period as well as the early recovery burst~suppression EEG successfully from the base-line EEG[22].

3.1.2 Alzheimer's Disorder

Alzheimer's is a neurodegenerative condition that affects the nerve cells and is the leading cause of dementia, which involves a variety of syndromes such as loss of memory and trouble with thinking, balance or concentration, problem-solving, or speech and learning. Proteins accumulate and develop plaques or tangles inside the brain during this disorder. That weakens the bond between nerve cells, and consequently the nerve cells die and brain tissues are lost. Almost 44 million individuals worldwide have Alzheimer's or associated dementia. Just 1 in 4 is diagnosed with it. While an effective therapy or medication for Alzheimer has not yet been found, it is plausible to decrease its consequences if identified at an early stage. The testing of Alzheimer is currently conducted through an electroencephalography. As bio signatures of dementia, information theoretic approaches have appeared as a hugely powerful method of measuring variations in the EEG. Tsallis entropy leads among the most prominent theoretic information methods for assessing variations in the EEG.

P Zhao et al. have researched about EEG interpretation in Alzheimer's disorder using information theoretic approaches. He took into account the two datasets A and B, then used LZW (Lempel-Ziv-Welch) algorithm and the Tsallis entropy model to yield compression ratios and normalized entropies for each participant, respectively. The findings suggest that both parameters (compression ratio and normalized Tsallis entropy) are lesser in Alzheimer's patient than for the healthy individual group and indicate that information theoretic approach produces a highly beneficial technique to produce EEG markers for Alzheimer[10].

The use of Tsallis entropy as a screening tool for Alzheimer's disorder has also been studied by A.H. Al-nuaimi et al. For each EEG electrode and for every participant from a reference dataset that includes dementia and normal individuals, the Tsallis entropy was calculated. To highlight the variations between the entropy for normal individual and dementia patients, the entropy values calculated were then normalized. These normalized Tsallis entropy values were used to construct reference feature vectors, one for dementia patients and one for normal individuals. And thereafter, to differentiate between Alzheimer patients and healthy or normal individuals, the feature vector for the new dataset was compared with the reference vectors using K-means clustering. In order to obtain the Alzheimer's biomarkers, the entire EEG processing was split into two phases (development phase and test phase) and Tsallis entropy was estimated in both phases. With the sensitivity of 100% & 85.7%, specificity of 50% & 70.9%, accuracy of 84.6% & 78.8% and precision of 72.7% & 77.4% for dataset A & B respectively, it can be implied that the method can form the basis as a first line decision - making aid for dementia evaluation[42].

P. Lazar et al. carried out a work on Alzheimer's EEG analysis in empirical mode decomposition (EMD) domain using Tsallis thresholding i.e. EMD was implemented as an alternate approach to process Alzheimer signal. The accuracy and reliability of EMD depends solely on a threshold parameter, for which Tsallis entropy has been used in his work. The input EEG signal was decomposed by EMD into finite intrinsic mode functions and a residue signal, then with use of Tsallis entropy the optimal threshold value was calculated. Further, a neural network, which is an information processing paradigm, is trained, which gives the classified output either as AD-affected or as normal brain. The results show that classification rate is improved with Tsallis entropy-based thresholding[44].

3.3.3 Epilepsy

Epilepsy is a neurological condition in which repetitive episodes of seizures occurs. While seizures can be identified by means of an electroencephalogram (EEG), it is quite hard to distinguish the various stages of epilepsy like normal, preictal and ictal by visual observation, because of the non-linear and non-stationary characteristics and also the long-term recordings of the EEG signals. In literature, numerous studies have been performed to auto-detect the seizures for both short-term and long-term EEG recordings, Tsallis entropy as one of the approaches.

M. Thilagaraj et al. in their study used Tsallis entropy feature with five different classifiers; Naïve Bayes classifier (NBC), radial basis function (RBF), decision trees: functional tree (DTFT), KNN and adaboost. Tenfold cross validation technique was used for evaluating the classifiers with objective to classify abnormal and normal states. Study concluded that Tsallis entropy is simple yet the fastest method with the least computation time of 0.9ms and can be considered for real time detection tasks as it offers 92.67-100% accuracies for binary (two-stages) problems[41].

N. Arunkumar in his study for automatic detection of epileptic seizures have employed Tsallis entropy features together with other features like permutation entropy and Kolmogorov complexity along with five different classifiers. Maximum accuracy of 89.33% was achieved with sensitivity and specificity of 85% and 84% respectively, with DT classifier. However, the method has been tested on a limited size of data and needed to get checked with other large database for consistent result[38].

Kai Fu et al. used Tsallis and other entropy features along with SVM classifier with RBH kernel in the Hilbert marginal spectrum domain instead of the commonly used Fourier spectrum domain for automatic seizure detection. The outcomes of the study show that the marginal spectrum approach offers higher precision than Fourier spectrum analysis. It seems from the findings that the approach suggested is not substantially better than the Fourier one for parameters other than accuracy. However, since the results of the classification correspond to the dataset, feature extraction and learning algorithm apart from the methods of signal analysis, it is assumed that the method discussed is a potential tool for detecting seizures in EEG signals[40]. All the works discussed above have used Bonn university dataset, details of which is given in Table 1.

3.1.4 Depth of anesthesia

General anesthesia is essential in the surgery unit to ensure the safety, comfort and successful surgery of patients. Anesthetic drugs majorly target the central nervous system (CNS) which control most bodily functions. The electroencephalogram (EEG) which originates in the CNS represents the neuronal functions of the brain and has been extensively used as a reference parameter to measure the effect of anesthesia. The mean amplitude of the EEG steadily increases with deepening anesthesia and then this amplitude decreases in the recovery state. With the emergence of data analysis, different approaches have been used to assess the effects of anesthesia.

Zhenhu liang et al. in his work to evaluate depth of anesthesia using entropy used Tsallis wavelet entropy and Tsallis permutation entropy together with 12 other extensive and non-extensive entropy features. EEG dataset during sevoflurane and isoflurane-induced anesthesia has been utilized for this study, details of which is given in table 1. The results showed that Tsallis Permutation entropy (TPE) and Renyi permutation entropy (RPE) were better than Shannon permutation entropy in determining the effect of anesthesia. Similar findings were also seen in Tsallis wavelet entropy, Renyi wavelet entropy and Shannon wavelet entropy respectively. No research using TPE or RPE have ever been conducted to monitor the depth of Anesthesia before. The exceptional performance reveals their possible utility in studying depth of anesthesia[23].

3.1.5 Fatigue/Drowsiness

The term fatigue is used to describe a condition or state of mind that lacks alertness and have minimal mental or physical work outputs, usually followed by drowsiness. This Drowsiness can give rise to severe fatalities in certain profession like that of pilots, drivers, electric power and steel plant employees. It is therefore of critical interest and imperative need to formulate methods that can sense drowsiness/fatigues. Across a range of neuroimaging modalities which can be applied for fatigue detection, EEG is regarded as the most prominent and accurate one. Significant number of research articles are published to study the probability of implementing EEG device and its dataset to identify fatigues or drowsiness.

S.kar et al. in his paper studied various fatigue representing variables based on higher order entropy methods of EEG data in the wavelet domain. It's been reported that with the growth in fatigue rate, the entropy value rises. The work affirms that the extensive entropies (Shannon's Entropy and Renyi Entropy) have significant achievement in identifying the fatigue levels than the non-extensive ones (Tsallis entropy)[24]. Complete details of the data used for this work in given in table 1.

A. Zhang et al. also presented a study on Tsallis entropy approach to detect the drowsiness. In his work EEG data recorded from 20 healthy volunteers during the awake and drowsiness state has been used. All four types of EEG rhythms were subsequently extracted, and then complexity of each rhythm was studied with Tsallis entropy. The findings suggest that the Tsallis entropy of EEG theta rhythm in the awake state is significantly greater than the state of drowsiness. So, the Tsallis entropy of theta rhythm can be used to accurately detect the drowsiness in real-time[48].

3.1.6 Mental State Recognition

Information regarding inner mental states can be a precious asset in numerous cases. It could be utilized to avoid potential hazards like traffic fatalities by tracking mental fatigues in drivers. EEG headsets along with other wearable devices can be used to track psychological state in patients with certain psychological disorders like anxiety or depression. Other potential application includes the measurement of various forms of reaction in response to an external stimulus. It can be used to customize the content or style and to conduct social analysis if a person's reaction (whether emotional or rational) towards a speech or advertisement is known. Devices like MRI scanners or medical grade EEG are being used in most of the latest research on mental state monitoring. There is still a need for a solution that can determine the mental state of the consumer in real-time and in a regular, everyday environment.

R. Richer et al. in their work incorporate and examines various methods to detect different mental state in real time by generating scores that measure how focused or relaxed a person is, using a simple EEG headband. The data used is described in table 1. Data were pre-processed then naïve score computation and entropy-based score computation were used for mental state recognition. Entropy based measures included Shannon entropy, Renyi entropy (for alpha=3), Tsallis entropy (for alpha=3) and Kullback-Leibler divergence. The findings suggest that for the Focused state, the Tsallis-based method outperforms the Renyi-based method, while for the Relaxed state, the Renyi-based method performed best. It reveals that the Tsallis method provides the highest sensitivity, while the Renyi method reached the highest precision, and it also indicates that mental state identification can be done in real-time[25].

3.1.7 Cyclic Alternating Pattern (CAP)

Studies suggests that, through quantitative study of sleep database, various disorders can be identified. Identification and evaluation of cyclic alternating pattern (CAP) is an integral part of the sleep analysis. And it must be acknowledged that EEG analysis is the most effective approach to track cognitive function specifically during the sleep period. Sleep EEG database not only offers cyclic alternating patterns (CAPs) features but also other details of sleep phases such as sleep and wake states of rapid and non-rapid eye movement sleep. CAPs are intermittent activities in EEG that interprets the sustained arousal instability oscillating between higher and lower arousal stages characterized as phase A and phase B respectively. CAPs show variations in disruptive or unstable situations caused by internal (insomnia, depression, epilepsy, periodic limb movements, circadian constraints) and external (noise, ambient temperature) sources. The dataset used in this study is taken from Physionet data repository, furthers information is provided in Table 1.

F. Karimzadeh et al. examined a set of entropy-based methods along with three classifiers; support vector machine (SVM), k-nearest neighbor (KNN), and linear discriminant analysis (LDA) to differentiate CAP and non-CAP in an EEG database. In his research, the sleep EEG of 4 healthy volunteers and 4 patients were examined by the standard and proposed models to evaluate all of it. Entropy based features included Tsallis entropy together with Shannon entropy, spectral entropy, sample entropy, Higuchi fractal dimension, Kolmogorov entropy and band feature with Kolmogorov entropy were calculated. Then best relevant features were selected through a heuristic algorithm called sequential forward selection (SFS), then the three classifiers were used to classify the EEG signal. Findings shows that conventional features like band powers and Hjorth parameter can't measure the migration of brain state from CAP to non-CAP precisely and entropy can better detect the migration between these brain states. The results further show that Shannon entropy outperformed the Tsallis entropy, because range of variation of Shannon entropy is higher than that of Tsallis entropy[49].

3.1.8 Working Memory Load

Assessing the level of cognitive memory load while practicing a cognitive task is of utmost importance for various factors; to minimize the error in decision-making, for advancement of efficient user interfaces, to prevent memory exhaustion and retain efficiency and potency throughout any cognitive activities, also in professions with critical/high mental pressures; like people involved in the military and emergency/intervention medicine fields. Various approaches are available today for assessing working memory load, like behavioural/physiological techniques or performance-based/subjective ratings approach. Of these, EEG is ranked as the best physiological approach, providing better precision and tolerance when evaluating memory load. A number of linear and non-linear methods are being implemented to calculate the working memory load using EEG data.

P. Zarjam et al. in their study investigated the applicability of wavelet-based complexity assessment approach of EEG signals to examine variations in working memory load during the execution of a cognitive task of various degrees of difficulty/load. Wavelet-complexity assessments approach related with different entropy methods - Shannon, Tsallis, Escort-Tsallis and Renyi entropy have been employed to differentiate between seven load levels of working memory. Details of dataset used for the study is given in table 1. Source localization work suggests that task load levels mainly influence the frontal and occipital regions. For every entropy function, measured complexity data rendered that complexity value rises with the escalation in task load. Extracted features were then fed into a Artificial Neural Network (ANN) classifier. Tsallis entropy gave an accuracy of 94.18% in frontal region and 88.36% in occipital region, but lags behind the accuracy of Shannon's entropy[50].

3.1.9 Sleep State Separation

A variety of extremely complicated behaviors arise during sleeping stage in the brain. Significant research has recently been conducted to analyze the different sleep states and its link to all other psychological processes. Yet, information regarding sleep stages is hardly acknowledged. The number of individuals having sleep disorders is very high. Sleep disorders not only cause various health problems to the individuals but also harden their day-to-day activities to a great extent. Previous findings also reported that sleep may have a significant role in memory consolidation where some memories are developed while other lesser valuable memories are erased. Thus, providing an effective framework for sleep supervision and sleep behaviour analysis is of great significance. The automated identification of sleep phases from EEG dataset is a big challenge and several methods have been proposed for the same.

N. A. Loafgren et al. conducted a study on sleep stage separation via entropy estimation using a markov model of EEG. If the samples are correlated, the signal is more predictable and probability density function (PDF) of one sample will usually be affected by the previous sample. Therefore, direct entropy estimation will overestimate the entropy when consecutive samples are correlated. To approach this issue, the authors considered the signals as markov model. Then measuring entropy from this markov model of EEG transition matrix is taken into consideration which contains the information regarding the transitions among various states. Loafgren employed four algorithms; Shannon entropy, Shannon entropy of a markov model, Tsallis entropy, Tsallis entropy of markov model to study sleep stage separation. The results suggest that the sleep stage separation can be improved by using the extra information given in the transition matrix as compared to just using the probability density function. The separation expressed by the average T statistic is more than doubled when the entropy is based on a Markov Model of the EEG. However, the difference between the performance of Shannon entropy and Tsallis entropy is less pronounced[45].

3.1.10 Parkinson's disease

Parkinson's disease (PD) is the second most common neurodegenerative disease resulting primarily from the death of dopaminergic neurons in the substantia nigra. It is estimated to affect nearly 2 percent of those over age 65. According to the National Parkinson Foundation (NPF), over the entire course of their illness, 50 to 80 percent of those suffering from Parkinson's disease gradually develop dementia. The estimated duration from the occurrence of difficulties with mobility to dementia progression is around ten years. Accurately diagnosing Parkinson's Disease – especially in its early stages – requires experienced practitioners. Thus, providing tools for detecting early changes in brain activity that are as easy to use as taking one's own blood pressure is important.

S.M. Keller et al. carried out research on effectiveness of computational EEG in studying Parkinson's disease. The study was focused on how to complement and improve existing signal power-based classification methods for classifying individuals with Parkinson's disease from the healthy ones by (i) introducing additional entropy-based features (Tsallis entropy with q=2), (ii) including eyes open and eyes closed states into the measurement process and (iii) using the Berger effect as a potential feature indicative of abnormal brain activity. Dataset used for this work is described in table 1. Best result is achieved with a combination of eyes closed and eyes open measurements using Tsallis entropy (q=2) and band power features[51].

3.2 Role of Tsallis Parameter 'q'

Literature showed the significance of Tsallis entropy in extracting information from complex non-extensive systems. However, the Tsallis parameter 'q' is what renders the peculiarity to TsEn estimate. The Tsallis parameter 'q' functions like a zoom lens for the biomedical datasets like EEG which can reflect long as well as short-range (burst or spikes) rhythm changes in data with low and high 'q' values respectively. Inspite of the significant role of 'q' in information extraction from complex non-extensive systems through Tsallis entropy, no technique of optimizing its value has been developed yet. Analysts often undertake contrasting values of q to explore EEG or any other biomedical signals, and refine the range of q based on certain precedent and the nature of data being studied. Former studies on Tsallis studies also shows that with the parameter q, the value of Tsallis entropy reduces constantly whereas 'the spike-encounter-potential' increases with q. It can also be inferred that Tsallis entropy is associated to neuronal conditions and can thus be used to provide early diagnostic information accurately and effectively. Table 2 summarizes different values of Tsallis parameter 'q' in different works by the researchers, which can be useful for further studies using Tsallis entropy.

Table 2: Summary of all possible values of Tsallis parameter 'q' used in different studies

Study Area	Q value	References
Brain Injury from cardiac arrest	0.5, 1, 3, 5 1.5, 3, 4.5 1.5, 3, 5 3	D. Zhang et al. [37] S. Tong et al. [36] A. Bezerianos et al. [22] S. Tong et al. [30]
Fatigue/Drowsiness	2 0.6, 1.2, 3, 5	S. Kar et al. [24] A. Zhang et al. [48]
Epilepsy	2 5 2	N. Arunkumar et al. [38] A. Capurro et al. [5] K. Fu et al. [40]
Mental State Recognition	3	R. Richer et al. [25]
Alzheimer	0.5 0.5	C. Coronel et al. [58] Ali H. Al-nuaimi et al. [42]
Stroke-related Mild Cognitive Impairment & Vascular Dementia	2	N.K. Al-Qazzaz et al. [59]
qEEG for diagnosis accuracy	6	A. Lay-Ekuakille et al. [60]
Somatosensory system	10	P. Lazar et al. [44]
Working Memory Load	0.1, 0.9	N.A. Lofgren et al. [45]
Parkinson's disease	2	S.M. Keller et al. [51]

4.RESEARCH DIRECTION

Present review in the field of Tsallis entropy-based EEG analysis suggests ways of improving the research in two directions; first concerning the algorithm or the method, i.e., Tsallis entropy, and second in the field of EEG research.

For EEG to be perfectly suitable for clinical and non-clinical use, there are several obvious future directions. First of all is the optimization of the number of electrodes used. There are 32, 64, 128 and 256 electrode variation EEG devices available for study. Considering data from all the electrodes makes experimental time setup and the computation time an issue. Minimizing the use of the number of electrodes would be ideal for a more convenient application. The second direction of research addresses the size of the sample or datasets. As of now, EEG datasets for almost every research application is relatively smaller than needed. A larger dataset is needed to examine or efficiently analyze the neurological disorders or to produce and implement any algorithm. The study for certain disorders suggests the need for a standard procedure to record EEG signals in both eyes open and eyes closed state. Future works are needed in examining the already developed EEG indicators against known biomarkers for a larger dataset. Third, investigating the longitudinal changes in EEG over relatively short periods at the individual level is needed to enhance the EEG based research outcomes. Fourth, data fusion should also be considered a potential field of research for better diagnosis of disorders and advancement in EEG-based BCI applications.

Possible future directions concerning the method, i.e., the non-extensive Tsallis entropy, can be the method's validation in a more realistic environment and its extension in other undiscovered application such as in the development of real time emotion recognition system or any other BCI application where a significant improvement in accuracy is possible. Also, in studies concerning detection of diabetes in its early stages, autism, other mental disabilities like ADHD, dyslexia, schizophrenia. Future work will always be open to fuse or integrate other features with Tsallis entropy to expect better outcomes.

5.CONCLUSION

The complexity is a distinguishing yet enigmatic characteristic of any physiological system [61]. The stable or balanced systems show variation in complexity related to long range interactions within the system along with other different nonlinear interactions whereas the complexity disintegrates with instability or disorders in the system. There is still a great necessity of quantitative measures which can calibrate medical dataset in multiple ways, thereby simplifying the study of brainwaves obtained from complex brain dynamics. For multiple issues, the Tsallis entropy approximation accurately estimated the entropy of most EEG Signals. The role of Tsallis entropy in biomedical data analysis domain is crucial for numerous causes, primarily because the technique is employed to study real-world data as real-world data. This offers a thorough assessment of any theory than the simulated data. The background of EEG research constitutes a further explanation for the importance of TsEn in EEG findings. EEG studies have not historically developed any therapeutic approaches which are sufficiently reliable for its direct manifestation in medical diagnosis except for seizures and sleep disorders. Many researchers had surrendered upon its effectiveness and claimed that statistical significance of EEG in medical research is unattainable. Our present review, however, indicates that such a despondency cannot be upheld. Rather EEG research in medical diagnostics only lacked techniques like Tsallis entropy for assessing the information in the EEG data. The purpose of the review conducted is to demonstrate the importance of i. EEG research in the advancement of medical and BCI research, and ii. application of Tsallis entropy approach for the processing of existing intricate EEG signals. Study also concludes that despite of having high significance in complexity analysis of systems, optimization of Tsallis non-extensive parameter `q' remains to be studied further. Study also shows that non-extensive Tsallis entropy approach gives high distinction of various states of a system than its conventional counterpart i.e. the Shannon entropy, which further suggests the construction of fully automated spike detection sensors based on Tsallis entropy algorithm. A multi-diagnosis software package can also be developed and installed in different diagnostic centers. Prior to the implementations, however, the technique would have to be tested extensively for uniformity in precision, specificity and sensitivity with other diverse databases. In order to broadly assess the reliability of this method, it is still essential to acquire large databases.

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COMPETING INTEREST

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AUTHORS' CONTRIBUTIONS

Pragati Patel: Conceptualization, Computation, Formal analysis, Writing - original draft .
Contact Details: pragatipatel.001@pondiuni.ac.in, ResearchGate: https://www.researchgate.net/profile/Pragati-Patel-2, Linkedin: https://www.linkedin.com/in/pragati-patel-8076631b8/ , Orcid: 0000-0003-1256-8923

Ramesh Naidu Annavarapu: Reviewing, Editing and Supervision.
All authors read and approved the final manuscript.

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