

An investigation of Max-Min Fairness Power Control in Cell-Free Massive MIMO Networks

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ABSTRACT

The impact of max-min fairness power control on the performance of cell-free massive multiple-input multiple-output (CF-mMIMO) and small-cell mMIMO (SC-mMIMO) are studied in this work. We consider a CF-mMIMO network with multi-antennas at access points (APs) and single-antenna users in correlated channels. Then, the throughput of this network, data power control at the downlink and uplink transmission is exploited. The closed-form expressions for downlink and uplink throughput leading to max-min power control algorithms are used. Max-min power control ensures uniformly quality of service (QoS). The numerical results show that, the throughput of CF-mMIMO outperforms the network with SC-mMIMO. Moreover, additional antennas at APs improve the CF-mMIMO network's throughput when there are users in the network than SC-mMIMO.

Index Terms— CF-mMIMO, SC-mMIMO, throughput.

1. INTRODUCTION

Massive multiple-input multiple-output (mMIMO), where a base station with many antennas at the same serves many users in the same time-frequency resource, is a promising future wireless access technology that can provide high throughput and reliability [1]. The increase of the number of mobile end users has caused speedy development of mMIMO network known as cell-free massive MIMO (CF-mMIMO). Since CF-mMIMO combines the mMIMO, network MIMO, distributed MIMO, (coherent) cooperative multipoint joint processing (CoMP) and distributed antenna systems (DAS) concepts which means there are no cell boundaries or cells [2]. The cell-free massive MIMO is a scalable network of mMIMO, where a large number of access points (APs), which are geographically located and distributed [1-4]. CF-mMIMO acquires all benefits from mMIMO (favorable propagation when using multiple antennas at APs, and channel hardening [5]) and network MIMO (increased macro-diversity gain), and hence, it can offer very coverage probability and high throughput. These pros and cons can be achieved with local channel acquisition at each AP and simple signal processing. In addition, with the cell-free topology, the excessive handover issue in small-cell mMIMO (SC-mMIMO) networks can be resolved. Therefore, CF-mMIMO has attracted a lot of research interest currently [1-7]. To the best of our knowledge, the aforementioned studies the performance of CF-mMIMO and SC-mMIMO with single-antenna users.

Power control is an important aspect of CF-mMIMO in order to balance the effects of mutual interference and amplifying the power of the desired signals [8]. Power control is challenging since the power allocated to increase the quality of service (QoS) for one user will contribute to interference at the other users. Power control is central in CF-mMIMO since it controls the intra-user and the inter-user interference, near-far effects to optimize the objective or utility functions such as the max-min fairness, maximum product signal interference plus noise ratio (SINR), minimize inverse SINR and the sum Spectral Efficiency (SE) [3], [9-11]. Ideally, power control is done at the APs and users under the assumption that the AP perfectly knows all large-scale fading coefficients. Then, the optimal power control coefficients will be sent to the APs for the downlink transmission and to the Users for the uplink transmission. Power control in wireless networks has been studied for decades, but one big issue with existing algorithms is the complexity when deploying the algorithms in small-scale MIMO networks due to the fast variations of small-scale fading which require the power control to change very often [2]. The closed-form expressions of downlink and uplink throughputs are derived. Moreover, effect of the number of antennas at APs and users are exploited through the use of max-min fairness power control in this work. Globally optimal solutions computed by solving a sequence of second-order cone programs (SOCPs) for the downlink, and a sequence of linear programs for the uplink in [2, 8]. We devised max-min fairness power control algorithm that maximize the minimum of all user throughput. We quantitatively compare the performance of CF-mMIMO to that of SC-mMIMO networks under correlated shadow fading models.

The rest of paper is organized as follows. In Section 2, we describe the CF-mMIMO network model. The max-min fairness is presented in Section 2 and SC-mMIMO network is also discussed in Section 2. We provide numerical results and discussions in Section 3 and finally conclude the paper in Section 4.

2. CF-mMIMO NETWORK MODEL

We consider a CF-mMIMO network with M APs randomly located in a large area and K users. All APs and users are equipped with N antennas and single antenna respectively. Furthermore, central processing unit (CPU) connects all APs through a backhaul network as depicted Figure 1. We assume that all M APs with N antennas simultaneously serve all K users in the same time-frequency resource.

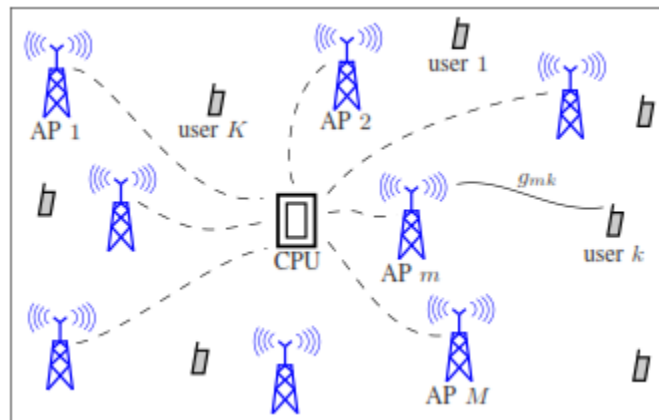


Figure 1: Cell-Free Massive MIMO network [2].

The transmission from the APs to/from the users proceeds via time-division duplex (TDD) protocol operation. Each coherence interval is divided into three phases: uplink pilot, downlink payload data transmission, and uplink payload data transmission. In the uplink pilot phase, the users send pilot sequences to the APs and each AP estimates the channel to all users [1-8]. The so-obtained channel estimates are used to precode the transmit signals in the downlink, and to detect the signals transmitted from the users in the uplink. In this work, to avoid sharing of channel state information between the APs, we considered the downlink (DL) and uplink (UL) channels.

$$y_{dl} = \sum_{m=1}^M (g_{mk}^m)^H s_m + n_{mk} \quad (1)$$

$$y_{ul} = \sum_{k=1}^K g_{mk} s_{mk} + n_{mk} \quad (2)$$

Let s_m represents the data signal transmitted by AP m to users, s_{mk} represents data signal transmitted from user k to AP m , and g_{mk} denote the channel coefficient between the k th user and the m th AP. The channel g_{mk} is modelled as follows:

$$g_{mk} = \gamma_{mk} h_{mk} \quad (3)$$

Where h_{mk} represents the small-scale fading, and γ_{mk} represents the large-scale fading.

M indicates the number of APs, $(.)^H$ indicates the Hermitian transpose matrix operator and the received DL signal $y_{mk} \in \mathbb{C}^{N \times K}$ at UE k in AP is modeled below:

$$h_{mk}^m = \begin{bmatrix} h_{11} & h_{21} & h_{31} & \dots & h_{m1} \\ h_{12} & h_{22} & h_{32} & \dots & h_{m2} \\ h_{13} & h_{23} & h_{33} & \dots & h_{m3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{m1} & h_{m2} & h_{m3} & \dots & h_{mk} \end{bmatrix}$$

$h_{mk}^m \in \mathbb{C}^{N \times K}$ is the mMIMO DL channel matrices, where \mathbb{C} denotes a complex value matrix, M is the number of BS antennas represents the number of rows in the matrix and K is the number of UEs represents the number of columns in the matrix, m superscript denotes the AP's index and mk subscripts denote k th UE in AP m .

There are K single-antenna UEs in the network and the channel between AP m and UE k is denoted by $h_{mk} \in \mathbb{C}^N$, where \mathbb{C} denotes a complex value matrix and N is the number of AP antennas.

$$h_{mk} = N_C(0, R_{mk}) \tag{4}$$

h_{mk} is the correlated Rayleigh channel and $R_{mk} \in \mathbb{C}^{N \times N}$ is the spatial correlation matrix, where mk subscripts represent k th UE in AP m .

Access Point (AP) transmitting Cooperation

All the APs are connected via fronthaul connections to a CPU that has high computational resources. Hence, the APs can be viewed as remote-radio heads that cooperate to support coherent communication with the UEs. The fronthaul can consist of a mix of wired and wireless connections, organized a star or mesh topology [1]; the methods developed in this paper can be applied with any fronthaul topology. AP m transmits the signal s_m in (5) and can use the available channel estimates to precode the data signals locally, or can fully or partially delegate this task to the CPU. The benefit of using the CPU is that it can combine the inputs from all APs, but this must be balanced against the required amount of fronthaul signaling.

Fully Centralized Processing Unit (CPU)

The most advanced level of CF-mMIMO operation is when the M APs send their received pilot and transmitted data signals from the CPU to users, which takes care of the channel estimation and data signal precoding. In other words, the APs act as relays that forward all signals from the CPU [12].

The transmitted signal available at the UEs from CPU is expressed as

$$\begin{bmatrix} y_{1k} \\ \vdots \\ y_{mk} \end{bmatrix} = \sum_{m=1}^M \begin{bmatrix} (h_{1k}^1)^H \\ \vdots \\ (h_{mk}^m)^H \end{bmatrix} s_m + \begin{bmatrix} n_{1k} \\ \vdots \\ n_{mk} \end{bmatrix} \tag{5}$$

The collective channel is distributed as $h_{mk} = N_C(0, R_{mk})$.

Small-cell (SC)-mMIMO Network

For SC-mMIMO network, we assume that each user is served by only one AP. For each user, the available AP with the largest average received useful signal power is selected. If an AP has already been chosen by another user, this AP becomes unavailable. The simplest implementation level is when the signal from user k is precoded by using only the transmitted signal from one AP. In this case, the precoding can be done locally at the AP by using its own local channel estimates without exchange anything with the CPU. This makes the network truly distributed [12] and essentially turns CF-mMIMO into a SC-mMIMO network. The macro diversity achieved by selecting the best out of many APs could potentially make it competitive compared to conventional cellular mMIMO with larger cells. CF-mMIMO and SC-mMIMO were compared in [2], [12] with $N = 1$ and an AP selection based on the largest large-scale fading coefficient. In addition to this, the authors impose that each AP can only serve one UE.

Achievable Uplink throughput: The central processing unit detects the desired signal s_{mk} from r_u in (6) and (7). We assume that the central processing unit uses only statistical knowledge of the channel when performing the detection. Using a similar methodology, we obtain a rigorous closed form expression for the achievable uplink rate as follows.

$$r_u = \sum_{k=1}^K g_{mk} y_{ul} \quad (6)$$

$$r_u = DS_{mk} \cdot s_{mk} + PR_{mk} \cdot s_{mk} + \sum(CI_{mk} + NI_{mk}) \cdot s_{mk} + n_{mk} \quad (7)$$

Achievable Downlink throughput: We assume that each user has knowledge of the channel statistics but not of the channel realizations. The received signal r_{dk} in (8) can be written as

$$r_{dk} = DS_{mk} \cdot s_m + PR_{mk} \cdot s_m + \sum(CI_{mk} + NI_{mk}) \cdot s_m + n_{mk} \quad (8)$$

Here DS_{mk} , CI_{mk} , NI_{mk} , PR_{mk} and n_{mk} are for desired signals, coherent Interference, non-coherent Interference, precoding gain uncertainty and additive noise respectively. We treat the sum of the second, third, and fourth terms in (1) as “effective noise”. Since s_m is independent of DS and PR, interference, $I = \{|CI_{mk}|^2\} + \{|NI_{mk}|^2\}$. Thus, the first and the second terms of (7) and (8) are uncorrelated. A similar calculation shows that the third and fourth terms of (7) and (8) are uncorrelated. Therefore, the effective noise and the desired signal are uncorrelated. By using the fact that uncorrelated Gaussian noise represents the worst case, we obtain the following achievable rate of the k th user for DL or UL operation:

$$SINR_{mk}^d = \frac{\mathbb{E}\{|DS_{mk}|^2\}}{\sum(\mathbb{E}\{|CI_{mk}|^2\} + \mathbb{E}\{|NI_{mk}|^2\}) + \mathbb{E}\{|PR_{mk}|^2\} + 1} \quad (9a)$$

$$SINR_{mk}^u = \frac{\mathbb{E}\{|DS_{mk}|^2\}}{\sum(\mathbb{E}\{|CI_{mk}|^2\} + \mathbb{E}\{|NI_{mk}|^2\}) + \mathbb{E}\{|PR_{mk}|^2\} + \mathbb{E}\{|N_{mk}|^2\}} \quad (9b)$$

$$SE = \log_2(1 + SINR_{mk}) \quad (10)$$

Network throughput (bits/s) value is obtained by the multiplication of operational Bandwidth (Hz) and SE (bits/s/Hz).

$$Network\ throughput \left(\frac{bit}{s}\right) = Bandwidth\ (Hz) \times SE \left(\frac{bit}{Hz}\right) \quad (11)$$

Channel Estimator

For efficient usage of AP antennas, each AP is required to acquire knowledge of the channels from the users which are active in the coherence block [12]. The AP m estimates the channels knowledge from its UEs. The channel estimates can be determined from the uplink pilot signal Y_m^p received at BS j , which can be defined as [12] and [13] expressed in equation (7)

$$Y_m^p = \sum_{k=1}^K \sqrt{p_{mk}} g_{mk}^m \vartheta_{mk}^T + \sum_{\substack{m'=1 \\ m' \neq m}}^{M'} \sum_{i=1}^K \sqrt{p_{li}} g_{li}^m \vartheta_{li}^T + N_m^p \quad (12)$$

The first part in Equation (7) denotes the desired pilots in the cell, the second part denotes the interfering pilots from other adjacent cells and then the third part N_m^p denotes the receiver noise.

Matrix $N_m^p \in \mathbb{C}^{M \times \tau_p}$ contains independent identically distributed elements which follow a complex Gaussian distribution with zero mean and noise variance σ^2 . p_{mk} is deterministic uplink pilot signal. In the channel estimation phase, the aggregated received uplink pilot signals at AP m are denoted as $Y_m^p \in \mathbb{C}^{M \times \tau_p}$ where τ_p is the length of a pilot sequence (and also equals to the number of orthogonal pilot sequences available for the network. The mutually orthogonal uplink pilot matrix τ_p was

organized as columns at AP m , $\vartheta_m = [\vartheta_{m1}, \vartheta_{m2}, \vartheta_{m3}, \dots, \vartheta_{mk}] \in \mathbb{C}^{\tau_p \times K}$ which were transmitted by the k th user of the AP m . All pilot sequences are assumed to originate from a predefined orthogonal pilot book in which sequence $\vartheta_{li}^T \in \mathbb{C}^{\tau_p}$ was defined as below [13] expressed in equations (13a) to equation (13b):

BS j correlates Y_m^P with ϑ_{mk}^* to estimate $y_{mk}^{m,p}$.

$$y_{mk}^{m,p} = Y_m^P \vartheta_{mk}^* \tag{13a}$$

$$\vartheta_{mk}^H \vartheta_{ik} = \begin{cases} \tau_p & \text{when } m = i \\ 0 & \text{when } m \neq i \end{cases} \tag{13b}$$

No pilots are transmitted in the downlink of CF-mMIMO. The users do not need to estimate their effective channel gain, but instead rely on channel hardening, which makes this gain close to its expected value, a known deterministic constant [2,12]. This work considers the downlink and uplink, which consists of τ_c channel dedicated for pilots and $\tau_c - \tau_p$ channel uses for payload data.

Power Control and Pilot design

Power control is vital to handle the near-far effect, and hence, protect any given user from strong interference from other users. The power control is done at the CPU. The optimized power-control coefficients for the uplink and downlink are sent to the APs and users, respectively.

Pilot design

The AP allocates pilot signals to each user that belongs to its cell. The intra-user pilot signal interference and inter-user pilot signal interference are referred to as pilot interference. The pilot design is crucial in mMIMO networks [2],[5]-[6] since every BS obtains instantaneous channel state information (CSI) from UL pilot signals from each user, and then use them to construct the UL detection and DL precoding vectors. In prior works, the pilot design is divided into the two separate tasks:

1. Pilot assignment only, and
2. Pilot power control only.

1. *Pilot assignment only*: It consists of methods to assign each user with a pilot from an orthogonal pilot set to mitigate interference known as pilot contamination in the pilot transmission [2-12] and each user has the same power level. This is a challenging problem since different users are more or less susceptible to contamination. The best assignment solution is typically obtained by exhaustive search methods but such methods have exponential computational complexity.

2. *Pilot power control only*: It focuses on the pilot power control, while pilots are assigned in a predefined manner in each cell.

Combination of pilot assignment and pilot power control: This the combination of the two pilot designs. By utilizing imbalanced power allocation, pilot power control can give better channel estimation quality and reduce the coherent interference.

Data Power Control

We next show that CF-mMIMO can provide uniformly good service to all users, regardless of their geographical location, by using max-min power control. While power control in general is a well studied topic, the max-min power control problems that arise when optimizing CF-mMIMO are entirely new. The power control is performed at the CPU, and importantly, is done on the large scale fading time scale.

Max-min Fairness Data Power Control

In CF-mMIMO, a good uniformly distributed quality-of-service (QoS) for all users is one of the main targets. Thus, in this paper, we propose a max-min optimization problem, which maximizes the minimum throughput of user as follows.

Downlink: In the downlink, given realizations of the large-scale fading, we find the downlink power control coefficient μ , $m = 1, \dots, M$, $k = 1, \dots, K$, that maximize the minimum of the downlink throughput of all users, under the power constraint (14). At the optimum point, all users get the same rate. Mathematically, the max-min power control problem can be formulated as below:

$$\begin{aligned} & \max. \min_{mk} \text{throughput}_{mk} \\ & \text{subject to } \mu \geq 1, k = 1, \dots, K, m = 1, \dots, M \end{aligned} \quad (14b)$$

Uplink: In the uplink, the max-min power control problem can be formulated as below:

$$\begin{aligned} & \max. \min_k \text{throughput}_{mk} \\ & \text{subject to } 0 \leq \alpha \leq 1, k = 1, \dots, K \end{aligned} \quad (14b)$$

Where α is the uplink power control coefficient.

Pilot Assignment Algorithm:

1) *Initialization:* choose K pilot sequences $\vartheta_{m1}, \vartheta_{m2}, \vartheta_{m3}, \dots, \vartheta_{mk}$ using the random pilot assignment method. Choose the number of iterations, I , and set $i = 1$.

2) Compute throughput using (14). Find the user with the max-min throughput:

$$\max. \min_k \text{throughput}_{mk}$$

3) Update the pilot sequence for the k th user by choosing ϑ_{mk} .

$$\vartheta_{mk}^H \vartheta_{ik} = \begin{cases} \tau_p & \text{when } j = i \\ 0 & \text{when } j \neq i \end{cases}$$

4) Set $i = i + 1$. Stop if $i > I$. Otherwise, go to Step 2.

3. NUMERICAL RESULTS AND DISCUSSIONS

We quantitatively study the performance of CF-mMIMO, and compare it to that of SC-mMIMO networks. We specifically demonstrate the effects of shadow fading correlation. The M APs and K users are uniformly distributed at random within a square of size $D \times D$ km.

Table 1: Simulation Parameters

Parameter	Value
Number of Access Points (APs) M	20
Number of Antennas per AP N	5
AP antenna height	15m
Number of users	40
User antenna height	1.65m
Communication Bandwidth B	20 MHz
DL transmit power	100mW (-10dB)
UL transmit power	50mW (-13dB)

Pilot power	50mW (-13dB)
D, d ₁ , d ₂	1km, 0.01km, 0.05km
Number of Iterations	200

Network throughput

Network throughput is one of the vital metric to evaluate network performance in CF-mMIMO and SC-mMIMO networks. This metric is expressed in equation (11) and measures the quality of service (QoS) of the network. In order to examine the impact of throughput on the performance of the channel estimator (MMSE) as was introduced into the simulation as shown in figures 2–3. The simulation results were generated using the modified codes in [2].

We compared the performance of CF-mMIMO and SC-mMIMO networks with pilot assignment, max-min power control and no power control. Figure 2 compares the cumulative distribution function (CDF) of the UL throughput for CF-mMIMO and SC-mMIMO networks, with shadow fading correlation.

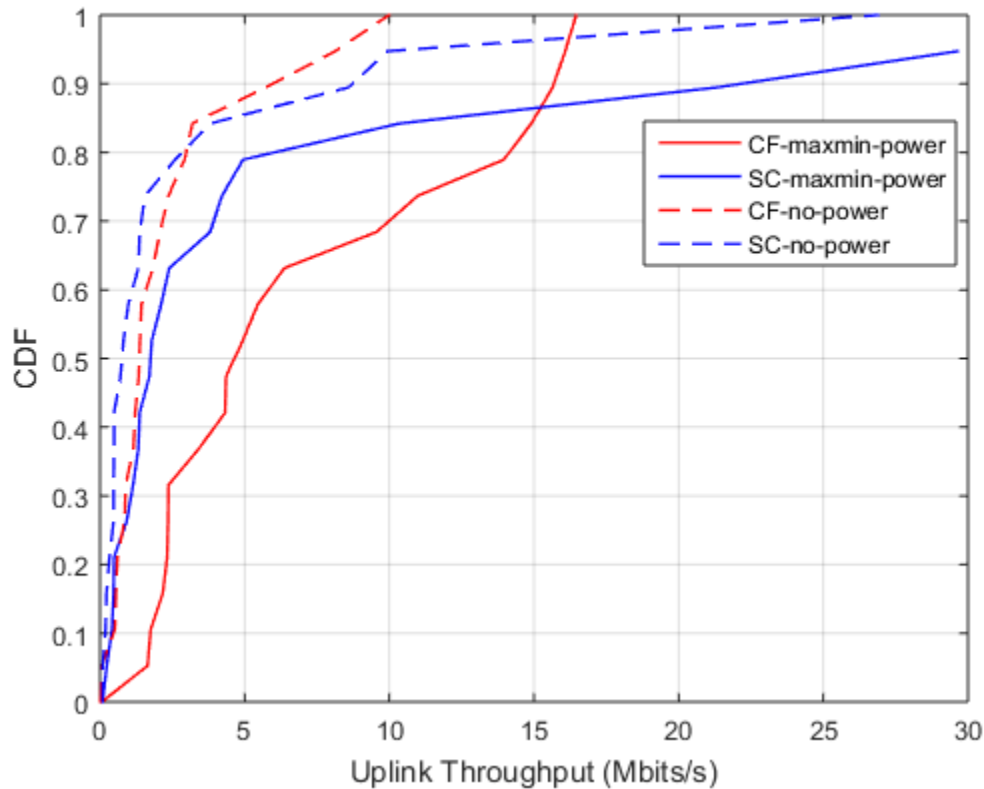


Figure 2: CDF vs. UL Throughput (Mbits/s)

CF-mMIMO significantly outperforms SC-mMIMO in 90%-likely throughput performance points (i.e., where the vertical axis is 0.1). The UL throughput of CF-mMIMO is much more concentrated around its median, compared with the SC-mMIMO networks. Without power control, the 90%-likely UL throughput of the CF-mMIMO is higher than that of the SC-mMIMO network. In particular, we can see that the SC-mMIMO networks are much more affected by shadow fading correlation than CF-mMIMO. This is due to the fact that when the shadowing coefficients are highly correlated, the gain from choosing the best APs in a SC-mMIMO network is reduced. With shadowing correlation, the 90%-likely throughput of the CF-mMIMO uplink is higher than that of the SC-mMIMO network.

We compared the performance of CF-mMIMO and SC-mMIMO networks with pilot assignment, max-min power control and no power control. Figure 3 compares the cumulative distribution function (CDF) of the DL throughput for CF-mMIMO and SC-mMIMO networks, with shadow fading correlation.

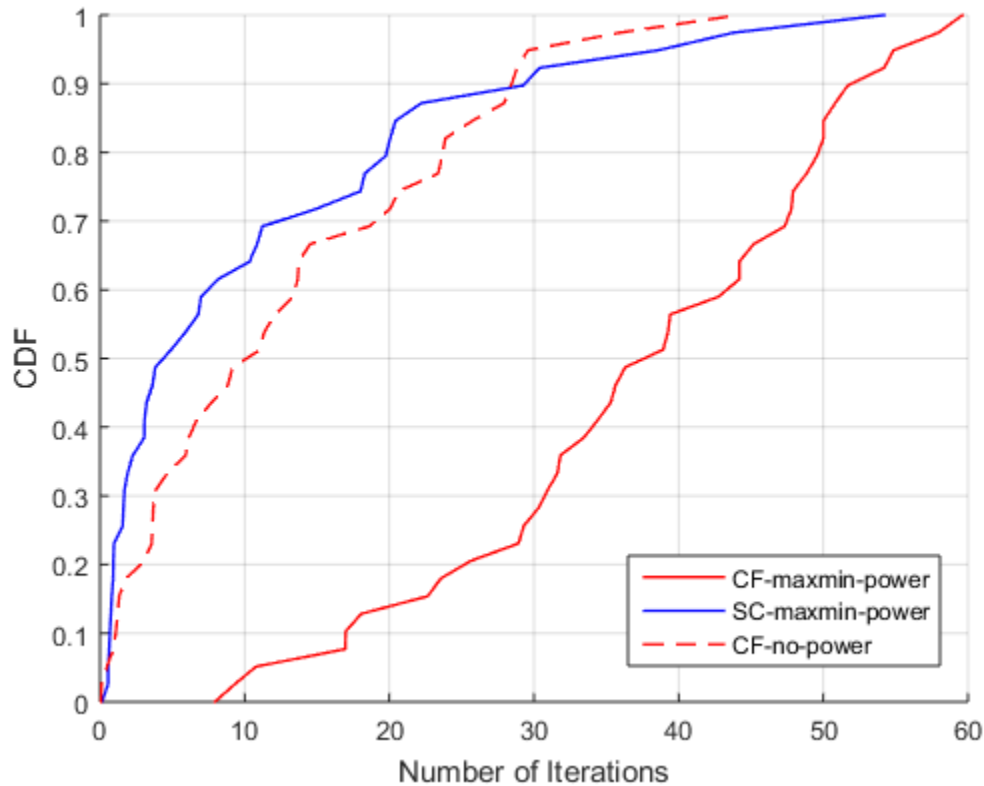


Figure 3: CDF vs. DL Throughput (Mbits/s)

CF-mMIMO excellently outperforms SC-mMIMO in 90%-likely performance. The DL throughput of CF-mMIMO is 17 times higher than that of SC-mMIMO networks (about 18.2 Mbits/s). Without max-min power control, the 90%-likely DL throughput of the CF-mMIMO is higher than that of the SC-mMIMO network.

4. CONCLUSION

In this paper, uplink and downlink throughput of CF-mMIMO and SC-mMIMO with multi-antennas at finite numbers of APs and single antenna at users are analyzed. A comparison between CF-mMIMO and SC-mMIMO networks were also performed, under correlated shadow fading, taking into account the effects of channel estimation, pilot assignment, and power control. The results show that CF-mMIMO networks can largely outperform SC-mMIMO networks in terms of throughput. In particular, CF-mMIMO networks are much more robust to correlation channel than SC-mMIMO networks. The 90%-likely throughputs of CF-mMIMO with max-min power control are an order of magnitude higher than those of the SC-mMIMO networks in DL and UL transmissions. In terms of implementation complexity, however, SC-mMIMO networks require much less backhaul than CF-mMIMO networks.

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